Computer Algebra
Independent Integration Tests

Summer 2023 edition

7-Inverse-hyperbolic-functions/7.1-Inverse-hyperbolic-sine/186-7.1.2-d-x-^m-a+b-arcsinh-c-x-^n

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [156]. This is test number [186].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (156)	0.00 (0)
Mathematica	100.00 (156)	0.00 (0)
Maple	69.87 (109)	30.13 (47)
Maxima	32.69 (51)	67.31 (105)
Sympy	30.77 (48)	69.23 (108)
Fricas	27.56 (43)	72.44 (113)
Giac	23.08 (36)	76.92 (120)
Mupad	19.23 (30)	80.77 (126)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

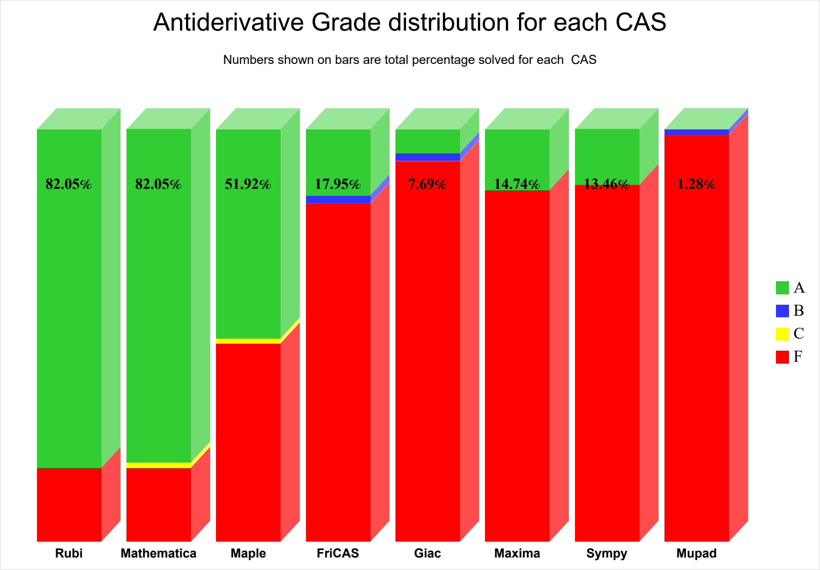
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

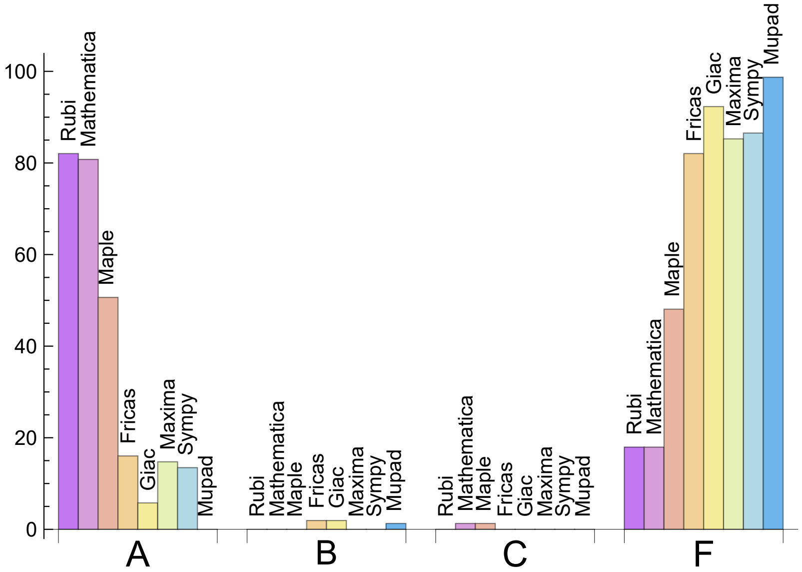
System	% A grade	% B grade	% C grade	% F grade
Rubi	82.051	0.000	0.000	17.949
Mathematica	80.769	0.000	1.282	17.949
Maple	50.641	0.000	1.282	48.077
Fricas	16.026	1.923	0.000	82.051
Maxima	14.744	0.000	0.000	85.256
Sympy	13.462	0.000	0.000	86.538
Giac	5.769	1.923	0.000	92.308
Mupad	0.000	1.282	0.000	98.718

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Maple	47	100.00	0.00	0.00
Fricas	113	38.94	0.00	61.06
Maxima	105	100.00	0.00	0.00
Sympy	108	98.15	1.85	0.00
Giac	120	63.33	3.33	33.33
Mupad	126	0.00	100.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maple	0.07
Rubi	0.14
Fricas	0.26
Giac	0.33
Mathematica	0.33
Maxima	0.34
Mupad	2.40
Sympy	4.08

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	13.23	1.09	12.00	1.08
Giac	36.00	1.31	12.00	1.20
Sympy	53.08	0.97	12.00	0.98
Fricas	70.07	1.22	59.00	1.20
Maple	72.95	1.04	56.00	0.94
Mathematica	93.59	0.96	74.00	0.99
Rubi	103.12	1.00	84.50	1.00
Maxima	155.24	12.55	43.00	1.00

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

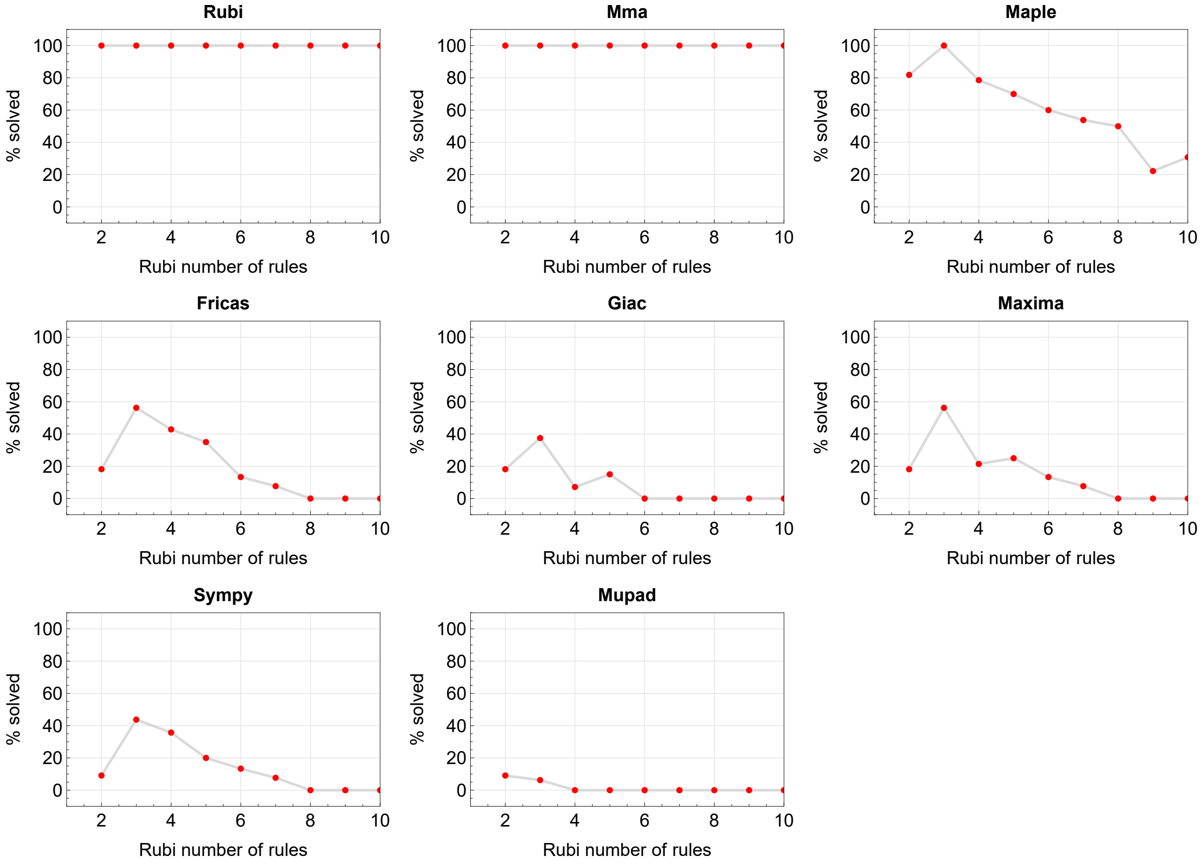


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

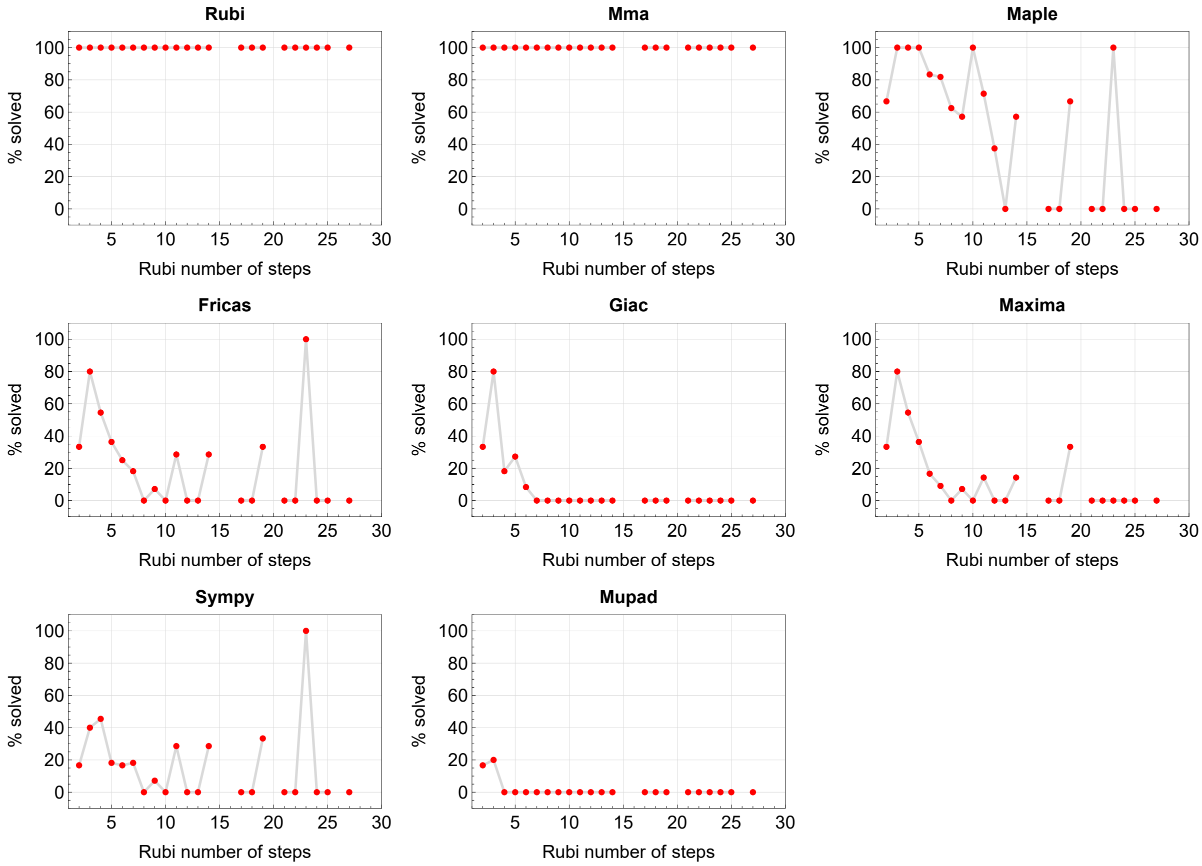


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

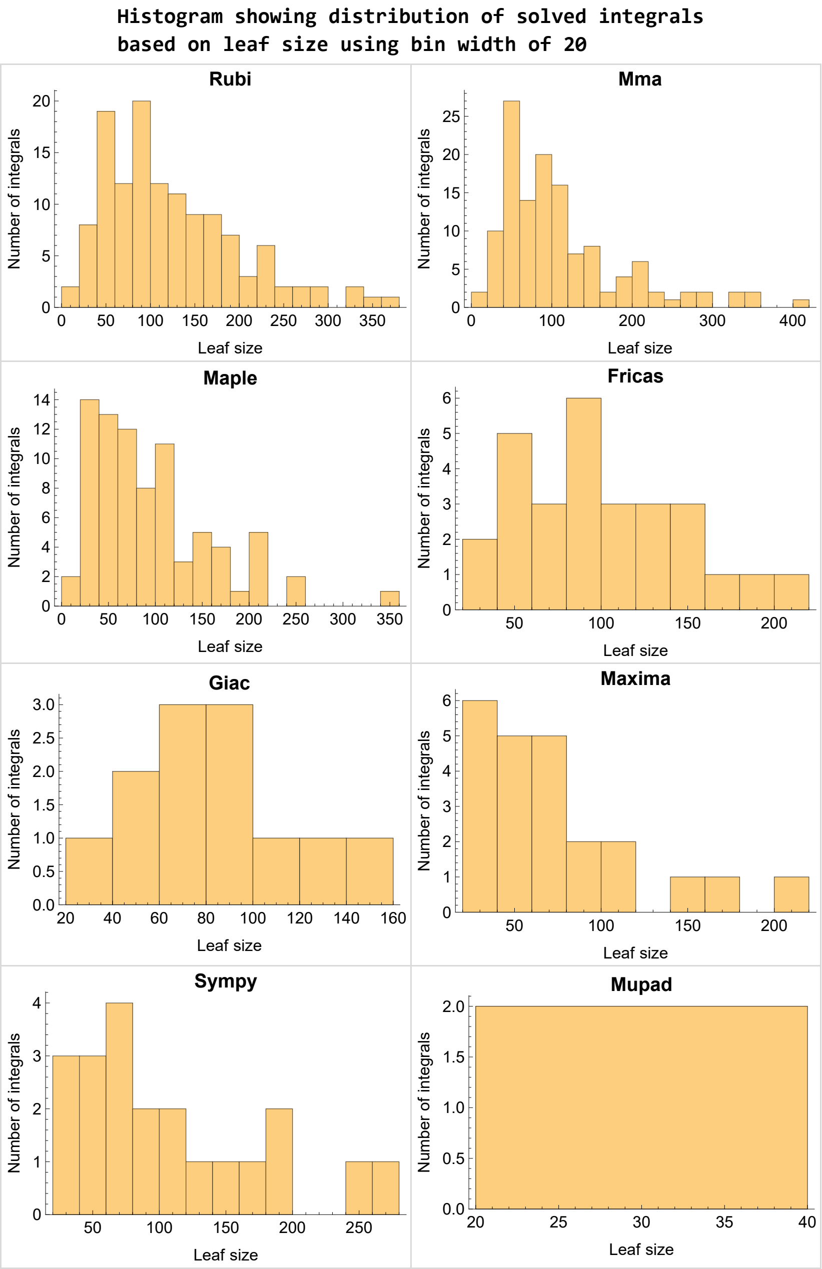


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

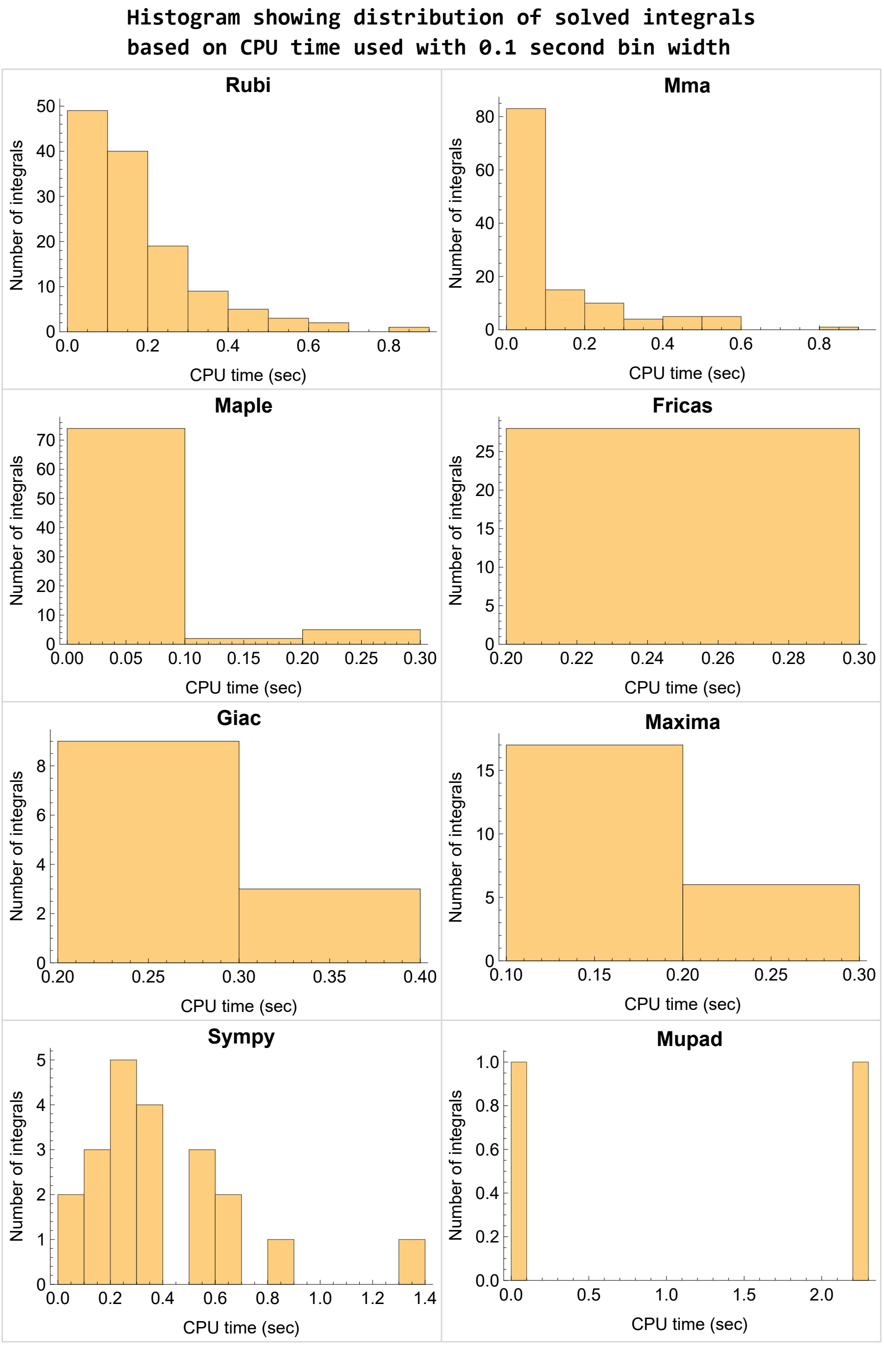


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

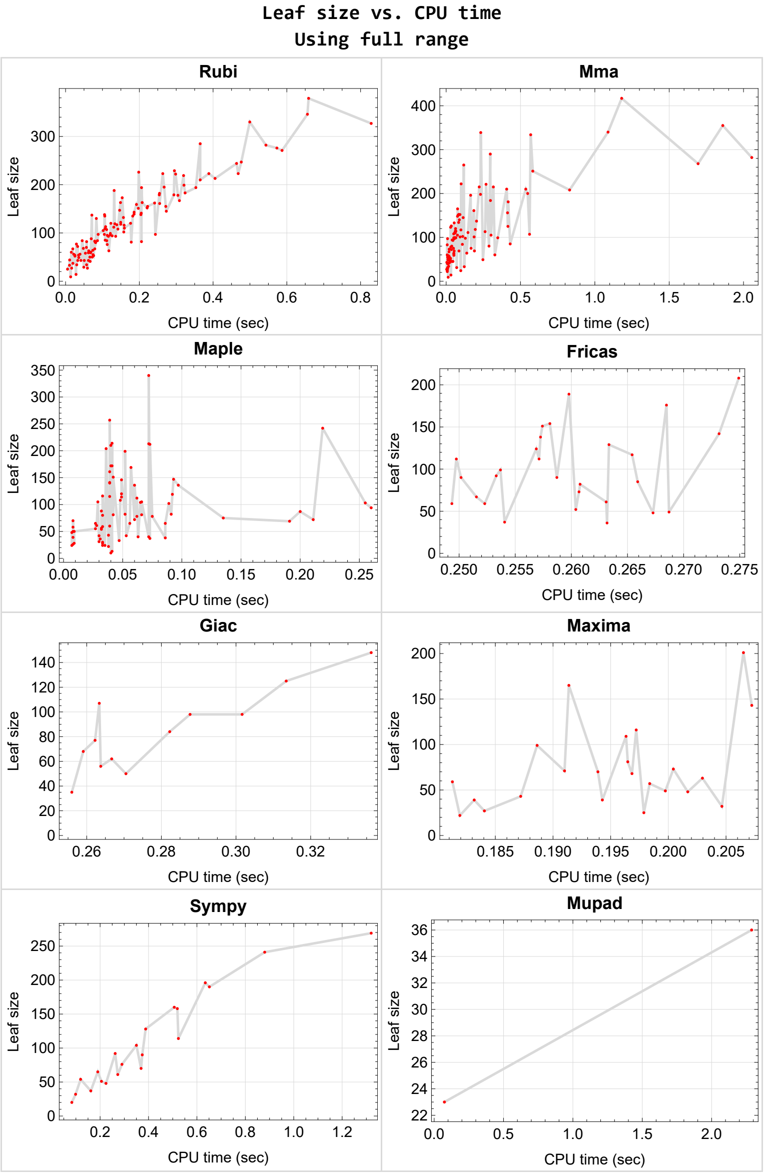


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{49, 50, 58, 59, 65, 66, 72, 73, 79, 85, 91, 97, 98, 104, 110, 116, 117, 118, 121, 122, 123, 124, 125, 126, 127, 128, 134, 135\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
  x, aa = expr.operator(), expr.operands()
  if x is None:
    return 1
  else:
    return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

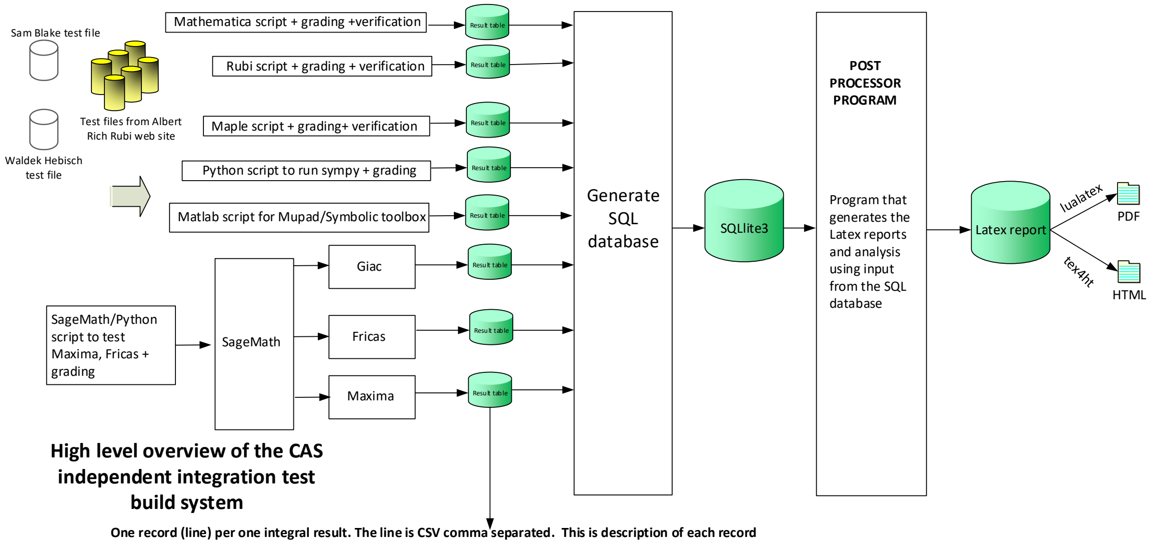
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



High level overview of the CAS independent integration test build system

One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form “{n,n,..}” which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

Nasser M. Abbasi
June 27, 2023
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 63, 64, 67, 68, 69, 70, 71, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 92, 93, 94, 95, 96, 99, 100, 101, 102, 103, 105, 106, 107, 108, 109, 111, 112, 113, 114, 115, 119, 120, 129, 130, 131, 132, 133, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 63, 64, 67, 68, 69, 70, 71, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 92, 93, 94, 95, 96, 99, 100, 101, 102, 103, 105, 106, 107, 108, 109, 111, 112, 113, 114, 115, 119, 120, 129, 130, 131, 132, 133, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156 }

B grade { }

C grade { 11, 40 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 63, 64, 67, 68, 69, 70, 71, 77, 78, 83, 84, 89, 90, 95, 96, 102, 103, 108, 109, 114, 115 }

B grade { }

C grade { 132, 133 }

F normal fail { 74, 75, 76, 80, 81, 82, 86, 87, 88, 92, 93, 94, 99, 100, 101, 105, 106, 107, 111, 112, 113, 119, 120, 129, 130, 131, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 8, 10, 12, 13, 14, 15, 16, 19, 21, 22, 23, 24, 25, 26, 32, 33, 34, 35, 36, 37 }

B grade { 7, 9, 11 }

C grade { }

F normal fail { 6, 17, 18, 20, 27, 28, 29, 30, 31, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 63, 64, 67, 68, 69, 70, 71, 119, 120, 129, 130, 131, 132, 133 }

F(-1) timedout fail { }

F(-2) exception fail { 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 123, 124, 125, 126, 127, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156 }

Maxima

A grade { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 19, 21, 22, 24, 26, 33, 35, 37 }

B grade { }

C grade { }

F normal fail { 6, 17, 18, 20, 23, 25, 27, 28, 29, 30, 31, 32, 34, 36, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 63, 64, 67, 68, 69, 70, 71, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 92, 93, 94, 95, 96, 99, 100, 101, 102, 103, 105, 106, 107, 108, 109, 111, 112, 113, 114, 115, 119, 120, 129, 130, 131, 132, 133, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade $\{ 4, 5, 8, 9, 10, 11, 16, 26, 37 \}$

B grade $\{ 7, 19, 21 \}$

C grade $\{ \}$

F normal fail $\{ 6, 17, 18, 20, 25, 27, 28, 30, 38, 39, 41, 42, 44, 46, 47, 48, 51, 53, 55, 56, 57, 60, 62, 63, 64, 67, 69, 70, 71, 74, 76, 77, 78, 80, 82, 83, 84, 92, 94, 95, 96, 99, 101, 102, 103, 105, 107, 108, 109, 111, 113, 114, 115, 119, 120, 129, 131, 132, 133, 136, 137, 138, 140, 141, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156 \}$

F(-1) timedout fail $\{ 123, 124, 125, 128 \}$

F(-2) exception fail $\{ 1, 2, 3, 12, 13, 14, 15, 22, 23, 24, 29, 31, 32, 33, 34, 35, 36, 40, 43, 45, 52, 54, 61, 68, 75, 81, 86, 87, 88, 89, 90, 93, 100, 106, 112, 130, 139, 142, 143, 144 \}$

Mupad

A grade $\{ \}$

B grade $\{ 4, 5 \}$

C grade $\{ \}$

F normal fail $\{ \}$

F(-1) timedout fail $\{ 1, 2, 3, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 63, 64, 67, 68, 69, 70, 71, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 92, 93, 94, 95, 96, 99, 100, 101, 102, 103, 105, 106, 107, 108, 109, 111, 112, 113, 114, 115, 119, 120, 129, 130, 131, 132, 133, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156 \}$

F(-2) exception fail $\{ \}$

Sympy

A grade $\{ 1, 2, 3, 4, 5, 12, 13, 14, 15, 16, 22, 23, 24, 25, 26, 32, 33, 34, 35, 36, 37 \}$

B grade $\{ \}$

C grade $\{ \}$

F normal fail $\{ 6, 7, 8, 9, 10, 11, 17, 18, 19, 20, 21, 27, 28, 29, 30, 31, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 63, 64, 67, 68, 69, 70, 71, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 87, 88, 89, 90, 92, 93, 94, 95, 96, 99, 100, 101, 102, 103, 105, 106, 107, 108, 109, 111, 112, 113, 114, 115, 119, 120, 129, 130, 131, 132, 133, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156 \}$

F(-1) timedout fail $\{ 86, 123 \}$

F(-2) exception fail $\{ \}$

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	50	69	68	61	70	0	0
N.S.	1	1.00	0.69	0.96	0.94	0.85	0.97	0.00	0.00
time (sec)	N/A	0.033	0.027	0.191	0.197	0.263	0.369	0.000	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	49	58	59	59	61	0	0
N.S.	1	1.00	0.73	0.87	0.88	0.88	0.91	0.00	0.00
time (sec)	N/A	0.020	0.015	0.008	0.181	0.249	0.273	0.000	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	41	50	48	52	48	0	0
N.S.	1	1.00	0.79	0.96	0.92	1.00	0.92	0.00	0.00
time (sec)	N/A	0.025	0.019	0.009	0.202	0.260	0.224	0.000	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	40	39	39	48	37	68	36
N.S.	1	1.00	0.91	0.89	0.89	1.09	0.84	1.55	0.82
time (sec)	N/A	0.011	0.009	0.008	0.183	0.267	0.161	0.259	2.288

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	25	37	20	35	23
N.S.	1	1.00	1.00	0.96	1.00	1.48	0.80	1.40	0.92
time (sec)	N/A	0.005	0.008	0.007	0.198	0.254	0.083	0.256	0.074

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	94	0	0	0	0	0
N.S.	1	1.00	1.00	2.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.042	0.005	0.260	0.000	0.000	0.000	0.000	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	26	22	90	0	56	0
N.S.	1	1.00	1.00	0.96	0.81	3.33	0.00	2.07	0.00
time (sec)	N/A	0.016	0.004	0.008	0.182	0.259	0.000	0.264	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	28	28	27	36	0	50	0
N.S.	1	1.00	0.85	0.85	0.82	1.09	0.00	1.52	0.00
time (sec)	N/A	0.011	0.008	0.009	0.184	0.263	0.000	0.271	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	48	43	117	0	84	0
N.S.	1	1.00	1.00	0.89	0.80	2.17	0.00	1.56	0.00
time (sec)	N/A	0.025	0.010	0.007	0.187	0.265	0.000	0.282	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	40	50	49	49	0	77	0
N.S.	1	1.00	0.71	0.89	0.88	0.88	0.00	1.38	0.00
time (sec)	N/A	0.022	0.011	0.008	0.200	0.269	0.000	0.262	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	49	70	63	129	0	107	0
N.S.	1	1.00	0.64	0.91	0.82	1.68	0.00	1.39	0.00
time (sec)	N/A	0.029	0.012	0.008	0.203	0.263	0.000	0.263	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	75	103	99	99	114	0	0
N.S.	1	1.00	0.64	0.88	0.85	0.85	0.97	0.00	0.00
time (sec)	N/A	0.139	0.049	0.255	0.189	0.254	0.523	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	72	87	109	92	90	0	0
N.S.	1	1.00	0.75	0.91	1.14	0.96	0.94	0.00	0.00
time (sec)	N/A	0.121	0.035	0.200	0.196	0.253	0.374	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	59	72	70	82	76	0	0
N.S.	1	1.00	0.74	0.90	0.88	1.02	0.95	0.00	0.00
time (sec)	N/A	0.081	0.052	0.211	0.194	0.261	0.290	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	53	59	81	73	51	0	0
N.S.	1	1.00	0.90	1.00	1.37	1.24	0.86	0.00	0.00
time (sec)	N/A	0.068	0.024	0.033	0.196	0.261	0.206	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	36	32	59	32	62	0
N.S.	1	1.00	1.00	1.06	0.94	1.74	0.94	1.82	0.00
time (sec)	N/A	0.030	0.017	0.031	0.205	0.252	0.099	0.267	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	60	151	0	0	0	0	0
N.S.	1	1.00	1.00	2.52	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.068	0.007	0.042	0.000	0.000	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	75	108	0	0	0	0	0
N.S.	1	1.00	1.50	2.16	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.073	0.168	0.048	0.000	0.000	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	72	39	67	0	98	0
N.S.	1	1.00	1.00	1.67	0.91	1.56	0.00	2.28	0.00
time (sec)	N/A	0.051	0.025	0.060	0.194	0.252	0.000	0.302	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	125	136	0	0	0	0	0
N.S.	1	1.00	1.26	1.37	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.121	0.416	0.060	0.000	0.000	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	64	112	71	85	0	148	0
N.S.	1	1.00	0.75	1.32	0.84	1.00	0.00	1.74	0.00
time (sec)	N/A	0.107	0.052	0.062	0.191	0.266	0.000	0.336	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	120	172	165	151	196	0	0
N.S.	1	1.00	0.62	0.88	0.85	0.77	1.01	0.00	0.00
time (sec)	N/A	0.267	0.063	0.040	0.191	0.257	0.635	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	110	141	0	142	160	0	0
N.S.	1	1.00	0.67	0.87	0.00	0.87	0.98	0.00	0.00
time (sec)	N/A	0.208	0.061	0.039	0.000	0.273	0.506	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	93	116	116	124	128	0	0
N.S.	1	1.00	0.70	0.88	0.88	0.94	0.97	0.00	0.00
time (sec)	N/A	0.157	0.046	0.033	0.197	0.257	0.388	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	80	88	0	112	92	0	0
N.S.	1	1.00	0.82	0.91	0.00	1.15	0.95	0.00	0.00
time (sec)	N/A	0.105	0.038	0.032	0.000	0.257	0.262	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	58	55	57	90	54	98	0
N.S.	1	1.00	1.00	0.95	0.98	1.55	0.93	1.69	0.00
time (sec)	N/A	0.054	0.017	0.027	0.198	0.250	0.119	0.288	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	83	204	0	0	0	0	0
N.S.	1	1.00	1.00	2.46	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.079	0.007	0.036	0.000	0.000	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	117	161	0	0	0	0	0
N.S.	1	1.00	1.39	1.92	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.112	0.099	0.039	0.000	0.000	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	80	146	0	0	0	0	0
N.S.	1	1.00	0.86	1.57	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.116	0.288	0.049	0.000	0.000	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	268	212	0	0	0	0	0
N.S.	1	1.00	1.77	1.40	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.197	1.694	0.073	0.000	0.000	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	107	213	0	0	0	0	0
N.S.	1	1.00	0.67	1.34	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.192	0.561	0.072	0.000	0.000	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	276	165	242	0	208	269	0	0
N.S.	1	1.00	0.60	0.88	0.00	0.75	0.97	0.00	0.00
time (sec)	N/A	0.573	0.077	0.219	0.000	0.275	1.320	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	148	210	201	189	241	0	0
N.S.	1	1.00	0.61	0.86	0.82	0.77	0.99	0.00	0.00
time (sec)	N/A	0.464	0.076	0.040	0.207	0.260	0.880	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	133	172	0	176	190	0	0
N.S.	1	1.00	0.69	0.89	0.00	0.91	0.98	0.00	0.00
time (sec)	N/A	0.353	0.054	0.041	0.000	0.268	0.651	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	112	140	143	154	158	0	0
N.S.	1	1.00	0.69	0.86	0.88	0.95	0.98	0.00	0.00
time (sec)	N/A	0.241	0.060	0.039	0.207	0.258	0.519	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	94	105	0	138	104	0	0
N.S.	1	1.00	0.85	0.95	0.00	1.25	0.95	0.00	0.00
time (sec)	N/A	0.159	0.041	0.029	0.000	0.257	0.350	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	67	65	73	112	65	125	0
N.S.	1	1.00	1.00	0.97	1.09	1.67	0.97	1.87	0.00
time (sec)	N/A	0.079	0.019	0.027	0.200	0.250	0.190	0.313	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	97	257	0	0	0	0	0
N.S.	1	1.00	1.00	2.65	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.088	0.009	0.039	0.000	0.000	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	161	214	0	0	0	0	0
N.S.	1	1.00	1.34	1.78	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.123	0.186	0.041	0.000	0.000	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	113	199	0	0	0	0	0
N.S.	1	1.00	1.05	1.84	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.142	0.262	0.052	0.000	0.000	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	355	340	0	0	0	0	0
N.S.	1	1.00	1.59	1.52	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.264	1.861	0.072	0.000	0.000	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	40	40	0	0	0	0	0
N.S.	1	1.00	0.73	0.73	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.074	0.011	0.063	0.000	0.000	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	33	33	0	0	0	0	0
N.S.	1	1.00	0.77	0.77	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.062	0.122	0.047	0.000	0.000	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	31	31	0	0	0	0	0
N.S.	1	1.00	0.76	0.76	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.067	0.009	0.030	0.000	0.000	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	24	24	0	0	0	0	0
N.S.	1	1.00	0.83	0.83	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.049	0.098	0.035	0.000	0.000	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	22	22	0	0	0	0	0
N.S.	1	1.00	0.81	0.81	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.059	0.008	0.038	0.000	0.000	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	0	0	0	0	0
N.S.	1	1.00	1.00	0.93	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.028	0.032	0.041	0.000	0.000	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	0	0	0	0	0
N.S.	1	1.00	1.00	1.11	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.014	0.014	0.040	0.000	0.000	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	8	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	0.80	1.20	1.20
time (sec)	N/A	0.012	0.146	0.030	0.237	0.244	0.324	0.273	2.567

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	10	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	1.00	1.20	1.20
time (sec)	N/A	0.011	0.840	0.051	0.237	0.266	0.329	0.275	2.660

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	85	104	0	0	0	0	0
N.S.	1	1.00	1.04	1.27	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.057	0.431	0.065	0.000	0.000	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	78	78	0	0	0	0	0
N.S.	1	1.00	1.11	1.11	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.056	0.031	0.062	0.000	0.000	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	60	80	0	0	0	0	0
N.S.	1	1.00	0.88	1.18	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.048	0.328	0.033	0.000	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	56	54	0	0	0	0	0
N.S.	1	1.00	1.00	0.96	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.039	0.021	0.032	0.000	0.000	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	49	56	0	0	0	0	0
N.S.	1	1.00	0.91	1.04	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.035	0.247	0.032	0.000	0.000	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	29	28	0	0	0	0	0
N.S.	1	1.00	0.78	0.76	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.020	0.010	0.033	0.000	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	31	30	0	0	0	0	0
N.S.	1	1.00	0.91	0.88	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.057	0.071	0.033	0.000	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	200	12	10	12	12
N.S.	1	1.00	1.20	1.00	20.00	1.20	1.00	1.20	1.20
time (sec)	N/A	0.010	0.771	0.032	0.319	0.253	0.387	0.269	2.543

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	237	12	12	12	12
N.S.	1	1.00	1.20	1.00	23.70	1.20	1.20	1.20	1.20
time (sec)	N/A	0.012	5.279	0.043	0.361	0.250	0.465	0.276	2.670

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	102	120	0	0	0	0	0
N.S.	1	1.00	1.05	1.24	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.244	0.111	0.049	0.000	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	69	82	0	0	0	0	0
N.S.	1	1.00	0.84	1.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.206	0.190	0.052	0.000	0.000	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	64	81	0	0	0	0	0
N.S.	1	1.00	0.79	1.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.178	0.139	0.042	0.000	0.000	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	62	43	0	0	0	0	0
N.S.	1	1.00	0.98	0.68	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.117	0.051	0.038	0.000	0.000	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	47	42	0	0	0	0	0
N.S.	1	1.00	0.94	0.84	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.066	0.022	0.030	0.000	0.000	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	697	12	10	12	12
N.S.	1	1.00	1.20	1.00	69.70	1.20	1.00	1.20	1.20
time (sec)	N/A	0.010	0.473	0.033	0.625	0.248	0.464	0.274	2.491

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	822	12	12	12	12
N.S.	1	1.00	1.20	1.00	82.20	1.20	1.20	1.20	1.20
time (sec)	N/A	0.010	4.652	0.044	0.728	0.258	0.576	0.282	2.514

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	156	169	0	0	0	0	0
N.S.	1	1.00	1.01	1.09	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.222	0.413	0.057	0.000	0.000	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	105	114	0	0	0	0	0
N.S.	1	1.00	0.74	0.81	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.205	0.302	0.049	0.000	0.000	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	99	115	0	0	0	0	0
N.S.	1	1.00	0.72	0.83	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.204	0.347	0.039	0.000	0.000	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	84	60	0	0	0	0	0
N.S.	1	1.00	0.88	0.63	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.113	0.109	0.039	0.000	0.000	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	69	61	0	0	0	0	0
N.S.	1	1.00	0.91	0.80	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.113	0.086	0.028	0.000	0.000	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	1611	12	10	12	12
N.S.	1	1.00	1.20	1.00	161.10	1.20	1.00	1.20	1.20
time (sec)	N/A	0.010	1.987	0.031	1.292	0.262	0.607	0.270	2.598

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	1885	12	12	12	12
N.S.	1	1.00	1.20	1.00	188.50	1.20	1.20	1.20	1.20
time (sec)	N/A	0.013	7.511	0.046	1.505	0.253	0.803	0.272	2.581

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	182	182	152	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.255	0.093	0.000	0.000	0.000	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	139	139	99	0	0	0	0	0	0
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.186	0.059	0.000	0.000	0.000	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	120	120	99	0	0	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.176	0.057	0.000	0.000	0.000	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	52	75	0	0	0	0	0
N.S.	1	1.00	0.56	0.81	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.136	0.029	0.135	0.000	0.000	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	45	42	0	0	0	0	0
N.S.	1	1.00	0.85	0.79	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.076	0.046	0.053	0.000	0.000	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	10	12	0	10	12	12
N.S.	1	1.00	1.17	0.83	1.00	0.00	0.83	1.00	1.00
time (sec)	N/A	0.014	0.163	0.039	0.350	0.000	0.412	0.425	2.501

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	330	330	152	0	0	0	0	0	0
N.S.	1	1.00	0.46	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.500	0.085	0.000	0.000	0.000	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	199	199	99	0	0	0	0	0	0
N.S.	1	1.00	0.50	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.321	0.063	0.000	0.000	0.000	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	179	179	99	0	0	0	0	0	0
N.S.	1	1.00	0.55	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.254	0.052	0.000	0.000	0.000	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	52	102	0	0	0	0	0
N.S.	1	1.00	0.43	0.84	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.149	0.021	0.089	0.000	0.000	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	47	65	0	0	0	0	0
N.S.	1	1.00	0.58	0.80	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.080	0.026	0.086	0.000	0.000	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	10	12	0	10	12	12
N.S.	1	1.00	1.17	0.83	1.00	0.00	0.83	1.00	1.00
time (sec)	N/A	0.010	0.136	0.040	0.367	0.000	1.666	0.572	2.460

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	379	379	152	0	0	0	0	0	0
N.S.	1	1.00	0.40	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.659	0.085	0.000	0.000	0.000	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	247	247	99	0	0	0	0	0	0
N.S.	1	1.00	0.40	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.476	0.057	0.000	0.000	0.000	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	210	210	99	0	0	0	0	0	0
N.S.	1	1.00	0.47	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.365	0.055	0.000	0.000	0.000	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-2)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	52	136	0	0	0	0	0
N.S.	1	1.00	0.34	0.89	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.221	0.029	0.097	0.000	0.000	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-2)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	45	78	0	0	0	0	0
N.S.	1	1.00	0.48	0.83	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.126	0.040	0.075	0.000	0.000	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	10	12	0	10	12	12
N.S.	1	1.00	1.17	0.83	1.00	0.00	0.83	1.00	1.00
time (sec)	N/A	0.010	0.140	0.041	0.352	0.000	19.286	0.600	2.370

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	163	163	151	0	0	0	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.149	0.081	0.000	0.000	0.000	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	109	109	99	0	0	0	0	0	0
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.107	0.065	0.000	0.000	0.000	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	105	105	99	0	0	0	0	0	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.102	0.062	0.000	0.000	0.000	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	52	37	0	0	0	0	0
N.S.	1	1.00	0.83	0.59	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.063	0.018	0.073	0.000	0.000	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	47	24	0	0	0	0	0
N.S.	1	1.00	1.09	0.56	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.033	0.027	0.033	0.000	0.000	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	10	12	0	12	12	12
N.S.	1	1.00	1.17	0.83	1.00	0.00	1.00	1.00	1.00
time (sec)	N/A	0.010	0.166	0.042	0.354	0.000	0.434	0.423	2.345

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	10	12	0	14	12	12
N.S.	1	1.00	1.17	0.83	1.00	0.00	1.17	1.00	1.00
time (sec)	N/A	0.011	1.337	0.053	0.357	0.000	0.553	0.444	2.328

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	188	188	265	0	0	0	0	0	0
N.S.	1	1.00	1.41	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.132	0.118	0.000	0.000	0.000	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	138	138	126	0	0	0	0	0	0
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.106	0.035	0.000	0.000	0.000	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	130	130	140	0	0	0	0	0	0
N.S.	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.083	0.089	0.000	0.000	0.000	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	73	82	0	0	0	0	0
N.S.	1	1.00	0.87	0.98	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.045	0.020	0.091	0.000	0.000	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	69	65	0	0	0	0	0
N.S.	1	1.00	1.08	1.02	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.074	0.054	0.056	0.000	0.000	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	10	12	0	12	12	12
N.S.	1	1.00	1.17	0.83	1.00	0.00	1.00	1.00	1.00
time (sec)	N/A	0.010	0.158	0.042	0.354	0.000	0.964	0.272	2.294

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	223	223	339	0	0	0	0	0	0
N.S.	1	1.00	1.52	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.389	0.233	0.000	0.000	0.000	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	167	167	184	0	0	0	0	0	0
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.309	0.298	0.000	0.000	0.000	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	161	161	222	0	0	0	0	0	0
N.S.	1	1.00	1.38	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.255	0.101	0.000	0.000	0.000	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	98	119	0	0	0	0	0
N.S.	1	1.00	0.83	1.01	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.150	0.129	0.092	0.000	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	105	81	0	0	0	0	0
N.S.	1	1.00	1.25	0.96	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.086	0.076	0.066	0.000	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	10	12	0	12	12	12
N.S.	1	1.00	1.17	0.83	1.00	0.00	1.00	1.00	1.00
time (sec)	N/A	0.010	0.165	0.042	0.363	0.000	7.076	0.281	2.350

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	285	285	334	0	0	0	0	0	0
N.S.	1	1.00	1.17	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.365	0.569	0.000	0.000	0.000	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	229	229	210	0	0	0	0	0	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.295	0.536	0.000	0.000	0.000	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	222	222	221	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.298	0.268	0.000	0.000	0.000	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	118	147	0	0	0	0	0
N.S.	1	1.00	0.80	1.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.147	0.196	0.093	0.000	0.000	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	111	105	0	0	0	0	0
N.S.	1	1.00	0.99	0.94	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.131	0.147	0.066	0.000	0.000	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	10	12	0	12	12	12
N.S.	1	1.00	1.17	0.83	1.00	0.00	1.00	1.00	1.00
time (sec)	N/A	0.010	0.164	0.040	0.342	0.000	67.437	0.282	2.318

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	134	12	10	12	12
N.S.	1	1.00	1.20	1.00	13.40	1.20	1.00	1.20	1.20
time (sec)	N/A	0.070	0.397	0.478	0.472	0.271	5.765	0.409	2.558

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	134	12	10	12	12
N.S.	1	1.00	1.20	1.00	13.40	1.20	1.00	1.20	1.20
time (sec)	N/A	0.081	0.377	0.335	0.483	0.256	3.047	0.345	2.880

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	137	137	123	0	0	0	0	0	0
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.071	0.030	0.000	0.000	0.000	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	60	60	55	0	0	0	0	0	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.016	0.019	0.000	0.000	0.000	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	8	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	0.80	1.20	1.20
time (sec)	N/A	0.012	0.181	0.257	0.244	0.246	0.454	0.286	2.413

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	268	12	10	12	12
N.S.	1	1.00	1.20	1.00	26.80	1.20	1.00	1.20	1.20
time (sec)	N/A	0.011	0.205	0.260	0.530	0.243	0.851	0.268	2.431

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	F(-1)	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	10	12	0	0	0	12
N.S.	1	1.00	1.17	0.83	1.00	0.00	0.00	0.00	1.00
time (sec)	N/A	0.018	0.310	0.059	0.348	0.000	0.000	0.000	2.453

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	10	12	0	12	0	12
N.S.	1	1.00	1.17	0.83	1.00	0.00	1.00	0.00	1.00
time (sec)	N/A	0.010	0.300	0.061	0.350	0.000	64.646	0.000	2.417

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	10	12	0	12	0	12
N.S.	1	1.00	1.17	0.83	1.00	0.00	1.00	0.00	1.00
time (sec)	N/A	0.011	0.369	0.053	0.360	0.000	1.204	0.000	2.435

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	10	12	0	12	12	12
N.S.	1	1.00	1.17	0.83	1.00	0.00	1.00	1.00	1.00
time (sec)	N/A	0.010	0.379	0.054	0.359	0.000	0.607	0.467	2.439

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	10	12	0	12	12	12
N.S.	1	1.00	1.17	0.83	1.00	0.00	1.00	1.00	1.00
time (sec)	N/A	0.010	0.342	0.049	0.358	0.000	4.078	0.328	2.455

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	14	14	12	0	14
N.S.	1	1.00	1.17	1.00	1.17	1.17	1.00	0.00	1.17
time (sec)	N/A	0.015	0.412	0.045	0.351	0.269	3.978	0.000	2.438

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	173	173	145	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.154	0.114	0.000	0.000	0.000	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	119	119	99	0	0	0	0	0	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.134	0.052	0.000	0.000	0.000	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	113	113	97	0	0	0	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.109	0.055	0.000	0.000	0.000	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	57	38	0	0	0	0	0
N.S.	1	1.00	0.97	0.64	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.062	0.011	0.086	0.000	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	45	40	0	0	0	0	0
N.S.	1	1.00	0.92	0.82	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.036	0.026	0.072	0.000	0.000	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	8	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	0.80	1.20	1.20
time (sec)	N/A	0.012	0.233	0.034	0.354	0.267	0.482	0.426	2.576

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	10	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	1.00	1.20	1.20
time (sec)	N/A	0.014	0.901	0.036	0.362	0.278	0.686	0.408	2.471

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	213	213	215	0	0	0	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.405	0.317	0.000	0.000	0.000	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	145	145	111	0	0	0	0	0	0
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.273	0.059	0.000	0.000	0.000	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	102	102	101	0	0	0	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.158	0.188	0.000	0.000	0.000	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	282	282	215	0	0	0	0	0	0
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.543	0.223	0.000	0.000	0.000	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	179	179	114	0	0	0	0	0	0
N.S.	1	1.00	0.64	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.294	0.065	0.000	0.000	0.000	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	135	135	251	0	0	0	0	0	0
N.S.	1	1.00	1.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.182	0.583	0.000	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	327	327	198	0	0	0	0	0	0
N.S.	1	1.00	0.61	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.829	0.231	0.000	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	223	223	115	0	0	0	0	0	0
N.S.	1	1.00	0.52	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.468	0.062	0.000	0.000	0.000	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	155	155	282	0	0	0	0	0	0
N.S.	1	1.00	1.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.272	2.056	0.000	0.000	0.000	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	194	194	196	0	0	0	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.206	0.165	0.000	0.000	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	107	107	108	0	0	0	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.111	0.051	0.000	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	88	88	101	0	0	0	0	0	0
N.S.	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.070	0.094	0.000	0.000	0.000	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	226	226	290	0	0	0	0	0	0
N.S.	1	1.00	1.28	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.198	0.297	0.000	0.000	0.000	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	135	135	134	0	0	0	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.106	0.084	0.000	0.000	0.000	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	116	116	137	0	0	0	0	0	0
N.S.	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.159	0.204	0.000	0.000	0.000	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	271	271	340	0	0	0	0	0	0
N.S.	1	1.00	1.25	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.587	1.089	0.000	0.000	0.000	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	183	183	200	0	0	0	0	0	0
N.S.	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.324	0.548	0.000	0.000	0.000	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	143	143	181	0	0	0	0	0	0
N.S.	1	1.00	1.27	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.186	0.417	0.000	0.000	0.000	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	346	346	417	0	0	0	0	0	0
N.S.	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.656	1.181	0.000	0.000	0.000	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	219	219	208	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.320	0.830	0.000	0.000	0.000	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	178	178	210	0	0	0	0	0	0
N.S.	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.305	0.407	0.000	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [30] had the largest ratio of [1]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	3	1.00	8	0.375
2	A	4	3	1.00	8	0.375
3	A	4	3	1.00	8	0.375
4	A	3	3	1.00	6	0.500
5	A	2	2	1.00	4	0.500
6	A	5	5	1.00	8	0.625
7	A	4	4	1.00	8	0.500
8	A	2	2	1.00	8	0.250
9	A	5	5	1.00	8	0.625
10	A	3	3	1.00	8	0.375
11	A	6	5	1.00	8	0.625
12	A	7	5	1.00	10	0.500
13	A	6	4	1.00	10	0.400
14	A	5	5	1.00	10	0.500
15	A	4	4	1.00	8	0.500
16	A	3	3	1.00	6	0.500
17	A	6	6	1.00	10	0.600
18	A	7	5	1.00	10	0.500
19	A	3	3	1.00	10	0.300
20	A	9	7	1.00	10	0.700
21	A	5	5	1.00	10	0.500
22	A	14	7	1.00	10	0.700
23	A	11	5	1.00	10	0.500
24	A	9	7	1.00	10	0.700

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	6	5	1.00	8	0.625
26	A	4	3	1.00	6	0.500
27	A	7	7	1.00	10	0.700
28	A	9	6	1.00	10	0.600
29	A	7	7	1.00	10	0.700
30	A	14	10	1.00	10	1.000
31	A	10	9	1.00	10	0.900
32	A	23	4	1.00	10	0.400
33	A	19	6	1.00	10	0.600
34	A	14	4	1.00	10	0.400
35	A	11	6	1.00	10	0.600
36	A	7	4	1.00	8	0.500
37	A	5	3	1.00	6	0.500
38	A	8	7	1.00	10	0.700
39	A	11	7	1.00	10	0.700
40	A	8	8	1.00	10	0.800
41	A	19	10	1.00	10	1.000
42	A	7	3	1.00	10	0.300
43	A	6	3	1.00	10	0.300
44	A	6	3	1.00	10	0.300
45	A	5	3	1.00	10	0.300
46	A	5	3	1.00	10	0.300
47	A	4	4	1.00	8	0.500
48	A	2	2	1.00	6	0.333
49	N/A	0	0	1.00	10	0.000
50	N/A	0	0	1.00	10	0.000
51	A	6	2	1.00	10	0.200
52	A	5	2	1.00	10	0.200
53	A	5	2	1.00	10	0.200
54	A	4	2	1.00	10	0.200
55	A	4	2	1.00	10	0.200
56	A	2	2	1.00	8	0.250
57	A	3	3	1.00	6	0.500
58	N/A	0	0	1.00	10	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
59	N/A	0	0	1.00	10	0.000
60	A	14	5	1.00	10	0.500
61	A	12	6	1.00	10	0.600
62	A	10	6	1.00	10	0.600
63	A	7	7	1.00	8	0.875
64	A	4	4	1.00	6	0.667
65	N/A	0	0	1.00	10	0.000
66	N/A	0	0	1.00	10	0.000
67	A	12	4	1.00	10	0.400
68	A	9	4	1.00	10	0.400
69	A	10	6	1.00	10	0.600
70	A	5	5	1.00	8	0.625
71	A	5	4	1.00	6	0.667
72	N/A	0	0	1.00	10	0.000
73	N/A	0	0	1.00	10	0.000
74	A	19	7	1.00	12	0.583
75	A	14	7	1.00	12	0.583
76	A	14	7	1.00	12	0.583
77	A	9	7	1.00	10	0.700
78	A	7	6	1.00	8	0.750
79	N/A	0	0	1.00	12	0.000
80	A	41	10	1.00	12	0.833
81	A	25	10	1.00	12	0.833
82	A	22	10	1.00	12	0.833
83	A	11	10	1.00	10	1.000
84	A	8	7	1.00	8	0.875
85	N/A	0	0	1.00	12	0.000
86	A	44	10	1.00	12	0.833
87	A	27	9	1.00	12	0.750
88	A	24	10	1.00	12	0.833
89	A	12	9	1.00	10	0.900
90	A	9	7	1.00	8	0.875
91	N/A	0	0	1.00	12	0.000
92	A	18	6	1.00	12	0.500

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
93	A	13	6	1.00	12	0.500
94	A	13	6	1.00	12	0.500
95	A	8	7	1.00	10	0.700
96	A	6	5	1.00	8	0.625
97	N/A	0	0	1.00	12	0.000
98	N/A	0	0	1.00	12	0.000
99	A	17	5	1.00	12	0.417
100	A	12	5	1.00	12	0.417
101	A	12	5	1.00	12	0.417
102	A	6	5	1.00	10	0.500
103	A	7	6	1.00	8	0.750
104	N/A	0	0	1.00	12	0.000
105	A	34	8	1.00	12	0.667
106	A	24	9	1.00	12	0.750
107	A	22	9	1.00	12	0.750
108	A	11	10	1.00	10	1.000
109	A	8	7	1.00	8	0.875
110	N/A	0	0	1.00	12	0.000
111	A	32	7	1.00	12	0.583
112	A	21	7	1.00	12	0.583
113	A	22	9	1.00	12	0.750
114	A	9	8	1.00	10	0.800
115	A	9	7	1.00	8	0.875
116	N/A	0	0	1.00	12	0.000
117	N/A	0	0	1.00	10	0.000
118	N/A	0	0	1.00	10	0.000
119	A	2	2	1.00	10	0.200
120	A	2	2	1.00	8	0.250
121	N/A	0	0	1.00	10	0.000
122	N/A	0	0	1.00	10	0.000
123	N/A	0	0	1.00	12	0.000
124	N/A	0	0	1.00	12	0.000
125	N/A	0	0	1.00	12	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
126	N/A	0	0	1.00	12	0.000
127	N/A	0	0	1.00	12	0.000
128	N/A	0	0	1.00	12	0.000
129	A	12	4	1.00	10	0.400
130	A	9	4	1.00	10	0.400
131	A	9	4	1.00	10	0.400
132	A	6	5	1.00	8	0.625
133	A	4	3	1.00	6	0.500
134	N/A	0	0	1.00	10	0.000
135	N/A	0	0	1.00	10	0.000
136	A	14	7	1.00	16	0.438
137	A	9	7	1.00	14	0.500
138	A	7	6	1.00	12	0.500
139	A	22	10	1.00	16	0.625
140	A	11	10	1.00	14	0.714
141	A	8	7	1.00	12	0.583
142	A	24	10	1.00	16	0.625
143	A	12	9	1.00	14	0.643
144	A	9	7	1.00	12	0.583
145	A	13	6	1.00	16	0.375
146	A	8	7	1.00	14	0.500
147	A	6	5	1.00	12	0.417
148	A	12	5	1.00	16	0.312
149	A	6	5	1.00	14	0.357
150	A	7	6	1.00	12	0.500
151	A	22	9	1.00	16	0.562
152	A	11	10	1.00	14	0.714
153	A	8	7	1.00	12	0.583
154	A	22	9	1.00	16	0.562
155	A	9	8	1.00	14	0.571
156	A	9	7	1.00	12	0.583

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int x^4 \text{arcsinh}(ax)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 72

$$\int x^4 \text{arcsinh}(ax)\,dx = -\frac{\sqrt{1+a^2x^2}}{5a^5} + \frac{2(1+a^2x^2)^{3/2}}{15a^5} - \frac{(1+a^2x^2)^{5/2}}{25a^5} + \frac{1}{5}x^5\text{arcsinh}(ax)$$

```
[Out] 2/15*(a^2*x^2+1)^(3/2)/a^5-1/25*(a^2*x^2+1)^(5/2)/a^5+1/5*x^5*arcsinh(a*x)-
1/5*(a^2*x^2+1)^(1/2)/a^5
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5776, 272, 45}

$$\int x^4 \text{arcsinh}(ax)\,dx = -\frac{(a^2x^2+1)^{5/2}}{25a^5} + \frac{2(a^2x^2+1)^{3/2}}{15a^5} - \frac{\sqrt{a^2x^2+1}}{5a^5} + \frac{1}{5}x^5\text{arcsinh}(ax)$$

```
[In] Int[x^4*ArcSinh[a*x],x]
```

```
[Out] -1/5*Sqrt[1 + a^2*x^2]/a^5 + (2*(1 + a^2*x^2)^(3/2))/(15*a^5) - (1 + a^2*x^
2)^(5/2)/(25*a^5) + (x^5*ArcSinh[a*x])/5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{5}x^5\text{arcsinh}(ax) - \frac{1}{5}a\int \frac{x^5}{\sqrt{1+a^2x^2}}\,dx \\
&= \frac{1}{5}x^5\text{arcsinh}(ax) - \frac{1}{10}a\text{Subst}\left(\int \frac{x^2}{\sqrt{1+a^2x}}\,dx, x, x^2\right) \\
&= \frac{1}{5}x^5\text{arcsinh}(ax) - \frac{1}{10}a\text{Subst}\left(\int \left(\frac{1}{a^4\sqrt{1+a^2x}} - \frac{2\sqrt{1+a^2x}}{a^4} + \frac{(1+a^2x)^{3/2}}{a^4}\right)\,dx, x, x^2\right) \\
&= -\frac{\sqrt{1+a^2x^2}}{5a^5} + \frac{2(1+a^2x^2)^{3/2}}{15a^5} - \frac{(1+a^2x^2)^{5/2}}{25a^5} + \frac{1}{5}x^5\text{arcsinh}(ax)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.69

$$\int x^4\text{arcsinh}(ax)\,dx = -\frac{\sqrt{1+a^2x^2}(8-4a^2x^2+3a^4x^4)}{75a^5} + \frac{1}{5}x^5\text{arcsinh}(ax)$$

```
[In] Integrate[x^4*ArcSinh[a*x],x]
[Out] -1/75*(Sqrt[1 + a^2*x^2]*(8 - 4*a^2*x^2 + 3*a^4*x^4))/a^5 + (x^5*ArcSinh[a*
x])/5
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{\frac{a^5x^5\operatorname{arcsinh}(ax)}{5}-\frac{a^4x^4\sqrt{a^2x^2+1}}{25}+\frac{4a^2x^2\sqrt{a^2x^2+1}}{75}-\frac{8\sqrt{a^2x^2+1}}{75}}{a^5}$	69
default	$\frac{\frac{a^5x^5\operatorname{arcsinh}(ax)}{5}-\frac{a^4x^4\sqrt{a^2x^2+1}}{25}+\frac{4a^2x^2\sqrt{a^2x^2+1}}{75}-\frac{8\sqrt{a^2x^2+1}}{75}}{a^5}$	69
parts	$\frac{x^5\operatorname{arcsinh}(ax)}{5}-\frac{a\left(\frac{x^4\sqrt{a^2x^2+1}}{5a^2}-\frac{4\left(\frac{x^2\sqrt{a^2x^2+1}}{3a^2}-\frac{2\sqrt{a^2x^2+1}}{3a^4}\right)}{5a^2}\right)}{5}$	75

```
[In] int(x^4*arcsinh(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^5*(1/5*a^5*x^5*arcsinh(a*x)-1/25*a^4*x^4*(a^2*x^2+1)^(1/2)+4/75*a^2*x^2
*(a^2*x^2+1)^(1/2)-8/75*(a^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.85

$$\int x^4\operatorname{arcsinh}(ax)\,dx=\frac{15\,a^5x^5\log\left(ax+\sqrt{a^2x^2+1}\right)-(3\,a^4x^4-4\,a^2x^2+8)\sqrt{a^2x^2+1}}{75\,a^5}$$

```
[In] integrate(x^4*arcsinh(a*x),x, algorithm="fricas")
```

```
[Out] 1/75*(15*a^5*x^5*log(a*x + sqrt(a^2*x^2 + 1)) - (3*a^4*x^4 - 4*a^2*x^2 + 8)
*sqrt(a^2*x^2 + 1))/a^5
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.97

$$\int x^4\operatorname{arcsinh}(ax)\,dx=\begin{cases}\frac{x^5\operatorname{asinh}(ax)}{5}-\frac{x^4\sqrt{a^2x^2+1}}{25a}+\frac{4x^2\sqrt{a^2x^2+1}}{75a^3}-\frac{8\sqrt{a^2x^2+1}}{75a^5} & \text{for } a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

```
[In] integrate(x**4*asinh(a*x),x)
```

```
[Out] Piecewise((x**5*asinh(a*x)/5 - x**4*sqrt(a**2*x**2 + 1)/(25*a) + 4*x**2*sqr
t(a**2*x**2 + 1)/(75*a**3) - 8*sqrt(a**2*x**2 + 1)/(75*a**5), Ne(a, 0)), (0
, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.94

$$\int x^4\text{arcsinh}(ax)\,dx = \frac{1}{5}\,x^5\,\text{arsinh}\,(ax) - \frac{1}{75}\left(\frac{3\,\sqrt{a^2x^2+1}x^4}{a^2} - \frac{4\,\sqrt{a^2x^2+1}x^2}{a^4} + \frac{8\,\sqrt{a^2x^2+1}}{a^6}\right)a$$

```
[In] integrate(x^4*arcsinh(a*x),x, algorithm="maxima")
```

```
[Out] 1/5*x^5*arcsinh(a*x) - 1/75*(3*sqrt(a^2*x^2 + 1)*x^4/a^2 - 4*sqrt(a^2*x^2 +
 1)*x^2/a^4 + 8*sqrt(a^2*x^2 + 1)/a^6)*a
```

Giac [F(-2)]

Exception generated.

$$\int x^4\text{arcsinh}(ax)\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^4*arcsinh(a*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^4\text{arcsinh}(ax)\,dx = \int x^4\,\text{asinh}(a\,x)\,dx$$

```
[In] int(x^4*asinh(a*x),x)
```

```
[Out] int(x^4*asinh(a*x), x)
```

3.2 $\int x^3 \mathbf{arcsinh}(ax)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 67

$$\int x^3\text{arcsinh}(ax)\,dx=\frac{3x\sqrt{1+a^2x^2}}{32a^3}-\frac{x^3\sqrt{1+a^2x^2}}{16a}-\frac{3\text{arcsinh}(ax)}{32a^4}+\frac{1}{4}x^4\text{arcsinh}(ax)$$

```
[Out] -3/32*arcsinh(a*x)/a^4+1/4*x^4*arcsinh(a*x)+3/32*x*(a^2*x^2+1)^(1/2)/a^3-1/
16*x^3*(a^2*x^2+1)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5776, 327, 221}

$$\int x^3\text{arcsinh}(ax)\,dx=-\frac{3\text{arcsinh}(ax)}{32a^4}-\frac{x^3\sqrt{a^2x^2+1}}{16a}+\frac{3x\sqrt{a^2x^2+1}}{32a^3}+\frac{1}{4}x^4\text{arcsinh}(ax)$$

```
[In] Int[x^3*ArcSinh[a*x],x]
```

```
[Out] (3*x*Sqrt[1 + a^2*x^2])/(32*a^3) - (x^3*Sqrt[1 + a^2*x^2])/(16*a) - (3*ArcS
inh[a*x])/(32*a^4) + (x^4*ArcSinh[a*x])/4
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{4}x^4\text{arcsinh}(ax) - \frac{1}{4}a\int \frac{x^4}{\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{x^3\sqrt{1+a^2x^2}}{16a} + \frac{1}{4}x^4\text{arcsinh}(ax) + \frac{3\int \frac{x^2}{\sqrt{1+a^2x^2}}\,dx}{16a} \\
&= \frac{3x\sqrt{1+a^2x^2}}{32a^3} - \frac{x^3\sqrt{1+a^2x^2}}{16a} + \frac{1}{4}x^4\text{arcsinh}(ax) - \frac{3\int \frac{1}{\sqrt{1+a^2x^2}}\,dx}{32a^3} \\
&= \frac{3x\sqrt{1+a^2x^2}}{32a^3} - \frac{x^3\sqrt{1+a^2x^2}}{16a} - \frac{3\text{arcsinh}(ax)}{32a^4} + \frac{1}{4}x^4\text{arcsinh}(ax)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.73

$$\int x^3\text{arcsinh}(ax)\,dx = \frac{ax(3-2a^2x^2)\sqrt{1+a^2x^2} + (-3+8a^4x^4)\,\text{arcsinh}(ax)}{32a^4}$$

```
[In] Integrate[x^3*ArcSinh[a*x],x]
[Out] (a*x*(3 - 2*a^2*x^2)*Sqrt[1 + a^2*x^2] + (-3 + 8*a^4*x^4)*ArcSinh[a*x])/(32
*a^4)
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.87

method	result	size
derivativedivides	$\frac{\frac{a^4x^4\,\text{arcsinh}(ax)}{4}-\frac{a^3x^3\sqrt{a^2x^2+1}}{16}+\frac{3ax\sqrt{a^2x^2+1}}{32}-\frac{3\,\text{arcsinh}(ax)}{32}}{a^4}$	58
default	$\frac{\frac{a^4x^4\,\text{arcsinh}(ax)}{4}-\frac{a^3x^3\sqrt{a^2x^2+1}}{16}+\frac{3ax\sqrt{a^2x^2+1}}{32}-\frac{3\,\text{arcsinh}(ax)}{32}}{a^4}$	58
parts	$\frac{x^4\,\text{arcsinh}(ax)}{4}-\frac{a\left(\frac{x^3\sqrt{a^2x^2+1}}{4a^2}-\frac{3\left(\frac{x\sqrt{a^2x^2+1}}{2a^2}-\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{2a^2\sqrt{a^2}}\right)}{4a^2}\right)}{4}$	90

```
[In] int(x^3*arcsinh(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*(1/4*a^4*x^4*arcsinh(a*x)-1/16*a^3*x^3*(a^2*x^2+1)^(1/2)+3/32*a*x*(a^
2*x^2+1)^(1/2)-3/32*arcsinh(a*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.88

$$\int x^3\text{arcsinh}(ax)\,dx=\frac{(8\,a^4x^4-3)\log\left(ax+\sqrt{a^2x^2+1}\right)-(2\,a^3x^3-3\,ax)\sqrt{a^2x^2+1}}{32\,a^4}$$

```
[In] integrate(x^3*arcsinh(a*x),x, algorithm="fricas")
```

```
[Out] 1/32*((8*a^4*x^4 - 3)*log(a*x + sqrt(a^2*x^2 + 1)) - (2*a^3*x^3 - 3*a*x)*sq
rt(a^2*x^2 + 1))/a^4
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.91

$$\int x^3\text{arcsinh}(ax)\,dx=\begin{cases}\frac{x^4\,\text{asinh}\,(ax)}{4}-\frac{x^3\sqrt{a^2x^2+1}}{16a}+\frac{3x\sqrt{a^2x^2+1}}{32a^3}-\frac{3\,\text{asinh}\,(ax)}{32a^4} & \text{for } a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

```
[In] integrate(x**3*asinh(a*x),x)
```

```
[Out] Piecewise((x**4*asinh(a*x)/4 - x**3*sqrt(a**2*x**2 + 1)/(16*a) + 3*x*sqrt(a
**2*x**2 + 1)/(32*a**3) - 3*asinh(a*x)/(32*a**4), Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.88

$$\int x^3\text{arcsinh}(ax)\,dx = \frac{1}{4}\,x^4\,\text{arsinh}\,(ax) - \frac{1}{32}\left(\frac{2\,\sqrt{a^2x^2+1}x^3}{a^2} - \frac{3\,\sqrt{a^2x^2+1}x}{a^4} + \frac{3\,\text{arsinh}\,(ax)}{a^5}\right)a$$

```
[In] integrate(x^3*arcsinh(a*x),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arcsinh(a*x) - 1/32*(2*sqrt(a^2*x^2 + 1)*x^3/a^2 - 3*sqrt(a^2*x^2 +
 1)*x/a^4 + 3*arcsinh(a*x)/a^5)*a
```

Giac [F(-2)]

Exception generated.

$$\int x^3\text{arcsinh}(ax)\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3*arcsinh(a*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^3\text{arcsinh}(ax)\,dx = \int x^3\,\text{asinh}(a\,x)\,dx$$

```
[In] int(x^3*asinh(a*x),x)
```

```
[Out] int(x^3*asinh(a*x), x)
```

3.3 $\int x^2 \mathbf{arcsinh}(ax)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 52

$$\int x^2\text{arcsinh}(ax)\,dx=\frac{\sqrt{1+a^2x^2}}{3a^3}-\frac{(1+a^2x^2)^{3/2}}{9a^3}+\frac{1}{3}x^3\text{arcsinh}(ax)$$

```
[Out] -1/9*(a^2*x^2+1)^(3/2)/a^3+1/3*x^3*arcsinh(a*x)+1/3*(a^2*x^2+1)^(1/2)/a^3
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5776, 272, 45}

$$\int x^2\text{arcsinh}(ax)\,dx=-\frac{(a^2x^2+1)^{3/2}}{9a^3}+\frac{\sqrt{a^2x^2+1}}{3a^3}+\frac{1}{3}x^3\text{arcsinh}(ax)$$

```
[In] Int[x^2*ArcSinh[a*x],x]
```

```
[Out] Sqrt[1 + a^2*x^2]/(3*a^3) - (1 + a^2*x^2)^(3/2)/(9*a^3) + (x^3*ArcSinh[a*x]
)/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}x^3\text{arcsinh}(ax) - \frac{1}{3}a\int \frac{x^3}{\sqrt{1+a^2x^2}}\,dx \\ &= \frac{1}{3}x^3\text{arcsinh}(ax) - \frac{1}{6}a\text{Subst}\left(\int \frac{x}{\sqrt{1+a^2x}}\,dx, x, x^2\right) \\ &= \frac{1}{3}x^3\text{arcsinh}(ax) - \frac{1}{6}a\text{Subst}\left(\int \left(-\frac{1}{a^2\sqrt{1+a^2x}} + \frac{\sqrt{1+a^2x}}{a^2}\right)\,dx, x, x^2\right) \\ &= \frac{\sqrt{1+a^2x^2}}{3a^3} - \frac{(1+a^2x^2)^{3/2}}{9a^3} + \frac{1}{3}x^3\text{arcsinh}(ax)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.79

$$\int x^2\text{arcsinh}(ax)\,dx = \frac{1}{9}\left(\frac{(2-a^2x^2)\sqrt{1+a^2x^2}}{a^3} + 3x^3\text{arcsinh}(ax)\right)$$

```
[In] Integrate[x^2*ArcSinh[a*x],x]
[Out] (((2 - a^2*x^2)*Sqrt[1 + a^2*x^2])/a^3 + 3*x^3*ArcSinh[a*x])/9
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{\frac{a^3x^3\operatorname{arcsinh}(ax)}{3}-\frac{a^2x^2\sqrt{a^2x^2+1}}{9}+\frac{2\sqrt{a^2x^2+1}}{9}}{a^3}$	50
default	$\frac{\frac{a^3x^3\operatorname{arcsinh}(ax)}{3}-\frac{a^2x^2\sqrt{a^2x^2+1}}{9}+\frac{2\sqrt{a^2x^2+1}}{9}}{a^3}$	50
parts	$\frac{x^3\operatorname{arcsinh}(ax)}{3}-\frac{a\left(\frac{x^2\sqrt{a^2x^2+1}}{3a^2}-\frac{2\sqrt{a^2x^2+1}}{3a^4}\right)}{3}$	50

```
[In] int(x^2*arcsinh(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(1/3*a^3*x^3*arcsinh(a*x)-1/9*a^2*x^2*(a^2*x^2+1)^(1/2)+2/9*(a^2*x^2+
1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int x^2\operatorname{arcsinh}(ax)\,dx=\frac{3\,a^3x^3\log\left(ax+\sqrt{a^2x^2+1}\right)-\sqrt{a^2x^2+1}(a^2x^2-2)}{9\,a^3}$$

```
[In] integrate(x^2*arcsinh(a*x),x, algorithm="fricas")
```

```
[Out] 1/9*(3*a^3*x^3*log(a*x + sqrt(a^2*x^2 + 1)) - sqrt(a^2*x^2 + 1)*(a^2*x^2 -
2))/a^3
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.92

$$\int x^2\operatorname{arcsinh}(ax)\,dx=\begin{cases}\frac{x^3\operatorname{asinh}(ax)}{3}-\frac{x^2\sqrt{a^2x^2+1}}{9a}+\frac{2\sqrt{a^2x^2+1}}{9a^3} & \text{for } a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

```
[In] integrate(x**2*asinh(a*x),x)
```

```
[Out] Piecewise((x**3*asinh(a*x)/3 - x**2*sqrt(a**2*x**2 + 1)/(9*a) + 2*sqrt(a**2
*x**2 + 1)/(9*a**3), Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.92

$$\int x^2\text{arcsinh}(ax)\,dx = \frac{1}{3}\,x^3\,\text{arsinh}\,(ax) - \frac{1}{9}\,a\left(\frac{\sqrt{a^2x^2+1}x^2}{a^2} - \frac{2\,\sqrt{a^2x^2+1}}{a^4}\right)$$

```
[In] integrate(x^2*arcsinh(a*x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arcsinh(a*x) - 1/9*a*(sqrt(a^2*x^2 + 1)*x^2/a^2 - 2*sqrt(a^2*x^2 +
1)/a^4)
```

Giac [F(-2)]

Exception generated.

$$\int x^2\text{arcsinh}(ax)\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2*arcsinh(a*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^2\text{arcsinh}(ax)\,dx = \int x^2\,\text{asinh}(a\,x)\,dx$$

```
[In] int(x^2*asinh(a*x),x)
```

```
[Out] int(x^2*asinh(a*x), x)
```

3.4 $\int x \operatorname{arcsinh}(ax)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 44

$$\int x\operatorname{arcsinh}(ax)\,dx=-\frac{x\sqrt{1+a^2x^2}}{4a}+\frac{\operatorname{arcsinh}(ax)}{4a^2}+\frac{1}{2}x^2\operatorname{arcsinh}(ax)$$

```
[Out] 1/4*arcsinh(a*x)/a^2+1/2*x^2*arcsinh(a*x)-1/4*x*(a^2*x^2+1)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5776, 327, 221}

$$\int x\operatorname{arcsinh}(ax)\,dx=\frac{\operatorname{arcsinh}(ax)}{4a^2}-\frac{x\sqrt{a^2x^2+1}}{4a}+\frac{1}{2}x^2\operatorname{arcsinh}(ax)$$

```
[In] Int[x*ArcSinh[a*x],x]
```

```
[Out] -1/4*(x*Sqrt[1 + a^2*x^2])/a + ArcSinh[a*x]/(4*a^2) + (x^2*ArcSinh[a*x])/2
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}x^2\operatorname{arcsinh}(ax) - \frac{1}{2}a\int \frac{x^2}{\sqrt{1+a^2x^2}}\,dx\\ &= -\frac{x\sqrt{1+a^2x^2}}{4a} + \frac{1}{2}x^2\operatorname{arcsinh}(ax) + \frac{\int \frac{1}{\sqrt{1+a^2x^2}}\,dx}{4a}\\ &= -\frac{x\sqrt{1+a^2x^2}}{4a} + \frac{\operatorname{arcsinh}(ax)}{4a^2} + \frac{1}{2}x^2\operatorname{arcsinh}(ax)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.91

$$\int x\operatorname{arcsinh}(ax)\,dx = \frac{-ax\sqrt{1+a^2x^2} + (1+2a^2x^2)\operatorname{arcsinh}(ax)}{4a^2}$$

```
[In] Integrate[x*ArcSinh[a*x],x]
[Out] (-(a*x*Sqrt[1 + a^2*x^2]) + (1 + 2*a^2*x^2)*ArcSinh[a*x])/(4*a^2)
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{\frac{a^2x^2\operatorname{arcsinh}(ax)}{2}-\frac{ax\sqrt{a^2x^2+1}}{4}+\frac{\operatorname{arcsinh}(ax)}{4}}{a^2}$	39
default	$\frac{\frac{a^2x^2\operatorname{arcsinh}(ax)}{2}-\frac{ax\sqrt{a^2x^2+1}}{4}+\frac{\operatorname{arcsinh}(ax)}{4}}{a^2}$	39
parts	$\frac{x^2\operatorname{arcsinh}(ax)}{2}-\frac{a\left(\frac{x\sqrt{a^2x^2+1}}{2a^2}-\frac{\ln\left(\frac{a^2x}{\sqrt{a^2}}+\sqrt{a^2x^2+1}\right)}{2a^2\sqrt{a^2}}\right)}{2}$	65

```
[In] int(x*arcsinh(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(1/2*a^2*x^2*arcsinh(a*x)-1/4*a*x*(a^2*x^2+1)^(1/2)+1/4*arcsinh(a*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.09

$$\int x\text{arcsinh}(ax)\,dx = -\frac{\sqrt{a^2x^2+1}ax - (2\,a^2x^2+1)\log\left(ax+\sqrt{a^2x^2+1}\right)}{4\,a^2}$$

```
[In] integrate(x*arcsinh(a*x),x, algorithm="fricas")
[Out] -1/4*(sqrt(a^2*x^2 + 1)*a*x - (2*a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1)))
/a^2
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int x\text{arcsinh}(ax)\,dx = \begin{cases} \frac{x^2\,\text{asinh}\,(ax)}{2} - \frac{x\sqrt{a^2x^2+1}}{4a} + \frac{\text{asinh}\,(ax)}{4a^2} & \text{for}\ a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

```
[In] integrate(x*asinh(a*x),x)
[Out] Piecewise((x**2*asinh(a*x)/2 - x*sqrt(a**2*x**2 + 1)/(4*a) + asinh(a*x)/(4*
a**2), Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int x\text{arcsinh}(ax)\,dx = \frac{1}{2}\,x^2\,\text{arsinh}\,(ax) - \frac{1}{4}\,a\left(\frac{\sqrt{a^2x^2+1}x}{a^2} - \frac{\text{arsinh}\,(ax)}{a^3}\right)$$

```
[In] integrate(x*arcsinh(a*x),x, algorithm="maxima")
[Out] 1/2*x^2*arcsinh(a*x) - 1/4*a*(sqrt(a^2*x^2 + 1)*x/a^2 - arcsinh(a*x)/a^3)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.55

$$\int x\text{arcsinh}(ax)\,dx = \frac{1}{2}\,x^2\log\left(ax+\sqrt{a^2x^2+1}\right) - \frac{1}{4}\,a\left(\frac{\sqrt{a^2x^2+1}x}{a^2}+\frac{\log\left(-x|a|+\sqrt{a^2x^2+1}\right)}{a^2|a|}\right)$$

```
[In] integrate(x*arcsinh(a*x),x, algorithm="giac")
```

```
[Out] 1/2*x^2*log(a*x + sqrt(a^2*x^2 + 1)) - 1/4*a*(sqrt(a^2*x^2 + 1)*x/a^2 + log
(-x*abs(a) + sqrt(a^2*x^2 + 1))/(a^2*abs(a)))
```

Mupad [B] (verification not implemented)

Time = 2.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int x\text{arcsinh}(ax)\,dx = x\,\text{asinh}(a\,x)\,\left(\frac{x}{2}+\frac{1}{4\,a^2\,x}\right)-\frac{x\,\sqrt{a^2\,x^2+1}}{4\,a}$$

```
[In] int(x*asinh(a*x),x)
```

```
[Out] x*asinh(a*x)*(x/2 + 1/(4*a^2*x)) - (x*(a^2*x^2 + 1)^(1/2))/(4*a)
```

3.5 $\int \operatorname{arcsinh}(ax)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 25

$$\int \operatorname{arcsinh}(ax)\,dx = -\frac{\sqrt{1+a^2x^2}}{a} + x\operatorname{arcsinh}(ax)$$

```
[Out] x*arcsinh(a*x)-(a^2*x^2+1)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5772, 267}

$$\int \operatorname{arcsinh}(ax)\,dx = x\operatorname{arcsinh}(ax) - \frac{\sqrt{a^2x^2+1}}{a}$$

```
[In] Int[ArcSinh[a*x],x]
```

```
[Out] -(Sqrt[1 + a^2*x^2]/a) + x*ArcSinh[a*x]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= x\text{arcsinh}(ax) - a\int \frac{x}{\sqrt{1+a^2x^2}}\,dx\\ &= -\frac{\sqrt{1+a^2x^2}}{a} + x\text{arcsinh}(ax)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \text{arcsinh}(ax)\,dx = -\frac{\sqrt{1+a^2x^2}}{a} + x\text{arcsinh}(ax)$$

```
[In] Integrate[ArcSinh[a*x],x]
[Out] -(Sqrt[1 + a^2*x^2]/a) + x*ArcSinh[a*x]
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

method	result	size
parts	$x\ \text{arcsinh}\left(ax\right)-\frac{\sqrt{a^{2}x^{2}+1}}{a}$	24
derivativedivides	$\frac{ax\ \text{arcsinh}(ax)-\sqrt{a^{2}x^{2}+1}}{a}$	26
default	$\frac{ax\ \text{arcsinh}(ax)-\sqrt{a^{2}x^{2}+1}}{a}$	26

```
[In] int(arcsinh(a*x),x,method=_RETURNVERBOSE)
[Out] x*arcsinh(a*x)-(a^2*x^2+1)^(1/2)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.48

$$\int \text{arcsinh}(ax)\,dx = \frac{ax\log\left(ax+\sqrt{a^2x^2+1}\right)-\sqrt{a^2x^2+1}}{a}$$

```
[In] integrate(arcsinh(a*x),x, algorithm="fricas")
[Out] (a*x*log(a*x + sqrt(a^2*x^2 + 1)) - sqrt(a^2*x^2 + 1))/a
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \operatorname{arcsinh}(ax)\,dx = \begin{cases} x \operatorname{asinh}{(ax)} - \frac{\sqrt{a^2x^2+1}}{a} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

```
[In] integrate(asinh(a*x),x)
[Out] Piecewise((x*asinh(a*x) - sqrt(a**2*x**2 + 1)/a, Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \operatorname{arcsinh}(ax)\,dx = \frac{ax \operatorname{arsinh}{(ax)} - \sqrt{a^2x^2+1}}{a}$$

```
[In] integrate(arcsinh(a*x),x, algorithm="maxima")
[Out] (a*x*arcsinh(a*x) - sqrt(a^2*x^2 + 1))/a
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.40

$$\int \operatorname{arcsinh}(ax)\,dx = x \log\left(ax + \sqrt{a^2x^2+1}\right) - \frac{\sqrt{a^2x^2+1}}{a}$$

```
[In] integrate(arcsinh(a*x),x, algorithm="giac")
[Out] x*log(a*x + sqrt(a^2*x^2 + 1)) - sqrt(a^2*x^2 + 1)/a
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \operatorname{arcsinh}(ax)\,dx = x \operatorname{asinh}(a\,x) - \frac{\sqrt{a^2\,x^2+1}}{a}$$

```
[In] int(asinh(a*x),x)
[Out] x*asinh(a*x) - (a^2*x^2 + 1)^(1/2)/a
```

3.6 $\int \frac{\operatorname{arcsinh}(ax)}{x}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 43

$$\int \frac{\operatorname{arcsinh}(ax)}{x}\,dx = -\frac{1}{2}\operatorname{arcsinh}(ax)^2 + \operatorname{arcsinh}(ax)\log\left(1-e^{2\operatorname{arcsinh}(ax)}\right) + \frac{1}{2}\operatorname{PolyLog}\left(2,e^{2\operatorname{arcsinh}(ax)}\right)$$

```
[Out] -1/2*arcsinh(a*x)^2+arcsinh(a*x)*ln(1-(a*x+(a^2*x^2+1)^(1/2))^2)+1/2*polylo
g(2,(a*x+(a^2*x^2+1)^(1/2))^2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5775, 3797, 2221, 2317, 2438}

$$\int \frac{\operatorname{arcsinh}(ax)}{x}\,dx = \frac{1}{2}\operatorname{PolyLog}\left(2,e^{2\operatorname{arcsinh}(ax)}\right) - \frac{1}{2}\operatorname{arcsinh}(ax)^2 + \operatorname{arcsinh}(ax)\log\left(1-e^{2\operatorname{arcsinh}(ax)}\right)$$

```
[In] Int[ArcSinh[a*x]/x,x]
```

```
[Out] -1/2*ArcSinh[a*x]^2 + ArcSinh[a*x]*Log[1 - E^(2*ArcSinh[a*x])] + PolyLog[2,
 E^(2*ArcSinh[a*x])]/2
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
```

```
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int x\coth(x)\,dx, x, \operatorname{arcsinh}(ax)\right)\\
&= -\frac{1}{2}\operatorname{arcsinh}(ax)^2 - 2\text{Subst}\left(\int \frac{e^{2x}x}{1-e^{2x}}\,dx, x, \operatorname{arcsinh}(ax)\right)\\
&= -\frac{1}{2}\operatorname{arcsinh}(ax)^2 + \operatorname{arcsinh}(ax)\log\left(1-e^{2\operatorname{arcsinh}(ax)}\right)\\
&\quad - \text{Subst}\left(\int \log\left(1-e^{2x}\right)\,dx, x, \operatorname{arcsinh}(ax)\right)\\
&= -\frac{1}{2}\operatorname{arcsinh}(ax)^2 + \operatorname{arcsinh}(ax)\log\left(1-e^{2\operatorname{arcsinh}(ax)}\right)\\
&\quad - \frac{1}{2}\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2\operatorname{arcsinh}(ax)}\right)\\
&= -\frac{1}{2}\operatorname{arcsinh}(ax)^2 + \operatorname{arcsinh}(ax)\log\left(1-e^{2\operatorname{arcsinh}(ax)}\right) + \frac{1}{2}\,\text{PolyLog}\left(2, e^{2\operatorname{arcsinh}(ax)}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}(ax)}{x}\,dx = -\frac{1}{2}\operatorname{arcsinh}(ax)^2 + \operatorname{arcsinh}(ax)\log\left(1-e^{2\operatorname{arcsinh}(ax)}\right) + \frac{1}{2}\operatorname{PolyLog}\left(2, e^{2\operatorname{arcsinh}(ax)}\right)$$

```
[In] Integrate[ArcSinh[a*x]/x,x]
```

```
[Out] -1/2*ArcSinh[a*x]^2 + ArcSinh[a*x]*Log[1 - E^(2*ArcSinh[a*x])] + PolyLog[2,
 E^(2*ArcSinh[a*x])]/2
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.19

method	result
derivativedivides	$-\frac{\operatorname{arcsinh}(ax)^2}{2} + \operatorname{arcsinh}(ax)\ln\left(1+ax+\sqrt{a^2x^2+1}\right) + \operatorname{polylog}\left(2, -ax-\sqrt{a^2x^2+1}\right)$
default	$-\frac{\operatorname{arcsinh}(ax)^2}{2} + \operatorname{arcsinh}(ax)\ln\left(1+ax+\sqrt{a^2x^2+1}\right) + \operatorname{polylog}\left(2, -ax-\sqrt{a^2x^2+1}\right)$

```
[In] int(arcsinh(a*x)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*arcsinh(a*x)^2+arcsinh(a*x)*ln(1+a*x+(a^2*x^2+1)^(1/2))+polylog(2,-a*x
-(a^2*x^2+1)^(1/2))+arcsinh(a*x)*ln(1-a*x-(a^2*x^2+1)^(1/2))+polylog(2,a*x+
(a^2*x^2+1)^(1/2))
```

Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax)}{x}\,dx = \int \frac{\operatorname{arsinh}(ax)}{x}\,dx$$

```
[In] integrate(arcsinh(a*x)/x,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x)/x, x)
```

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)}{x}\,dx = \int \frac{\operatorname{asinh}\left(ax\right)}{x}\,dx$$

```
[In] integrate(asinh(a*x)/x,x)
[Out] Integral(asinh(a*x)/x, x)
```

Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax)}{x}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)}{x}\,dx$$

```
[In] integrate(arcsinh(a*x)/x,x, algorithm="maxima")
[Out] integrate(arcsinh(a*x)/x, x)
```

Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax)}{x}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)}{x}\,dx$$

```
[In] integrate(arcsinh(a*x)/x,x, algorithm="giac")
[Out] integrate(arcsinh(a*x)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)}{x}\,dx = \int \frac{\operatorname{asinh}(a\,x)}{x}\,dx$$

```
[In] int(asinh(a*x)/x,x)
[Out] int(asinh(a*x)/x, x)
```

3.7 $\int \frac{\operatorname{arcsinh}(ax)}{x^2}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 27

$$\int \frac{\operatorname{arcsinh}(ax)}{x^2}\,dx = -\frac{\operatorname{arcsinh}(ax)}{x} - a\operatorname{arctanh}\left(\sqrt{1+a^2x^2}\right)$$

```
[Out] -arcsinh(a*x)/x-a*arctanh((a^2*x^2+1)^(1/2))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5776, 272, 65, 214}

$$\int \frac{\operatorname{arcsinh}(ax)}{x^2}\,dx = -a\operatorname{arctanh}\left(\sqrt{a^2x^2+1}\right) - \frac{\operatorname{arcsinh}(ax)}{x}$$

```
[In] Int[ArcSinh[a*x]/x^2,x]
```

```
[Out] -(ArcSinh[a*x]/x) - a*ArcTanh[Sqrt[1 + a^2*x^2]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\operatorname{arcsinh}(ax)}{x} + a\int \frac{1}{x\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{\operatorname{arcsinh}(ax)}{x} + \frac{1}{2}a\text{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx, x, x^2\right) \\
&= -\frac{\operatorname{arcsinh}(ax)}{x} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx, x, \sqrt{1+a^2x^2}\right)}{a} \\
&= -\frac{\operatorname{arcsinh}(ax)}{x} - a\text{arctanh}\left(\sqrt{1+a^2x^2}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}(ax)}{x^2}\,dx = -\frac{\operatorname{arcsinh}(ax)}{x} - a\text{arctanh}\left(\sqrt{1+a^2x^2}\right)$$

```
[In] Integrate[ArcSinh[a*x]/x^2,x]
[Out] -(ArcSinh[a*x]/x) - a*ArcTanh[Sqrt[1 + a^2*x^2]]
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

method	result	size
parts	$-\frac{\operatorname{arcsinh}(ax)}{x}-a\operatorname{arctanh}\left(\frac{1}{\sqrt{a^{2}x^{2}+1}}\right)$	26
derivativedivides	$a\left(-\frac{\operatorname{arcsinh}(ax)}{ax}-\operatorname{arctanh}\left(\frac{1}{\sqrt{a^{2}x^{2}+1}}\right)\right)$	30
default	$a\left(-\frac{\operatorname{arcsinh}(ax)}{ax}-\operatorname{arctanh}\left(\frac{1}{\sqrt{a^{2}x^{2}+1}}\right)\right)$	30

```
[In] int(arcsinh(a*x)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -arcsinh(a*x)/x-a*arctanh(1/(a^2*x^2+1)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(25) = 50$.

Time = 0.26 (sec) , antiderivative size = 90, normalized size of antiderivative = 3.33

$$\int \frac{\operatorname{arcsinh}(ax)}{x^2}\,dx =$$

$$-\frac{ax\log\left(-ax+\sqrt{a^2x^2+1}+1\right)-ax\log\left(-ax+\sqrt{a^2x^2+1}-1\right)-(x-1)\log\left(ax+\sqrt{a^2x^2+1}\right)-x\,\mathrm{lo}}{x}$$

```
[In] integrate(arcsinh(a*x)/x^2,x, algorithm="fricas")
```

```
[Out] -(a*x*log(-a*x + sqrt(a^2*x^2 + 1) + 1) - a*x*log(-a*x + sqrt(a^2*x^2 + 1)
 - 1) - (x - 1)*log(a*x + sqrt(a^2*x^2 + 1)) - x*log(-a*x + sqrt(a^2*x^2 + 1
)))/x
```

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)}{x^2}\,dx = \int \frac{\operatorname{asinh}(ax)}{x^2}\,dx$$

```
[In] integrate(asinh(a*x)/x**2,x)
```

```
[Out] Integral(asinh(a*x)/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{\operatorname{arcsinh}(ax)}{x^2}\,dx = -a\operatorname{arsinh}\left(\frac{1}{a|x|}\right) - \frac{\operatorname{arsinh}(ax)}{x}$$

```
[In] integrate(arcsinh(a*x)/x^2,x, algorithm="maxima")
[Out] -a*arcsinh(1/(a*abs(x))) - arcsinh(a*x)/x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(25) = 50.

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.07

$$\int \frac{\operatorname{arcsinh}(ax)}{x^2}\,dx = -\frac{1}{2}a\left(\log\left(\sqrt{a^2x^2+1}+1\right) - \log\left(\sqrt{a^2x^2+1}-1\right)\right) - \frac{\log\left(ax+\sqrt{a^2x^2+1}\right)}{x}$$

```
[In] integrate(arcsinh(a*x)/x^2,x, algorithm="giac")
[Out] -1/2*a*(log(sqrt(a^2*x^2 + 1) + 1) - log(sqrt(a^2*x^2 + 1) - 1)) - log(a*x
+ sqrt(a^2*x^2 + 1))/x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)}{x^2}\,dx = \int \frac{\operatorname{asinh}(a\,x)}{x^2}\,dx$$

```
[In] int(asinh(a*x)/x^2,x)
[Out] int(asinh(a*x)/x^2, x)
```

3.8 $\int \frac{\operatorname{arcsinh}(ax)}{x^3}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 33

$$\int \frac{\operatorname{arcsinh}(ax)}{x^3}\,dx = -\frac{a\sqrt{1+a^2x^2}}{2x} - \frac{\operatorname{arcsinh}(ax)}{2x^2}$$

```
[Out] -1/2*arcsinh(a*x)/x^2-1/2*a*(a^2*x^2+1)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5776, 270}

$$\int \frac{\operatorname{arcsinh}(ax)}{x^3}\,dx = -\frac{a\sqrt{a^2x^2+1}}{2x} - \frac{\operatorname{arcsinh}(ax)}{2x^2}$$

```
[In] Int[ArcSinh[a*x]/x^3,x]
```

```
[Out] -1/2*(a*Sqrt[1 + a^2*x^2])/x - ArcSinh[a*x]/(2*x^2)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
```

```
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{arcsinh}(ax)}{2x^2} + \frac{1}{2}a\int \frac{1}{x^2\sqrt{1+a^2x^2}}\,dx\\ &= -\frac{a\sqrt{1+a^2x^2}}{2x} - \frac{\text{arcsinh}(ax)}{2x^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int \frac{\text{arcsinh}(ax)}{x^3}\,dx = -\frac{ax\sqrt{1+a^2x^2}+\text{arcsinh}(ax)}{2x^2}$$

```
[In] Integrate[ArcSinh[a*x]/x^3,x]
[Out] -1/2*(a*x*Sqrt[1 + a^2*x^2] + ArcSinh[a*x])/x^2
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

method	result	size
parts	$-\frac{\text{arcsinh}(ax)}{2x^2}-\frac{a\sqrt{a^2x^2+1}}{2x}$	28
derivativedivides	$a^2\left(-\frac{\text{arcsinh}(ax)}{2a^2x^2}-\frac{\sqrt{a^2x^2+1}}{2ax}\right)$	37
default	$a^2\left(-\frac{\text{arcsinh}(ax)}{2a^2x^2}-\frac{\sqrt{a^2x^2+1}}{2ax}\right)$	37

```
[In] int(arcsinh(a*x)/x^3,x,method=_RETURNVERBOSE)
[Out] -1/2*arcsinh(a*x)/x^2-1/2*a*(a^2*x^2+1)^(1/2)/x
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

$$\int \frac{\operatorname{arcsinh}(ax)}{x^3}\,dx = -\frac{\sqrt{a^2x^2+1}ax + \log\left(ax+\sqrt{a^2x^2+1}\right)}{2\,x^2}$$

```
[In] integrate(arcsinh(a*x)/x^3,x, algorithm="fricas")
[Out] -1/2*(sqrt(a^2*x^2 + 1)*a*x + log(a*x + sqrt(a^2*x^2 + 1)))/x^2
```

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)}{x^3}\,dx = \int \frac{\operatorname{asinh}(ax)}{x^3}\,dx$$

```
[In] integrate(asinh(a*x)/x**3,x)
[Out] Integral(asinh(a*x)/x**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{\operatorname{arcsinh}(ax)}{x^3}\,dx = -\frac{\sqrt{a^2x^2+1}a}{2\,x} - \frac{\operatorname{arsinh}(ax)}{2\,x^2}$$

```
[In] integrate(arcsinh(a*x)/x^3,x, algorithm="maxima")
[Out] -1/2*sqrt(a^2*x^2 + 1)*a/x - 1/2*arcsinh(a*x)/x^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.52

$$\int \frac{\operatorname{arcsinh}(ax)}{x^3}\,dx = \frac{a|a|}{\left(x|a|-\sqrt{a^2x^2+1}\right)^2-1} - \frac{\log\left(ax+\sqrt{a^2x^2+1}\right)}{2\,x^2}$$

```
[In] integrate(arcsinh(a*x)/x^3,x, algorithm="giac")
[Out] a*abs(a)/((x*abs(a) - sqrt(a^2*x^2 + 1))^2 - 1) - 1/2*log(a*x + sqrt(a^2*x^
2 + 1))/x^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)}{x^3}\,dx = \int \frac{\operatorname{asinh}(a\,x)}{x^3}\,dx$$

```
[In] int(asinh(a*x)/x^3,x)
[Out] int(asinh(a*x)/x^3, x)
```

3.9 $\int \frac{\operatorname{arcsinh}(ax)}{x^4}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 54

$$\int \frac{\operatorname{arcsinh}(ax)}{x^4}\,dx=-\frac{a\sqrt{1+a^2x^2}}{6x^2}-\frac{\operatorname{arcsinh}(ax)}{3x^3}+\frac{1}{6}a^3\operatorname{arctanh}\left(\sqrt{1+a^2x^2}\right)$$

```
[Out] -1/3*arcsinh(a*x)/x^3+1/6*a^3*arctanh((a^2*x^2+1)^(1/2))-1/6*a*(a^2*x^2+1)^
(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5776, 272, 44, 65, 214}

$$\int \frac{\operatorname{arcsinh}(ax)}{x^4}\,dx=-\frac{a\sqrt{a^2x^2+1}}{6x^2}+\frac{1}{6}a^3\operatorname{arctanh}\left(\sqrt{a^2x^2+1}\right)-\frac{\operatorname{arcsinh}(ax)}{3x^3}$$

```
[In] Int[ArcSinh[a*x]/x^4,x]
```

```
[Out] -1/6*(a*Sqrt[1 + a^2*x^2])/x^2 - ArcSinh[a*x]/(3*x^3) + (a^3*ArcTanh[Sqrt[1
 + a^2*x^2]])/6
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] && !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\operatorname{arcsinh}(ax)}{3x^3}+\frac{1}{3}a\int\frac{1}{x^3\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{\operatorname{arcsinh}(ax)}{3x^3}+\frac{1}{6}a\text{Subst}\left(\int\frac{1}{x^2\sqrt{1+a^2x}}\,dx,x,x^2\right)\\
&= -\frac{a\sqrt{1+a^2x^2}}{6x^2}-\frac{\operatorname{arcsinh}(ax)}{3x^3}-\frac{1}{12}a^3\text{Subst}\left(\int\frac{1}{x\sqrt{1+a^2x}}\,dx,x,x^2\right)\\
&= -\frac{a\sqrt{1+a^2x^2}}{6x^2}-\frac{\operatorname{arcsinh}(ax)}{3x^3}-\frac{1}{6}a\text{Subst}\left(\int\frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx,x,\sqrt{1+a^2x^2}\right)\\
&= -\frac{a\sqrt{1+a^2x^2}}{6x^2}-\frac{\operatorname{arcsinh}(ax)}{3x^3}+\frac{1}{6}a^3\operatorname{arctanh}\left(\sqrt{1+a^2x^2}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}(ax)}{x^4}\,dx=-\frac{a\sqrt{1+a^2x^2}}{6x^2}-\frac{\operatorname{arcsinh}(ax)}{3x^3}+\frac{1}{6}a^3\operatorname{arctanh}\left(\sqrt{1+a^2x^2}\right)$$

```
[In] Integrate[ArcSinh[a*x]/x^4,x]
```

```
[Out] -1/6*(a*Sqrt[1 + a^2*x^2])/x^2 - ArcSinh[a*x]/(3*x^3) + (a^3*ArcTanh[Sqrt[1
 + a^2*x^2]])/6
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

method	result	size
parts	$-\frac{\operatorname{arcsinh}(ax)}{3x^3}+\frac{a\left(-\frac{\sqrt{a^2x^2+1}}{2x^2}+\frac{a^2\operatorname{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)}{2}\right)}{3}$	48
derivativedivides	$a^3\left(-\frac{\operatorname{arcsinh}(ax)}{3a^3x^3}-\frac{\sqrt{a^2x^2+1}}{6a^2x^2}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)}{6}\right)$	51
default	$a^3\left(-\frac{\operatorname{arcsinh}(ax)}{3a^3x^3}-\frac{\sqrt{a^2x^2+1}}{6a^2x^2}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{a^2x^2+1}}\right)}{6}\right)$	51

```
[In] int(arcsinh(a*x)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*arcsinh(a*x)/x^3+1/3*a*(-1/2/x^2*(a^2*x^2+1)^(1/2)+1/2*a^2*arctanh(1/(
a^2*x^2+1)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(44) = 88$.

Time = 0.27 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.17

$$\int \frac{\operatorname{arcsinh}(ax)}{x^4}\,dx$$
$$=\frac{a^3x^3\log\left(-ax+\sqrt{a^2x^2+1}+1\right)-a^3x^3\log\left(-ax+\sqrt{a^2x^2+1}-1\right)+2\,x^3\log\left(-ax+\sqrt{a^2x^2+1}\right)-\sqrt{a}}{6\,x^3}$$

```
[In] integrate(arcsinh(a*x)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(a^3*x^3*log(-a*x + sqrt(a^2*x^2 + 1) + 1) - a^3*x^3*log(-a*x + sqrt(a^
2*x^2 + 1) - 1) + 2*x^3*log(-a*x + sqrt(a^2*x^2 + 1)) - sqrt(a^2*x^2 + 1)*a
*x + 2*(x^3 - 1)*log(a*x + sqrt(a^2*x^2 + 1)))/x^3
```

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)}{x^4}\,dx = \int \frac{\operatorname{asinh}(ax)}{x^4}\,dx$$

```
[In] integrate(asinh(a*x)/x**4,x)
```

```
[Out] Integral(asinh(a*x)/x**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

$$\int \frac{\operatorname{arcsinh}(ax)}{x^4}\,dx = \frac{1}{6}\left(a^2 \operatorname{arsinh}\left(\frac{1}{a|x|}\right) - \frac{\sqrt{a^2x^2+1}}{x^2}\right)a - \frac{\operatorname{arsinh}(ax)}{3\,x^3}$$

```
[In] integrate(arcsinh(a*x)/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*(a^2*arcsinh(1/(a*abs(x))) - sqrt(a^2*x^2 + 1)/x^2)*a - 1/3*arcsinh(a*x
)/x^3
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.56

$$\int \frac{\operatorname{arcsinh}(ax)}{x^4}\,dx = \frac{a^4 \log\left(\sqrt{a^2x^2+1}+1\right) - a^4 \log\left(\sqrt{a^2x^2+1}-1\right) - \frac{2\sqrt{a^2x^2+1}a^2}{x^2}}{12\,a} - \frac{\log\left(ax+\sqrt{a^2x^2+1}\right)}{3\,x^3}$$

```
[In] integrate(arcsinh(a*x)/x^4,x, algorithm="giac")
```

```
[Out] 1/12*(a^4*log(sqrt(a^2*x^2 + 1) + 1) - a^4*log(sqrt(a^2*x^2 + 1) - 1) - 2*s
qrt(a^2*x^2 + 1)*a^2/x^2)/a - 1/3*log(a*x + sqrt(a^2*x^2 + 1))/x^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)}{x^4}\,dx = \int \frac{\operatorname{asinh}(a\,x)}{x^4}\,dx$$

```
[In] int(asinh(a*x)/x^4,x)
[Out] int(asinh(a*x)/x^4, x)
```

3.10 $\int \frac{\operatorname{arcsinh}(ax)}{x^5}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 56

$$\int \frac{\operatorname{arcsinh}(ax)}{x^5}\,dx=-\frac{a\sqrt{1+a^2x^2}}{12x^3}+\frac{a^3\sqrt{1+a^2x^2}}{6x}-\frac{\operatorname{arcsinh}(ax)}{4x^4}$$

```
[Out] -1/4*arcsinh(a*x)/x^4-1/12*a*(a^2*x^2+1)^(1/2)/x^3+1/6*a^3*(a^2*x^2+1)^(1/2
)/x
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5776, 277, 270}

$$\int \frac{\operatorname{arcsinh}(ax)}{x^5}\,dx=-\frac{a\sqrt{a^2x^2+1}}{12x^3}+\frac{a^3\sqrt{a^2x^2+1}}{6x}-\frac{\operatorname{arcsinh}(ax)}{4x^4}$$

```
[In] Int[ArcSinh[a*x]/x^5,x]
```

```
[Out] -1/12*(a*Sqrt[1 + a^2*x^2])/x^3 + (a^3*Sqrt[1 + a^2*x^2])/(6*x) - ArcSinh[a
*x]/(4*x^4)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{arcsinh}(ax)}{4x^4} + \frac{1}{4}a\int \frac{1}{x^4\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{a\sqrt{1+a^2x^2}}{12x^3} - \frac{\text{arcsinh}(ax)}{4x^4} - \frac{1}{6}a^3\int \frac{1}{x^2\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{a\sqrt{1+a^2x^2}}{12x^3} + \frac{a^3\sqrt{1+a^2x^2}}{6x} - \frac{\text{arcsinh}(ax)}{4x^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.71

$$\int \frac{\text{arcsinh}(ax)}{x^5}\,dx = \frac{ax\sqrt{1+a^2x^2}(-1+2a^2x^2) - 3\text{arcsinh}(ax)}{12x^4}$$

```
[In] Integrate[ArcSinh[a*x]/x^5,x]
[Out] (a*x*Sqrt[1 + a^2*x^2]*(-1 + 2*a^2*x^2) - 3*ArcSinh[a*x])/(12*x^4)
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.89

method	result	size
parts	$-\frac{\operatorname{arcsinh}(ax)}{4x^4}+\frac{a\left(-\frac{\sqrt{a^2x^2+1}}{3x^3}+\frac{2a^2\sqrt{a^2x^2+1}}{3x}\right)}{4}$	50
derivativedivides	$a^4\left(-\frac{\operatorname{arcsinh}(ax)}{4a^4x^4}-\frac{\sqrt{a^2x^2+1}}{12a^3x^3}+\frac{\sqrt{a^2x^2+1}}{6ax}\right)$	56
default	$a^4\left(-\frac{\operatorname{arcsinh}(ax)}{4a^4x^4}-\frac{\sqrt{a^2x^2+1}}{12a^3x^3}+\frac{\sqrt{a^2x^2+1}}{6ax}\right)$	56

```
[In] int(arcsinh(a*x)/x^5,x,method=_RETURNVERBOSE)
[Out] -1/4*arcsinh(a*x)/x^4+1/4*a*(-1/3/x^3*(a^2*x^2+1)^(1/2)+2/3*a^2/x*(a^2*x^2+
1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.88

$$\int \frac{\operatorname{arcsinh}(ax)}{x^5}\,dx = \frac{(2\,a^3x^3 - ax)\sqrt{a^2x^2+1} - 3\,\log\left(ax + \sqrt{a^2x^2+1}\right)}{12\,x^4}$$

```
[In] integrate(arcsinh(a*x)/x^5,x, algorithm="fricas")
[Out] 1/12*((2*a^3*x^3 - a*x)*sqrt(a^2*x^2 + 1) - 3*log(a*x + sqrt(a^2*x^2 + 1)))
/x^4
```

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)}{x^5}\,dx = \int \frac{\operatorname{asinh}\left(ax\right)}{x^5}\,dx$$

```
[In] integrate(asinh(a*x)/x**5,x)
[Out] Integral(asinh(a*x)/x**5, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.88

$$\int \frac{\operatorname{arcsinh}(ax)}{x^5}\,dx = \frac{1}{12}\left(\frac{2\sqrt{a^2x^2+1}a^2}{x} - \frac{\sqrt{a^2x^2+1}}{x^3}\right)a - \frac{\operatorname{arsinh}(ax)}{4\,x^4}$$

```
[In] integrate(arcsinh(a*x)/x^5,x, algorithm="maxima")
[Out] 1/12*(2*sqrt(a^2*x^2 + 1)*a^2/x - sqrt(a^2*x^2 + 1)/x^3)*a - 1/4*arcsinh(a*
x)/x^4
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.38

$$\int \frac{\operatorname{arcsinh}(ax)}{x^5}\,dx = \frac{\left(3\left(x|a| - \sqrt{a^2x^2+1}\right)^2 - 1\right)a^3|a|}{3\left(\left(x|a| - \sqrt{a^2x^2+1}\right)^2 - 1\right)^3} - \frac{\log\left(ax + \sqrt{a^2x^2+1}\right)}{4\,x^4}$$

```
[In] integrate(arcsinh(a*x)/x^5,x, algorithm="giac")
[Out] 1/3*(3*(x*abs(a) - sqrt(a^2*x^2 + 1))^2 - 1)*a^3*abs(a)/((x*abs(a) - sqrt(a
^2*x^2 + 1))^2 - 1)^3 - 1/4*log(a*x + sqrt(a^2*x^2 + 1))/x^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)}{x^5}\,dx = \int \frac{\operatorname{asinh}(a\,x)}{x^5}\,dx$$

```
[In] int(asinh(a*x)/x^5,x)
[Out] int(asinh(a*x)/x^5, x)
```

3.11 $\int \frac{\operatorname{arcsinh}(ax)}{x^6}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 77

$$\int \frac{\operatorname{arcsinh}(ax)}{x^6}\,dx = -\frac{a\sqrt{1+a^2x^2}}{20x^4}+\frac{3a^3\sqrt{1+a^2x^2}}{40x^2}-\frac{\operatorname{arcsinh}(ax)}{5x^5}-\frac{3}{40}a^5\operatorname{arctanh}\left(\sqrt{1+a^2x^2}\right)$$

```
[Out] -1/5*arcsinh(a*x)/x^5-3/40*a^5*arctanh((a^2*x^2+1)^(1/2))-1/20*a*(a^2*x^2+1
)^(1/2)/x^4+3/40*a^3*(a^2*x^2+1)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5776, 272, 44, 65, 214}

$$\int \frac{\operatorname{arcsinh}(ax)}{x^6}\,dx = -\frac{a\sqrt{a^2x^2+1}}{20x^4}-\frac{3}{40}a^5\operatorname{arctanh}\left(\sqrt{a^2x^2+1}\right)+\frac{3a^3\sqrt{a^2x^2+1}}{40x^2}-\frac{\operatorname{arcsinh}(ax)}{5x^5}$$

```
[In] Int[ArcSinh[a*x]/x^6,x]
```

```
[Out] -1/20*(a*Sqrt[1 + a^2*x^2])/x^4 + (3*a^3*Sqrt[1 + a^2*x^2])/(40*x^2) - ArcS
inh[a*x]/(5*x^5) - (3*a^5*ArcTanh[Sqrt[1 + a^2*x^2]])/40
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
```

```
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\operatorname{arcsinh}(ax)}{5x^5}+\frac{1}{5}a\int \frac{1}{x^5\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{\operatorname{arcsinh}(ax)}{5x^5}+\frac{1}{10}a\text{Subst}\left(\int \frac{1}{x^3\sqrt{1+a^2x}}\,dx,x,x^2\right)\\
&= -\frac{a\sqrt{1+a^2x^2}}{20x^4}-\frac{\operatorname{arcsinh}(ax)}{5x^5}-\frac{1}{40}\left(3a^3\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+a^2x}}\,dx,x,x^2\right)\\
&= -\frac{a\sqrt{1+a^2x^2}}{20x^4}+\frac{3a^3\sqrt{1+a^2x^2}}{40x^2}-\frac{\operatorname{arcsinh}(ax)}{5x^5}+\frac{1}{80}\left(3a^5\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx,x,x^2\right)\\
&= -\frac{a\sqrt{1+a^2x^2}}{20x^4}+\frac{3a^3\sqrt{1+a^2x^2}}{40x^2}-\frac{\operatorname{arcsinh}(ax)}{5x^5}\\
&\quad+\frac{1}{40}\left(3a^3\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx,x,\sqrt{1+a^2x^2}\right)
\end{aligned}
$$

$$= -\frac{a\sqrt{1+a^2x^2}}{20x^4} + \frac{3a^3\sqrt{1+a^2x^2}}{40x^2} - \frac{\operatorname{arcsinh}(ax)}{5x^5} - \frac{3}{40}a^5\operatorname{arctanh}\left(\sqrt{1+a^2x^2}\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.64

$$\int \frac{\operatorname{arcsinh}(ax)}{x^6}\,dx = -\frac{\operatorname{arcsinh}(ax)}{5x^5} - \frac{1}{5}a^5\sqrt{1+a^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},3,\frac{3}{2},1+a^2x^2\right)$$

```
[In] Integrate[ArcSinh[a*x]/x^6,x]
```

```
[Out] -1/5*ArcSinh[a*x]/x^5 - (a^5*Sqrt[1 + a^2*x^2]*Hypergeometric2F1[1/2, 3, 3/
2, 1 + a^2*x^2])/5
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$a^{5} \left(-\frac{\operatorname{arcsinh}\left(a x \right)}{5 a^{5} x^{5}}-\frac{\sqrt{a^{2} x^{2}+1}}{20 a^{4} x^{4}}+\frac{3 \sqrt{a^{2} x^{2}+1}}{40 a^{2} x^{2}}-\frac{3 \,\operatorname{arctanh}\left(\frac{1}{\sqrt{a^{2} x^{2}+1}}\right)}{40}\right)$	70
default	$a^{5} \left(-\frac{\operatorname{arcsinh}\left(a x \right)}{5 a^{5} x^{5}}-\frac{\sqrt{a^{2} x^{2}+1}}{20 a^{4} x^{4}}+\frac{3 \sqrt{a^{2} x^{2}+1}}{40 a^{2} x^{2}}-\frac{3 \,\operatorname{arctanh}\left(\frac{1}{\sqrt{a^{2} x^{2}+1}}\right)}{40}\right)$	70
parts	$-\frac{\operatorname{arcsinh}\left(a x \right)}{5 x^{5}}+\frac{a \left(-\frac{\sqrt{a^{2} x^{2}+1}}{4 x^{4}}-\frac{3 a^{2} \left(-\frac{\sqrt{a^{2} x^{2}+1}}{2 x^{2}}+\frac{a^{2} \operatorname{arctanh}\left(\frac{1}{\sqrt{a^{2} x^{2}+1}}\right)}{2}\right)}{4}\right)}{5}$	70

```
[In] int(arcsinh(a*x)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] a^5*(-1/5*arcsinh(a*x)/a^5/x^5-1/20/a^4/x^4*(a^2*x^2+1)^(1/2)+3/40/a^2/x^2*
(a^2*x^2+1)^(1/2)-3/40*arctanh(1/(a^2*x^2+1)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(63) = 126.

Time = 0.26 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.68

$$\int \frac{\operatorname{arcsinh}(ax)}{x^6}\,dx = -\frac{3\,a^5x^5\log\left(-ax+\sqrt{a^2x^2+1}+1\right) - 3\,a^5x^5\log\left(-ax+\sqrt{a^2x^2+1}-1\right) - 8\,x^5\log\left(-ax+\sqrt{a^2x^2+1}\right)}{40\,x^5}$$

```
[In] integrate(arcsinh(a*x)/x^6,x, algorithm="fricas")
```

```
[Out] -1/40*(3*a^5*x^5*log(-a*x + sqrt(a^2*x^2 + 1) + 1) - 3*a^5*x^5*log(-a*x + s
qrt(a^2*x^2 + 1) - 1) - 8*x^5*log(-a*x + sqrt(a^2*x^2 + 1)) - 8*(x^5 - 1)*l
og(a*x + sqrt(a^2*x^2 + 1)) - (3*a^3*x^3 - 2*a*x)*sqrt(a^2*x^2 + 1))/x^5
```

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)}{x^6}\,dx = \int \frac{\operatorname{asinh}(ax)}{x^6}\,dx$$

```
[In] integrate(asinh(a*x)/x**6,x)
```

```
[Out] Integral(asinh(a*x)/x**6, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.82

$$\int \frac{\operatorname{arcsinh}(ax)}{x^6}\,dx = -\frac{1}{40}\left(3\,a^4\operatorname{arsinh}\left(\frac{1}{a|x|}\right) - \frac{3\,\sqrt{a^2x^2+1}a^2}{x^2} + \frac{2\,\sqrt{a^2x^2+1}}{x^4}\right)a - \frac{\operatorname{arsinh}(ax)}{5\,x^5}$$

```
[In] integrate(arcsinh(a*x)/x^6,x, algorithm="maxima")
```

```
[Out] -1/40*(3*a^4*arcsinh(1/(a*abs(x))) - 3*sqrt(a^2*x^2 + 1)*a^2/x^2 + 2*sqrt(a
^2*x^2 + 1)/x^4)*a - 1/5*arcsinh(a*x)/x^5
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.39

$$\int \frac{\operatorname{arcsinh}(ax)}{x^6}\,dx$$
$$= -\frac{3\,a^6\log\left(\sqrt{a^2x^2+1}+1\right)-3\,a^6\log\left(\sqrt{a^2x^2+1}-1\right)-\frac{2\left(3\,(a^2x^2+1)^{\frac{3}{2}}a^6-5\,\sqrt{a^2x^2+1}a^6\right)}{a^4x^4}}{80\,a}$$
$$-\frac{\log\left(ax+\sqrt{a^2x^2+1}\right)}{5\,x^5}$$

```
[In] integrate(arcsinh(a*x)/x^6,x, algorithm="giac")
```

```
[Out] -1/80*(3*a^6*log(sqrt(a^2*x^2 + 1) + 1) - 3*a^6*log(sqrt(a^2*x^2 + 1) - 1)
- 2*(3*(a^2*x^2 + 1)^(3/2)*a^6 - 5*sqrt(a^2*x^2 + 1)*a^6)/(a^4*x^4))/a - 1/
5*log(a*x + sqrt(a^2*x^2 + 1))/x^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)}{x^6}\,dx = \int \frac{\operatorname{asinh}(a\,x)}{x^6}\,dx$$

```
[In] int(asinh(a*x)/x^6,x)
```

```
[Out] int(asinh(a*x)/x^6, x)
```

3.12 $\int x^4 \mathbf{arcsinh}(ax)^2\,dx$

Optimal result

Integrand size = 10, antiderivative size = 117

$$\begin{aligned}\int x^4\text{arcsinh}(ax)^2\,dx=&\frac{16x}{75a^4}-\frac{8x^3}{225a^2}+\frac{2x^5}{125}-\frac{16\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{75a^5}\\&+\frac{8x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{75a^3}\\&-\frac{2x^4\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{25a}+\frac{1}{5}x^5\text{arcsinh}(ax)^2\end{aligned}$$

```
[Out] 16/75*x/a^4-8/225*x^3/a^2+2/125*x^5+1/5*x^5*arcsinh(a*x)^2-16/75*arcsinh(a*
x)*(a^2*x^2+1)^(1/2)/a^5+8/75*x^2*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a^3-2/25*x
^4*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5776, 5812, 5798, 8, 30}

$$\begin{aligned}\int x^4\text{arcsinh}(ax)^2\,dx=&\frac{16x}{75a^4}-\frac{2x^4\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{25a}-\frac{8x^3}{225a^2}-\frac{16\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{75a^5}\\&+\frac{8x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{75a^3}+\frac{1}{5}x^5\text{arcsinh}(ax)^2+\frac{2x^5}{125}\end{aligned}$$

```
[In] Int[x^4*ArcSinh[a*x]^2,x]
```

```
[Out] (16*x)/(75*a^4) - (8*x^3)/(225*a^2) + (2*x^5)/125 - (16*Sqrt[1 + a^2*x^2]*A
rcSinh[a*x])/(75*a^5) + (8*x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(75*a^3) - (
2*x^4*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(25*a) + (x^5*ArcSinh[a*x]^2)/5
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{5}x^5\text{arcsinh}(ax)^2 - \frac{1}{5}(2a)\int \frac{x^5\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{2x^4\sqrt{1+a^2x^2}\,\text{arcsinh}(ax)}{25a} + \frac{1}{5}x^5\text{arcsinh}(ax)^2 + \frac{2\int x^4\,dx}{25} + \frac{8\int \frac{x^3\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx}{25a} \\
&= \frac{2x^5}{125} + \frac{8x^2\sqrt{1+a^2x^2}\,\text{arcsinh}(ax)}{75a^3} - \frac{2x^4\sqrt{1+a^2x^2}\,\text{arcsinh}(ax)}{25a} \\
&\quad + \frac{1}{5}x^5\text{arcsinh}(ax)^2 - \frac{16\int \frac{x\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx}{75a^3} - \frac{8\int x^2\,dx}{75a^2}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{8x^3}{225a^2} + \frac{2x^5}{125} - \frac{16\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{75a^5} + \frac{8x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{75a^3} \\
&\quad - \frac{2x^4\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{25a} + \frac{1}{5}x^5\text{arcsinh}(ax)^2 + \frac{16\int 1\,dx}{75a^4} \\
&= \frac{16x}{75a^4} - \frac{8x^3}{225a^2} + \frac{2x^5}{125} - \frac{16\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{75a^5} \\
&\quad + \frac{8x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{75a^3} - \frac{2x^4\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{25a} + \frac{1}{5}x^5\text{arcsinh}(ax)^2
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.64

$$
\int x^4\text{arcsinh}(ax)^2\,dx
= \frac{\frac{240x}{a^4} - \frac{40x^3}{a^2} + 18x^5 - \frac{30\sqrt{1+a^2x^2}(8-4a^2x^2+3a^4x^4)\text{arcsinh}(ax)}{a^5} + 225x^5\text{arcsinh}(ax)^2}{1125}
$$

```
[In] Integrate[x^4*ArcSinh[a*x]^2,x]
```

```
[Out] ((240*x)/a^4 - (40*x^3)/a^2 + 18*x^5 - (30*Sqrt[1 + a^2*x^2]*(8 - 4*a^2*x^2
 + 3*a^4*x^4)*ArcSinh[a*x])/a^5 + 225*x^5*ArcSinh[a*x]^2)/1125
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{\frac{a^5x^5\,\text{arcsinh}(ax)^2}{5}-\frac{16\,\text{arcsinh}(ax)\sqrt{a^2x^2+1}}{75}-\frac{2a^4x^4\,\text{arcsinh}(ax)\sqrt{a^2x^2+1}}{25}+\frac{8\,\text{arcsinh}(ax)\sqrt{a^2x^2+1}\,a^2x^2}{75}+\frac{16ax}{75}+\frac{2a^5x^5}{125}-\frac{8a^3x}{225}}{a^5}$
default	$\frac{\frac{a^5x^5\,\text{arcsinh}(ax)^2}{5}-\frac{16\,\text{arcsinh}(ax)\sqrt{a^2x^2+1}}{75}-\frac{2a^4x^4\,\text{arcsinh}(ax)\sqrt{a^2x^2+1}}{25}+\frac{8\,\text{arcsinh}(ax)\sqrt{a^2x^2+1}\,a^2x^2}{75}+\frac{16ax}{75}+\frac{2a^5x^5}{125}-\frac{8a^3x}{225}}{a^5}$

```
[In] int(x^4*arcsinh(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^5*(1/5*a^5*x^5*arcsinh(a*x)^2-16/75*arcsinh(a*x)*(a^2*x^2+1)^(1/2)-2/25
*a^4*x^4*arcsinh(a*x)*(a^2*x^2+1)^(1/2)+8/75*arcsinh(a*x)*(a^2*x^2+1)^(1/2)
*a^2*x^2+16/75*a*x+2/125*a^5*x^5-8/225*a^3*x^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.85

$$\int x^4 \operatorname{arcsinh}(ax)^2\,dx$$
$$= \frac{225\,a^5x^5\log\left(ax+\sqrt{a^2x^2+1}\right)^2 + 18\,a^5x^5 - 40\,a^3x^3 - 30\,(3\,a^4x^4 - 4\,a^2x^2 + 8)\sqrt{a^2x^2+1}\,\log\left(ax+\sqrt{a^2x^2}\right.}{1125\,a^5}$$

```
[In] integrate(x^4*arcsinh(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/1125*(225*a^5*x^5*log(a*x + sqrt(a^2*x^2 + 1))^2 + 18*a^5*x^5 - 40*a^3*x^
3 - 30*(3*a^4*x^4 - 4*a^2*x^2 + 8)*sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2
 + 1)) + 240*a*x)/a^5
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.97

$$\int x^4 \operatorname{arcsinh}(ax)^2\,dx$$
$$= \begin{cases} \frac{x^5\operatorname{asinh}^2(ax)}{5} + \frac{2x^5}{125} - \frac{2x^4\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{25a} - \frac{8x^3}{225a^2} + \frac{8x^2\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{75a^3} + \frac{16x}{75a^4} - \frac{16\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{75a^5} & \text{for } a \neq 0 \\ 0 & \text{otherwis} \end{cases}$$

```
[In] integrate(x**4*asinh(a*x)**2,x)
```

```
[Out] Piecewise((x**5*asinh(a*x)**2/5 + 2*x**5/125 - 2*x**4*sqrt(a**2*x**2 + 1)*a
sinh(a*x)/(25*a) - 8*x**3/(225*a**2) + 8*x**2*sqrt(a**2*x**2 + 1)*asinh(a*x
)/(75*a**3) + 16*x/(75*a**4) - 16*sqrt(a**2*x**2 + 1)*asinh(a*x)/(75*a**5),
 Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.85

$$\int x^4 \operatorname{arcsinh}(ax)^2\,dx$$
$$= \frac{1}{5}\,x^5 \operatorname{arsinh}(ax)^2 - \frac{2}{75}\left(\frac{3\sqrt{a^2x^2+1}x^4}{a^2} - \frac{4\sqrt{a^2x^2+1}x^2}{a^4} + \frac{8\sqrt{a^2x^2+1}}{a^6}\right)a\operatorname{arsinh}(ax)$$
$$+ \frac{2\,(9\,a^4x^5 - 20\,a^2x^3 + 120\,x)}{1125\,a^4}$$

```
[In] integrate(x^4*arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/5*x^5*arcsinh(a*x)^2 - 2/75*(3*sqrt(a^2*x^2 + 1)*x^4/a^2 - 4*sqrt(a^2*x^2
 + 1)*x^2/a^4 + 8*sqrt(a^2*x^2 + 1)/a^6)*a*arcsinh(a*x) + 2/1125*(9*a^4*x^5
 - 20*a^2*x^3 + 120*x)/a^4
```

Giac [F(-2)]

Exception generated.

$$\int x^4 \text{arcsinh}(ax)^2 \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^4*arcsinh(a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \text{arcsinh}(ax)^2 \, dx = \int x^4 \operatorname{asinh}(a\,x)^2 \,dx$$

```
[In] int(x^4*asinh(a*x)^2,x)
```

```
[Out] int(x^4*asinh(a*x)^2, x)
```

3.13 $\int x^3 \text{arcsinh}(ax)^2 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 96

$$\begin{aligned}\int x^3 \text{arcsinh}(ax)^2 \, dx = & -\frac{3x^2}{32a^2} + \frac{x^4}{32} + \frac{3x\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{16a^3} \\ & - \frac{x^3\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{8a} - \frac{3\text{arcsinh}(ax)^2}{32a^4} + \frac{1}{4}x^4\text{arcsinh}(ax)^2\end{aligned}$$

```
[Out] -3/32*x^2/a^2+1/32*x^4-3/32*arcsinh(a*x)^2/a^4+1/4*x^4*arcsinh(a*x)^2+3/16*
x*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a^3-1/8*x^3*arcsinh(a*x)*(a^2*x^2+1)^(1/2)
/a
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5776, 5812, 5783, 30}

$$\begin{aligned}\int x^3 \text{arcsinh}(ax)^2 \, dx = & -\frac{3\text{arcsinh}(ax)^2}{32a^4} - \frac{x^3\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{8a} - \frac{3x^2}{32a^2} \\ & + \frac{3x\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{16a^3} + \frac{1}{4}x^4\text{arcsinh}(ax)^2 + \frac{x^4}{32}\end{aligned}$$

```
[In] Int[x^3*ArcSinh[a*x]^2,x]
```

```
[Out] (-3*x^2)/(32*a^2) + x^4/32 + (3*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(16*a^3)
- (x^3*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(8*a) - (3*ArcSinh[a*x]^2)/(32*a^4)
+ (x^4*ArcSinh[a*x]^2)/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{4}x^4\text{arcsinh}(ax)^2 - \frac{1}{2}a\int \frac{x^4\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{x^3\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{8a} + \frac{1}{4}x^4\text{arcsinh}(ax)^2 + \frac{\int x^3\,dx}{8} + \frac{3\int \frac{x^2\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx}{8a} \\
&= \frac{x^4}{32} + \frac{3x\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{16a^3} - \frac{x^3\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{8a} \\
&\quad + \frac{1}{4}x^4\text{arcsinh}(ax)^2 - \frac{3\int \frac{\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx}{16a^3} - \frac{3\int x\,dx}{16a^2} \\
&= -\frac{3x^2}{32a^2} + \frac{x^4}{32} + \frac{3x\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{16a^3} \\
&\quad - \frac{x^3\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{8a} - \frac{3\text{arcsinh}(ax)^2}{32a^4} + \frac{1}{4}x^4\text{arcsinh}(ax)^2
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.75

$$\int x^3\text{arcsinh}(ax)^2\,dx = \frac{a^2x^2(-3+a^2x^2)-2ax\sqrt{1+a^2x^2}(-3+2a^2x^2)\,\text{arcsinh}(ax)+(-3+8a^4x^4)\,\text{arcsinh}(ax)^2}{32a^4}$$

```
[In] Integrate[x^3*ArcSinh[a*x]^2,x]
```

```
[Out] (a^2*x^2*(-3 + a^2*x^2) - 2*a*x*Sqrt[1 + a^2*x^2]*(-3 + 2*a^2*x^2)*ArcSinh[
a*x] + (-3 + 8*a^4*x^4)*ArcSinh[a*x]^2)/(32*a^4)
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\frac{\frac{a^4x^4\,\text{arcsinh}(ax)^2}{4}-\frac{a^3x^3\,\text{arcsinh}(ax)\sqrt{a^2x^2+1}}{8}+\frac{3\,\text{arcsinh}(ax)\sqrt{a^2x^2+1}\,ax}{16}-\frac{3\,\text{arcsinh}(ax)^2}{32}+\frac{a^4x^4}{32}-\frac{3a^2x^2}{32}-\frac{3}{32}}{a^4}$	87
default	$\frac{\frac{a^4x^4\,\text{arcsinh}(ax)^2}{4}-\frac{a^3x^3\,\text{arcsinh}(ax)\sqrt{a^2x^2+1}}{8}+\frac{3\,\text{arcsinh}(ax)\sqrt{a^2x^2+1}\,ax}{16}-\frac{3\,\text{arcsinh}(ax)^2}{32}+\frac{a^4x^4}{32}-\frac{3a^2x^2}{32}-\frac{3}{32}}{a^4}$	87

```
[In] int(x^3*arcsinh(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*(1/4*a^4*x^4*arcsinh(a*x)^2-1/8*a^3*x^3*arcsinh(a*x)*(a^2*x^2+1)^(1/2
)+3/16*arcsinh(a*x)*(a^2*x^2+1)^(1/2)*a*x-3/32*arcsinh(a*x)^2+1/32*a^4*x^4-
3/32*a^2*x^2-3/32)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.96

$$\int x^3\text{arcsinh}(ax)^2\,dx = \frac{a^4x^4-3\,a^2x^2+(8\,a^4x^4-3)\log\left(ax+\sqrt{a^2x^2+1}\right)^2-2\,(2\,a^3x^3-3\,ax)\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)}{32\,a^4}$$

```
[In] integrate(x^3*arcsinh(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/32*(a^4*x^4 - 3*a^2*x^2 + (8*a^4*x^4 - 3)*log(a*x + sqrt(a^2*x^2 + 1))^2
- 2*(2*a^3*x^3 - 3*a*x)*sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1)))/a^4
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.94

$$\int x^3 \text{arcsinh}(ax)^2\,dx = \begin{cases} \frac{x^4\operatorname{asinh}^2(ax)}{4} + \frac{x^4}{32} - \frac{x^3\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{8a} - \frac{3x^2}{32a^2} + \frac{3x\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{16a^3} - \frac{3\operatorname{asinh}^2(ax)}{32a^4} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

```
[In] integrate(x**3*asinh(a*x)**2,x)
```

```
[Out] Piecewise((x**4*asinh(a*x)**2/4 + x**4/32 - x**3*sqrt(a**2*x**2 + 1)*asinh(
a*x)/(8*a) - 3*x**2/(32*a**2) + 3*x*sqrt(a**2*x**2 + 1)*asinh(a*x)/(16*a**3
) - 3*asinh(a*x)**2/(32*a**4), Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.14

$$\begin{aligned}\int x^3 \text{arcsinh}(ax)^2\,dx &= \frac{1}{4}x^4\operatorname{arsinh}(ax)^2 + \frac{1}{32}\left(\frac{x^4}{a^2} - \frac{3x^2}{a^4} + \frac{3\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{a^6}\right)a^2 \\ &\quad - \frac{1}{16}\left(\frac{2\sqrt{a^2x^2+1}x^3}{a^2} - \frac{3\sqrt{a^2x^2+1}x}{a^4} + \frac{3\operatorname{arsinh}(ax)}{a^5}\right)a\operatorname{arsinh}(ax)\end{aligned}$$

```
[In] integrate(x^3*arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arcsinh(a*x)^2 + 1/32*(x^4/a^2 - 3*x^2/a^4 + 3*log(a*x + sqrt(a^2*x
^2 + 1))^2/a^6)*a^2 - 1/16*(2*sqrt(a^2*x^2 + 1)*x^3/a^2 - 3*sqrt(a^2*x^2 +
1)*x/a^4 + 3*arcsinh(a*x)/a^5)*a*arcsinh(a*x)
```

Giac [F(-2)]

Exception generated.

$$\int x^3 \text{arcsinh}(ax)^2\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3*arcsinh(a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^3\text{arcsinh}(ax)^2\,dx=\int x^3\,\mathrm{asinh}(a\,x)^2\,dx$$

```
[In] int(x^3*asinh(a*x)^2,x)
[Out] int(x^3*asinh(a*x)^2, x)
```

3.14 $\int x^2 \text{arcsinh}(ax)^2\,dx$

Optimal result

Integrand size = 10, antiderivative size = 80

$$\int x^2 \text{arcsinh}(ax)^2\,dx = -\frac{4x}{9a^2}+\frac{2x^3}{27}+\frac{4\sqrt{1+a^2x^2}\,\text{arcsinh}(ax)}{9a^3} - \frac{2x^2\sqrt{1+a^2x^2}\,\text{arcsinh}(ax)}{9a}+\frac{1}{3}x^3\text{arcsinh}(ax)^2$$

```
[Out] -4/9*x/a^2+2/27*x^3+1/3*x^3*arcsinh(a*x)^2+4/9*arcsinh(a*x)*(a^2*x^2+1)^(1/
2)/a^3-2/9*x^2*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5776, 5812, 5798, 8, 30}

$$\int x^2 \text{arcsinh}(ax)^2\,dx = -\frac{2x^2\sqrt{a^2x^2+1}\,\text{arcsinh}(ax)}{9a}-\frac{4x}{9a^2} + \frac{4\sqrt{a^2x^2+1}\,\text{arcsinh}(ax)}{9a^3}+\frac{1}{3}x^3\text{arcsinh}(ax)^2+\frac{2x^3}{27}$$

```
[In] Int[x^2*ArcSinh[a*x]^2,x]
```

```
[Out] (-4*x)/(9*a^2) + (2*x^3)/27 + (4*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(9*a^3) -
(2*x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(9*a) + (x^3*ArcSinh[a*x]^2)/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\operatorname{arcsinh}(ax)^2 - \frac{1}{3}(2a)\int \frac{x^3\operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{2x^2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{9a} + \frac{1}{3}x^3\operatorname{arcsinh}(ax)^2 + \frac{2\int x^2\,dx}{9} + \frac{4\int \frac{x\operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx}{9a} \\
&= \frac{2x^3}{27} + \frac{4\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{9a^3} - \frac{2x^2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{9a} + \frac{1}{3}x^3\operatorname{arcsinh}(ax)^2 - \frac{4\int 1\,dx}{9a^2} \\
&= -\frac{4x}{9a^2} + \frac{2x^3}{27} + \frac{4\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{9a^3} - \frac{2x^2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{9a} + \frac{1}{3}x^3\operatorname{arcsinh}(ax)^2
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.74

$$\int x^2\text{arcsinh}(ax)^2\,dx=\frac{1}{27}\left(2x\left(-\frac{6}{a^2}+x^2\right)-\frac{6(-2+a^2x^2)\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{a^3}\right.$$
$$\left.+\,9x^3\text{arcsinh}(ax)^2\right)$$

```
[In] Integrate[x^2*ArcSinh[a*x]^2,x]
```

```
[Out] (2*x*(-6/a^2 + x^2) - (6*(-2 + a^2*x^2)*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/a^3
 + 9*x^3*ArcSinh[a*x]^2)/27
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$\frac{\frac{a^3x^3\,\text{arcsinh}(ax)^2}{3}+\frac{4\,\text{arcsinh}(ax)\sqrt{a^2x^2+1}}{9}-\frac{2\,\text{arcsinh}(ax)\sqrt{a^2x^2+1}\,a^2x^2}{9}-\frac{4ax}{9}+\frac{2a^3x^3}{27}}{a^3}$	72
default	$\frac{\frac{a^3x^3\,\text{arcsinh}(ax)^2}{3}+\frac{4\,\text{arcsinh}(ax)\sqrt{a^2x^2+1}}{9}-\frac{2\,\text{arcsinh}(ax)\sqrt{a^2x^2+1}\,a^2x^2}{9}-\frac{4ax}{9}+\frac{2a^3x^3}{27}}{a^3}$	72

```
[In] int(x^2*arcsinh(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(1/3*a^3*x^3*arcsinh(a*x)^2+4/9*arcsinh(a*x)*(a^2*x^2+1)^(1/2)-2/9*ar
csinh(a*x)*(a^2*x^2+1)^(1/2)*a^2*x^2-4/9*a*x+2/27*a^3*x^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.02

$$\int x^2\text{arcsinh}(ax)^2\,dx$$
$$=\frac{9\,a^3x^3\log\left(ax+\sqrt{a^2x^2+1}\right)^2+2\,a^3x^3-6\,\sqrt{a^2x^2+1}(a^2x^2-2)\log\left(ax+\sqrt{a^2x^2+1}\right)-12\,ax}{27\,a^3}$$

```
[In] integrate(x^2*arcsinh(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/27*(9*a^3*x^3*log(a*x + sqrt(a^2*x^2 + 1))^2 + 2*a^3*x^3 - 6*sqrt(a^2*x^2
 + 1)*(a^2*x^2 - 2)*log(a*x + sqrt(a^2*x^2 + 1)) - 12*a*x)/a^3
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.95

$$\int x^2\text{arcsinh}(ax)^2\,dx = \begin{cases} \frac{x^3\operatorname{asinh}^2(ax)}{3} + \frac{2x^3}{27} - \frac{2x^2\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{9a} - \frac{4x}{9a^2} + \frac{4\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{9a^3} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

```
[In] integrate(x**2*asinh(a*x)**2,x)
```

```
[Out] Piecewise((x**3*asinh(a*x)**2/3 + 2*x**3/27 - 2*x**2*sqrt(a**2*x**2 + 1)*as
inh(a*x)/(9*a) - 4*x/(9*a**2) + 4*sqrt(a**2*x**2 + 1)*asinh(a*x)/(9*a**3),
Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.88

$$\int x^2\text{arcsinh}(ax)^2\,dx = \frac{1}{3}\,x^3\operatorname{arsinh}(ax)^2 - \frac{2}{9}\,a\left(\frac{\sqrt{a^2x^2+1}x^2}{a^2} - \frac{2\sqrt{a^2x^2+1}}{a^4}\right)\operatorname{arsinh}(ax) + \frac{2\,(a^2x^3-6\,x)}{27\,a^2}$$

```
[In] integrate(x^2*arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arcsinh(a*x)^2 - 2/9*a*(sqrt(a^2*x^2 + 1)*x^2/a^2 - 2*sqrt(a^2*x^2
+ 1)/a^4)*arcsinh(a*x) + 2/27*(a^2*x^3 - 6*x)/a^2
```

Giac [F(-2)]

Exception generated.

$$\int x^2\text{arcsinh}(ax)^2\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2*arcsinh(a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^2\text{arcsinh}(ax)^2\,dx=\int x^2\,\text{asinh}(a\,x)^2\,dx$$

```
[In] int(x^2*asinh(a*x)^2,x)
```

```
[Out] int(x^2*asinh(a*x)^2, x)
```

3.15 $\int x\text{arcsinh}(ax)^2\,dx$

Optimal result

Integrand size = 8, antiderivative size = 59

$$\int x\text{arcsinh}(ax)^2\,dx=\frac{x^2}{4}-\frac{x\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{2a}+\frac{\text{arcsinh}(ax)^2}{4a^2}+\frac{1}{2}x^2\text{arcsinh}(ax)^2$$

```
[Out] 1/4*x^2+1/4*arcsinh(a*x)^2/a^2+1/2*x^2*arcsinh(a*x)^2-1/2*x*arcsinh(a*x)*(a
^2*x^2+1)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5776, 5812, 5783, 30}

$$\int x\text{arcsinh}(ax)^2\,dx=-\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2a}+\frac{\text{arcsinh}(ax)^2}{4a^2}+\frac{1}{2}x^2\text{arcsinh}(ax)^2+\frac{x^2}{4}$$

```
[In] Int[x*ArcSinh[a*x]^2,x]
```

```
[Out] x^2/4 - (x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(2*a) + ArcSinh[a*x]^2/(4*a^2) +
 (x^2*ArcSinh[a*x]^2)/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
```

```
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}x^2\text{arcsinh}(ax)^2 - a\int \frac{x^2\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{x\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{2a} + \frac{1}{2}x^2\text{arcsinh}(ax)^2 + \frac{\int x\,dx}{2} + \frac{\int \frac{\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx}{2a} \\
&= \frac{x^2}{4} - \frac{x\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{2a} + \frac{\text{arcsinh}(ax)^2}{4a^2} + \frac{1}{2}x^2\text{arcsinh}(ax)^2
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int x\text{arcsinh}(ax)^2\,dx = \frac{a^2x^2 - 2ax\sqrt{1+a^2x^2}\text{arcsinh}(ax) + (1+2a^2x^2)\,\text{arcsinh}(ax)^2}{4a^2}$$

```
[In] Integrate[x*ArcSinh[a*x]^2,x]
```

```
[Out] (a^2*x^2 - 2*a*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x] + (1 + 2*a^2*x^2)*ArcSinh[a
*x]^2)/(4*a^2)
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{\frac{\operatorname{arcsinh}(ax)^2(a^2x^2+1)}{2}-\frac{\operatorname{arcsinh}(ax)\sqrt{a^2x^2+1}\,ax}{2}-\frac{\operatorname{arcsinh}(ax)^2}{4}+\frac{a^2x^2}{4}+\frac{1}{4}}{a^2}$	59
default	$\frac{\frac{\operatorname{arcsinh}(ax)^2(a^2x^2+1)}{2}-\frac{\operatorname{arcsinh}(ax)\sqrt{a^2x^2+1}\,ax}{2}-\frac{\operatorname{arcsinh}(ax)^2}{4}+\frac{a^2x^2}{4}+\frac{1}{4}}{a^2}$	59

```
[In] int(x*arcsinh(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(1/2*arcsinh(a*x)^2*(a^2*x^2+1)-1/2*arcsinh(a*x)*(a^2*x^2+1)^(1/2)*a*
x-1/4*arcsinh(a*x)^2+1/4*a^2*x^2+1/4)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.24

$$\int x\operatorname{arcsinh}(ax)^2\,dx = \frac{a^2x^2-2\sqrt{a^2x^2+1}ax\log\left(ax+\sqrt{a^2x^2+1}\right)+(2\,a^2x^2+1)\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{4\,a^2}$$

```
[In] integrate(x*arcsinh(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(a^2*x^2 - 2*sqrt(a^2*x^2 + 1)*a*x*log(a*x + sqrt(a^2*x^2 + 1)) + (2*a^
2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^2)/a^2
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

$$\int x\operatorname{arcsinh}(ax)^2\,dx = \begin{cases} \frac{x^2\operatorname{asinh}^2(ax)}{2}+\frac{x^2}{4}-\frac{x\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{2a}+\frac{\operatorname{asinh}^2(ax)}{4a^2} & \text{for } a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

```
[In] integrate(x*asinh(a*x)**2,x)
```

```
[Out] Piecewise((x**2*asinh(a*x)**2/2 + x**2/4 - x*sqrt(a**2*x**2 + 1)*asinh(a*x)
/(2*a) + asinh(a*x)**2/(4*a**2), Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.37

$$\int x\text{arcsinh}(ax)^2\,dx = \frac{1}{2}\,x^2\,\text{arsinh}\,(ax)^2 + \frac{1}{4}\,a^2\left(\frac{x^2}{a^2} - \frac{\log\left(ax + \sqrt{a^2x^2+1}\right)^2}{a^4}\right) - \frac{1}{2}\,a\left(\frac{\sqrt{a^2x^2+1}x}{a^2} - \frac{\text{arsinh}\,(ax)}{a^3}\right)\text{arsinh}\,(ax)$$

```
[In] integrate(x*arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arcsinh(a*x)^2 + 1/4*a^2*(x^2/a^2 - log(a*x + sqrt(a^2*x^2 + 1))^2/
a^4) - 1/2*a*(sqrt(a^2*x^2 + 1)*x/a^2 - arcsinh(a*x)/a^3)*arcsinh(a*x)
```

Giac [F(-2)]

Exception generated.

$$\int x\text{arcsinh}(ax)^2\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*arcsinh(a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x\text{arcsinh}(ax)^2\,dx = \int x\,\text{asinh}(a\,x)^2\,dx$$

```
[In] int(x*asinh(a*x)^2,x)
```

```
[Out] int(x*asinh(a*x)^2, x)
```

3.16 $\int \operatorname{arcsinh}(ax)^2\,dx$

Optimal result

Integrand size = 6, antiderivative size = 34

$$\int \operatorname{arcsinh}(ax)^2\,dx = 2x - \frac{2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{a} + x\operatorname{arcsinh}(ax)^2$$

```
[Out] 2*x+x*arcsinh(a*x)^2-2*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5772, 5798, 8}

$$\int \operatorname{arcsinh}(ax)^2\,dx = -\frac{2\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)}{a} + x\operatorname{arcsinh}(ax)^2 + 2x$$

```
[In] Int[ArcSinh[a*x]^2,x]
```

```
[Out] 2*x - (2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/a + x*ArcSinh[a*x]^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= x\text{arcsinh}(ax)^2 - (2a)\int \frac{x\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx\\ &= -\frac{2\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{a} + x\text{arcsinh}(ax)^2 + 2\int 1\,dx\\ &= 2x - \frac{2\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{a} + x\text{arcsinh}(ax)^2\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \text{arcsinh}(ax)^2\,dx = 2x - \frac{2\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{a} + x\text{arcsinh}(ax)^2$$

```
[In] Integrate[ArcSinh[a*x]^2,x]
[Out] 2*x - (2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/a + x*ArcSinh[a*x]^2
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$\frac{ax\,\text{arcsinh}(ax)^2-2\,\text{arcsinh}(ax)\sqrt{a^2x^2+1}+2ax}{a}$	36
default	$\frac{ax\,\text{arcsinh}(ax)^2-2\,\text{arcsinh}(ax)\sqrt{a^2x^2+1}+2ax}{a}$	36

```
[In] int(arcsinh(a*x)^2,x,method=_RETURNVERBOSE)
[Out] 1/a*(a*x*arcsinh(a*x)^2-2*arcsinh(a*x)*(a^2*x^2+1)^(1/2)+2*a*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.74

$$\int \operatorname{arcsinh}(ax)^2\,dx = \frac{ax\log\left(ax+\sqrt{a^2x^2+1}\right)^2+2\,ax-2\,\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)}{a}$$

```
[In] integrate(arcsinh(a*x)^2,x, algorithm="fricas")
```

```
[Out] (a*x*log(a*x + sqrt(a^2*x^2 + 1))^2 + 2*a*x - 2*sqrt(a^2*x^2 + 1)*log(a*x +
 sqrt(a^2*x^2 + 1)))/a
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \operatorname{arcsinh}(ax)^2\,dx = \begin{cases} x\operatorname{asinh}^2(ax)+2x-\frac{2\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{a} & \text{for } a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

```
[In] integrate(asinh(a*x)**2,x)
```

```
[Out] Piecewise((x*asinh(a*x)**2 + 2*x - 2*sqrt(a**2*x**2 + 1)*asinh(a*x)/a, Ne(a
, 0)), (0, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \operatorname{arcsinh}(ax)^2\,dx = x\operatorname{arsinh}(ax)^2+2\,x-\frac{2\,\sqrt{a^2x^2+1}\operatorname{arsinh}(ax)}{a}$$

```
[In] integrate(arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] x*arcsinh(a*x)^2 + 2*x - 2*sqrt(a^2*x^2 + 1)*arcsinh(a*x)/a
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.82

$$\int \operatorname{arcsinh}(ax)^2\,dx = x\log\left(ax+\sqrt{a^2x^2+1}\right)^2 + 2\,a\left(\frac{x}{a} - \frac{\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)}{a^2}\right)$$

```
[In] integrate(arcsinh(a*x)^2,x, algorithm="giac")
[Out] x*log(a*x + sqrt(a^2*x^2 + 1))^2 + 2*a*(x/a - sqrt(a^2*x^2 + 1)*log(a*x + s
qrt(a^2*x^2 + 1))/a^2)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{arcsinh}(ax)^2\,dx = \int \operatorname{asinh}(a\,x)^2\,dx$$

```
[In] int(asinh(a*x)^2,x)
[Out] int(asinh(a*x)^2, x)
```

3.17 $\int \frac{\operatorname{arcsinh}(ax)^2}{x}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 60

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x}\,dx = -\frac{1}{3}\operatorname{arcsinh}(ax)^3 + \operatorname{arcsinh}(ax)^2 \log\left(1-e^{2\operatorname{arcsinh}(ax)}\right) + \operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(2,e^{2\operatorname{arcsinh}(ax)}\right) - \frac{1}{2}\operatorname{PolyLog}\left(3,e^{2\operatorname{arcsinh}(ax)}\right)$$

```
[Out] -1/3*arcsinh(a*x)^3+arcsinh(a*x)^2*ln(1-(a*x+(a^2*x^2+1)^(1/2))^2)+arcsinh(
a*x)*polylog(2,(a*x+(a^2*x^2+1)^(1/2))^2)-1/2*polylog(3,(a*x+(a^2*x^2+1)^(1
/2))^2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5775, 3797, 2221, 2611, 2320, 6724}

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x}\,dx = \operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(2,e^{2\operatorname{arcsinh}(ax)}\right) - \frac{1}{2}\operatorname{PolyLog}\left(3,e^{2\operatorname{arcsinh}(ax)}\right) - \frac{1}{3}\operatorname{arcsinh}(ax)^3 + \operatorname{arcsinh}(ax)^2 \log\left(1-e^{2\operatorname{arcsinh}(ax)}\right)$$

```
[In] Int[ArcSinh[a*x]^2/x,x]
```

```
[Out] -1/3*ArcSinh[a*x]^3 + ArcSinh[a*x]^2*Log[1 - E^(2*ArcSinh[a*x])] + ArcSinh[
a*x]*PolyLog[2, E^(2*ArcSinh[a*x])] - PolyLog[3, E^(2*ArcSinh[a*x])]/2
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int x^2\coth(x)\,dx, x, \text{arcsinh}(ax)\right)\\ &= -\frac{1}{3}\text{arcsinh}(ax)^3 - 2\text{Subst}\left(\int \frac{e^{2x}x^2}{1-e^{2x}}\,dx, x, \text{arcsinh}(ax)\right)\end{aligned}$$

$$
\begin{aligned}
&= -\frac{1}{3}\text{arcsinh}(ax)^3 + \text{arcsinh}(ax)^2 \log\left(1 - e^{2\text{arcsinh}(ax)}\right) \\
&\quad - 2\text{Subst}\left(\int x \log\left(1 - e^{2x}\right)\, dx, x, \text{arcsinh}(ax)\right) \\
&= -\frac{1}{3}\text{arcsinh}(ax)^3 + \text{arcsinh}(ax)^2 \log\left(1 - e^{2\text{arcsinh}(ax)}\right) \\
&\quad + \text{arcsinh}(ax)\,\text{PolyLog}\left(2, e^{2\text{arcsinh}(ax)}\right) \\
&\quad - \text{Subst}\left(\int \text{PolyLog}\left(2, e^{2x}\right)\, dx, x, \text{arcsinh}(ax)\right) \\
&= -\frac{1}{3}\text{arcsinh}(ax)^3 + \text{arcsinh}(ax)^2 \log\left(1 - e^{2\text{arcsinh}(ax)}\right) \\
&\quad + \text{arcsinh}(ax)\,\text{PolyLog}\left(2, e^{2\text{arcsinh}(ax)}\right) \\
&\quad - \frac{1}{2}\text{Subst}\left(\int \frac{\text{PolyLog}(2, x)}{x}\, dx, x, e^{2\text{arcsinh}(ax)}\right) \\
&= -\frac{1}{3}\text{arcsinh}(ax)^3 + \text{arcsinh}(ax)^2 \log\left(1 - e^{2\text{arcsinh}(ax)}\right) \\
&\quad + \text{arcsinh}(ax)\,\text{PolyLog}\left(2, e^{2\text{arcsinh}(ax)}\right) - \frac{1}{2}\,\text{PolyLog}\left(3, e^{2\text{arcsinh}(ax)}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$
\begin{aligned}
\int \frac{\text{arcsinh}(ax)^2}{x}\, dx = &-\frac{1}{3}\text{arcsinh}(ax)^3 + \text{arcsinh}(ax)^2 \log\left(1 - e^{2\text{arcsinh}(ax)}\right) \\
&+ \text{arcsinh}(ax)\,\text{PolyLog}\left(2, e^{2\text{arcsinh}(ax)}\right) - \frac{1}{2}\,\text{PolyLog}\left(3, e^{2\text{arcsinh}(ax)}\right)
\end{aligned}
$$

```
[In] Integrate[ArcSinh[a*x]^2/x,x]
```

```
[Out] -1/3*ArcSinh[a*x]^3 + ArcSinh[a*x]^2*Log[1 - E^(2*ArcSinh[a*x])] + ArcSinh[
a*x]*PolyLog[2, E^(2*ArcSinh[a*x])] - PolyLog[3, E^(2*ArcSinh[a*x])]/2
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.52

method	result
derivativedivides	$-\frac{\text{arcsinh}(ax)^3}{3} + \text{arcsinh}\left(ax\right)^2 \ln\left(1 + ax + \sqrt{a^2x^2+1}\right) + 2\ \text{arcsinh}\left(ax\right) \text{polylog}\left(2, -ax -\right.$
default	$-\frac{\text{arcsinh}(ax)^3}{3} + \text{arcsinh}\left(ax\right)^2 \ln\left(1 + ax + \sqrt{a^2x^2+1}\right) + 2\ \text{arcsinh}\left(ax\right) \text{polylog}\left(2, -ax -\right.$

```
[In] int(arcsinh(a*x)^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*arcsinh(a*x)^3+arcsinh(a*x)^2*ln(1+a*x+(a^2*x^2+1)^(1/2))+2*arcsinh(a*
x)*polylog(2,-a*x-(a^2*x^2+1)^(1/2))-2*polylog(3,-a*x-(a^2*x^2+1)^(1/2))+ar
csinh(a*x)^2*ln(1-a*x-(a^2*x^2+1)^(1/2))+2*arcsinh(a*x)*polylog(2,a*x+(a^2*
x^2+1)^(1/2))-2*polylog(3,a*x+(a^2*x^2+1)^(1/2))
```

Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^2}{x}\,dx$$

```
[In] integrate(arcsinh(a*x)^2/x,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x)^2/x, x)
```

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x}\,dx = \int \frac{\operatorname{asinh}^2\left(ax\right)}{x}\,dx$$

```
[In] integrate(asinh(a*x)**2/x,x)
```

```
[Out] Integral(asinh(a*x)**2/x, x)
```

Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^2}{x}\,dx$$

```
[In] integrate(arcsinh(a*x)^2/x,x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^2/x, x)
```

Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^2}{x}\,dx$$

```
[In] integrate(arcsinh(a*x)^2/x,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^2/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x}\,dx = \int \frac{\operatorname{asinh}(a\,x)^2}{x}\,dx$$

```
[In] int(asinh(a*x)^2/x,x)
```

```
[Out] int(asinh(a*x)^2/x, x)
```

3.18 $\int \frac{\text{arcsinh}(ax)^2}{x^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 50

$$\int \frac{\text{arcsinh}(ax)^2}{x^2}\,dx = -\frac{\text{arcsinh}(ax)^2}{x} - 4a\text{arcsinh}(ax)\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right) - 2a\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(ax)}\right) + 2a\,\text{PolyLog}\left(2,e^{\text{arcsinh}(ax)}\right)$$

```
[Out] -arcsinh(a*x)^2/x-4*a*arcsinh(a*x)*arctanh(a*x+(a^2*x^2+1)^(1/2))-2*a*polyl
og(2,-a*x-(a^2*x^2+1)^(1/2))+2*a*polylog(2,a*x+(a^2*x^2+1)^(1/2))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5776, 5816, 4267, 2317, 2438}

$$\int \frac{\text{arcsinh}(ax)^2}{x^2}\,dx = -4a\text{arcsinh}(ax)\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right) - 2a\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(ax)}\right) + 2a\,\text{PolyLog}\left(2,e^{\text{arcsinh}(ax)}\right) - \frac{\text{arcsinh}(ax)^2}{x}$$

```
[In] Int[ArcSinh[a*x]^2/x^2,x]
```

```
[Out] -(ArcSinh[a*x]^2/x) - 4*a*ArcSinh[a*x]*ArcTanh[E^ArcSinh[a*x]] - 2*a*PolyLo
g[2, -E^ArcSinh[a*x]] + 2*a*PolyLog[2, E^ArcSinh[a*x]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\operatorname{arcsinh}(ax)^2}{x} + (2a)\int \frac{\operatorname{arcsinh}(ax)}{x\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{\operatorname{arcsinh}(ax)^2}{x} + (2a)\text{Subst}\Big(\int x\operatorname{csch}(x)\,dx, x, \operatorname{arcsinh}(ax)\Big) \\
&= -\frac{\operatorname{arcsinh}(ax)^2}{x} - 4a\operatorname{arcsinh}(ax)\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(ax)}\right) \\
&\quad - (2a)\text{Subst}\left(\int \log\left(1-e^x\right)\,dx, x, \operatorname{arcsinh}(ax)\right) \\
&\quad + (2a)\text{Subst}\left(\int \log\left(1+e^x\right)\,dx, x, \operatorname{arcsinh}(ax)\right)
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{\operatorname{arcsinh}(ax)^2}{x} - 4a\operatorname{arcsinh}(ax)\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(ax)}\right) \\
&\quad - (2a)\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{\operatorname{arcsinh}(ax)}\right) \\
&\quad + (2a)\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{\operatorname{arcsinh}(ax)}\right) \\
&= -\frac{\operatorname{arcsinh}(ax)^2}{x} - 4a\operatorname{arcsinh}(ax)\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(ax)}\right) \\
&\quad - 2a\,\text{PolyLog}\left(2, -e^{\operatorname{arcsinh}(ax)}\right) + 2a\,\text{PolyLog}\left(2, e^{\operatorname{arcsinh}(ax)}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.50

$$
\begin{aligned}
\int \frac{\operatorname{arcsinh}(ax)^2}{x^2}\,dx = a\Bigg(&-\operatorname{arcsinh}(ax)\left(\frac{\operatorname{arcsinh}(ax)}{ax} - 2\log\left(1-e^{-\operatorname{arcsinh}(ax)}\right)\right. \\
&\left. + 2\log\left(1+e^{-\operatorname{arcsinh}(ax)}\right)\right) + 2\,\text{PolyLog}\left(2, -e^{-\operatorname{arcsinh}(ax)}\right) \\
&- 2\,\text{PolyLog}\left(2, e^{-\operatorname{arcsinh}(ax)}\right)\Bigg)
\end{aligned}
$$

```
[In] Integrate[ArcSinh[a*x]^2/x^2,x]
```

```
[Out] a*(-(ArcSinh[a*x]*(ArcSinh[a*x]/(a*x) - 2*Log[1 - E^(-ArcSinh[a*x])] + 2*Lo
g[1 + E^(-ArcSinh[a*x])])) + 2*PolyLog[2, -E^(-ArcSinh[a*x])] - 2*PolyLog[2
, E^(-ArcSinh[a*x])])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.16

method	result
derivativedivides	$a\left(-\frac{\operatorname{arcsinh}(ax)^2}{ax} - 2\operatorname{arcsinh}(ax)\ln\left(1+ax+\sqrt{a^2x^2+1}\right) - 2\,\mathrm{polylog}\left(2, -ax-\sqrt{a^2x^2}\right.\right.$
default	$a\left(-\frac{\operatorname{arcsinh}(ax)^2}{ax} - 2\operatorname{arcsinh}(ax)\ln\left(1+ax+\sqrt{a^2x^2+1}\right) - 2\,\mathrm{polylog}\left(2, -ax-\sqrt{a^2x^2}\right.\right.$

```
[In] int(arcsinh(a*x)^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] a*(-arcsinh(a*x)^2/a/x-2*arcsinh(a*x)*ln(1+a*x+(a^2*x^2+1)^(1/2))-2*polylog
(2,-a*x-(a^2*x^2+1)^(1/2))+2*arcsinh(a*x)*ln(1-a*x-(a^2*x^2+1)^(1/2))+2*pol
ylog(2,a*x+(a^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^2}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^2}{x^2}\,dx$$

```
[In] integrate(arcsinh(a*x)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x)^2/x^2, x)
```

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^2}\,dx = \int \frac{\operatorname{asinh}^2\left(ax\right)}{x^2}\,dx$$

```
[In] integrate(asinh(a*x)**2/x**2,x)
```

```
[Out] Integral(asinh(a*x)**2/x**2, x)
```

Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^2}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^2}{x^2}\,dx$$

```
[In] integrate(arcsinh(a*x)^2/x^2,x, algorithm="maxima")
```

```
[Out] -log(a*x + sqrt(a^2*x^2 + 1))^2/x + integrate(2*(a^3*x^2 + sqrt(a^2*x^2 + 1
)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))/(a^3*x^4 + a*x^2 + (a^2*x^3 + x)*
sqrt(a^2*x^2 + 1)), x)
```

Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^2}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^2}{x^2}\,dx$$

```
[In] integrate(arcsinh(a*x)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^2/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^2}\,dx=\int \frac{{\mathrm{asinh}\left(a\,x\right)}^2}{x^2} \,d x$$

```
[In] int(asinh(a*x)^2/x^2,x)
```

```
[Out] int(asinh(a*x)^2/x^2, x)
```

3.19 $\int \frac{\mathbf{arcsinh}(ax)^2}{x^3} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 43

$$\int \frac{\text{arcsinh}(ax)^2}{x^3} \, dx = -\frac{a\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{x} - \frac{\text{arcsinh}(ax)^2}{2x^2} + a^2\log(x)$$

```
[Out] -1/2*arcsinh(a*x)^2/x^2+a^2*ln(x)-a*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5776, 5800, 29}

$$\int \frac{\text{arcsinh}(ax)^2}{x^3} \, dx = -\frac{a\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{x} + a^2\log(x) - \frac{\text{arcsinh}(ax)^2}{2x^2}$$

```
[In] Int[ArcSinh[a*x]^2/x^3,x]
```

```
[Out] -((a*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/x) - ArcSinh[a*x]^2/(2*x^2) + a^2*Log[
x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
```

```
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5800

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*
ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[
e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\operatorname{arcsinh}(ax)^2}{2x^2} + a\int \frac{\operatorname{arcsinh}(ax)}{x^2\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{a\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{x} - \frac{\operatorname{arcsinh}(ax)^2}{2x^2} + a^2\int \frac{1}{x}\,dx \\
&= -\frac{a\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{x} - \frac{\operatorname{arcsinh}(ax)^2}{2x^2} + a^2\log(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^3}\,dx = -\frac{a\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{x} - \frac{\operatorname{arcsinh}(ax)^2}{2x^2} + a^2\log(x)$$

```
[In] Integrate[ArcSinh[a*x]^2/x^3,x]
[Out] -((a*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/x) - ArcSinh[a*x]^2/(2*x^2) + a^2*Log[
x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.67

method	result
derivativedivides	$a^2 \left(-2\operatorname{arcsinh}(ax) - \frac{\operatorname{arcsinh}(ax)\left(-2a^2x^2+2ax\sqrt{a^2x^2+1}+\operatorname{arcsinh}(ax)\right)}{2a^2x^2} + \ln\left(\left(ax+\sqrt{a^2x^2+1}\right)^2\right.\right.$
default	$a^2 \left(-2\operatorname{arcsinh}(ax) - \frac{\operatorname{arcsinh}(ax)\left(-2a^2x^2+2ax\sqrt{a^2x^2+1}+\operatorname{arcsinh}(ax)\right)}{2a^2x^2} + \ln\left(\left(ax+\sqrt{a^2x^2+1}\right)^2\right.\right.$

```
[In] int(arcsinh(a*x)^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(-2*arcsinh(a*x)-1/2*arcsinh(a*x)*(-2*a^2*x^2+2*a*x*(a^2*x^2+1)^(1/2)+a
rcsinh(a*x))/a^2/x^2+ln((a*x+(a^2*x^2+1)^(1/2))^2-1))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.56

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^3}\,dx = \frac{2\,a^2x^2\log\left(x\right) - 2\,\sqrt{a^2x^2+1}ax\log\left(ax+\sqrt{a^2x^2+1}\right) - \log\left(ax+\sqrt{a^2x^2+1}\right)^2}{2\,x^2}$$

```
[In] integrate(arcsinh(a*x)^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^2*x^2*log(x) - 2*sqrt(a^2*x^2 + 1)*a*x*log(a*x + sqrt(a^2*x^2 + 1)
) - log(a*x + sqrt(a^2*x^2 + 1))^2)/x^2
```

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^3}\,dx = \int \frac{\operatorname{asinh}^2\left(ax\right)}{x^3}\,dx$$

```
[In] integrate(asinh(a*x)**2/x**3,x)
```

```
[Out] Integral(asinh(a*x)**2/x**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^3}\,dx = a^2\log\left(x\right) - \frac{\sqrt{a^2x^2+1}a\operatorname{arsinh}\left(ax\right)}{x} - \frac{\operatorname{arsinh}\left(ax\right)^2}{2\,x^2}$$

```
[In] integrate(arcsinh(a*x)^2/x^3,x, algorithm="maxima")
```

```
[Out] a^2*log(x) - sqrt(a^2*x^2 + 1)*a*arcsinh(a*x)/x - 1/2*arcsinh(a*x)^2/x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. 2(39) = 78.

Time = 0.30 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.28

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^3}\,dx = -\left(a\log\left(-x|a|+\sqrt{a^2x^2+1}\right)-a\log\left(|x|\right)-\frac{2\,|a|\log\left(ax+\sqrt{a^2x^2+1}\right)}{\left(x|a|-\sqrt{a^2x^2+1}\right)^2-1}\right)a - \frac{\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{2\,x^2}$$

```
[In] integrate(arcsinh(a*x)^2/x^3,x, algorithm="giac")
```

```
[Out] -(a*log(-x*abs(a) + sqrt(a^2*x^2 + 1)) - a*log(abs(x)) - 2*abs(a)*log(a*x +
 sqrt(a^2*x^2 + 1))/((x*abs(a) - sqrt(a^2*x^2 + 1))^2 - 1))*a - 1/2*log(a*x
 + sqrt(a^2*x^2 + 1))^2/x^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^3}\,dx = \int \frac{\operatorname{asinh}(a\,x)^2}{x^3}\,dx$$

```
[In] int(asinh(a*x)^2/x^3,x)
```

```
[Out] int(asinh(a*x)^2/x^3, x)
```

3.20 $\int \frac{\text{arcsinh}(ax)^2}{x^4}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 99

$$\int \frac{\text{arcsinh}(ax)^2}{x^4}\,dx = -\frac{a^2}{3x} - \frac{a\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{3x^2} - \frac{\text{arcsinh}(ax)^2}{3x^3} + \frac{2}{3}a^3\text{arcsinh}(ax)\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right) + \frac{1}{3}a^3\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(ax)}\right) - \frac{1}{3}a^3\,\text{PolyLog}\left(2,e^{\text{arcsinh}(ax)}\right)$$

```
[Out] -1/3*a^2/x-1/3*arcsinh(a*x)^2/x^3+2/3*a^3*arcsinh(a*x)*arctanh(a*x+(a^2*x^2
+1)^(1/2))+1/3*a^3*polylog(2,-a*x-(a^2*x^2+1)^(1/2))-1/3*a^3*polylog(2,a*x+
(a^2*x^2+1)^(1/2))-1/3*a*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {5776, 5809, 5816, 4267, 2317, 2438, 30}

$$\int \frac{\text{arcsinh}(ax)^2}{x^4}\,dx = \frac{2}{3}a^3\text{arcsinh}(ax)\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right) + \frac{1}{3}a^3\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(ax)}\right) - \frac{1}{3}a^3\,\text{PolyLog}\left(2,e^{\text{arcsinh}(ax)}\right) - \frac{a\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{3x^2} - \frac{a^2}{3x} - \frac{\text{arcsinh}(ax)^2}{3x^3}$$

```
[In] Int[ArcSinh[a*x]^2/x^4,x]
```

```
[Out] -1/3*a^2/x - (a*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(3*x^2) - ArcSinh[a*x]^2/(3
*x^3) + (2*a^3*ArcSinh[a*x]*ArcTanh[E^ArcSinh[a*x]])/3 + (a^3*PolyLog[2, -E
^ArcSinh[a*x]])/3 - (a^3*PolyLog[2, E^ArcSinh[a*x]])/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\operatorname{arcsinh}(ax)^2}{3x^3}+\frac{1}{3}(2a)\int\frac{\operatorname{arcsinh}(ax)}{x^3\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{a\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{3x^2}-\frac{\operatorname{arcsinh}(ax)^2}{3x^3}+\frac{1}{3}a^2\int\frac{1}{x^2}\,dx-\frac{1}{3}a^3\int\frac{\operatorname{arcsinh}(ax)}{x\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{a^2}{3x}-\frac{a\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{3x^2}-\frac{\operatorname{arcsinh}(ax)^2}{3x^3}-\frac{1}{3}a^3\text{Subst}\left(\int x\operatorname{csch}(x)\,dx,x,\operatorname{arcsinh}(ax)\right)\\
&= -\frac{a^2}{3x}-\frac{a\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{3x^2}-\frac{\operatorname{arcsinh}(ax)^2}{3x^3}\\
&\quad+\frac{2}{3}a^3\operatorname{arcsinh}(ax)\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(ax)}\right)\\
&\quad+\frac{1}{3}a^3\text{Subst}\left(\int\log\left(1-e^x\right)\,dx,x,\operatorname{arcsinh}(ax)\right)\\
&\quad-\frac{1}{3}a^3\text{Subst}\left(\int\log\left(1+e^x\right)\,dx,x,\operatorname{arcsinh}(ax)\right)\\
&= -\frac{a^2}{3x}-\frac{a\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{3x^2}-\frac{\operatorname{arcsinh}(ax)^2}{3x^3}+\frac{2}{3}a^3\operatorname{arcsinh}(ax)\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(ax)}\right)\\
&\quad+\frac{1}{3}a^3\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{\operatorname{arcsinh}(ax)}\right)-\frac{1}{3}a^3\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{\operatorname{arcsinh}(ax)}\right)\\
&= -\frac{a^2}{3x}-\frac{a\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{3x^2}-\frac{\operatorname{arcsinh}(ax)^2}{3x^3}\\
&\quad+\frac{2}{3}a^3\operatorname{arcsinh}(ax)\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(ax)}\right)\\
&\quad+\frac{1}{3}a^3\,\text{PolyLog}\left(2,-e^{\operatorname{arcsinh}(ax)}\right)-\frac{1}{3}a^3\,\text{PolyLog}\left(2,e^{\operatorname{arcsinh}(ax)}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.26

$$\int \frac{\text{arcsinh}(ax)^2}{x^4}\,dx = \\ -\frac{a^2x^2 + ax\sqrt{1+a^2x^2}\text{arcsinh}(ax) + \text{arcsinh}(ax)^2 + a^3x^3\text{arcsinh}(ax)\log\left(1-e^{-\text{arcsinh}(ax)}\right) - a^3x^3\text{arcsin}}{3x^3}$$

```
[In] Integrate[ArcSinh[a*x]^2/x^4,x]
```

```
[Out] -1/3*(a^2*x^2 + a*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x] + ArcSinh[a*x]^2 + a^3*x
^3*ArcSinh[a*x]*Log[1 - E^(-ArcSinh[a*x])] - a^3*x^3*ArcSinh[a*x]*Log[1 + E
^(-ArcSinh[a*x])] + a^3*x^3*PolyLog[2, -E^(-ArcSinh[a*x])] - a^3*x^3*PolyLo
g[2, E^(-ArcSinh[a*x])])/x^3
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.37

method	result
derivativedivides	$a^3\left(-\frac{\text{arcsinh}(ax)\sqrt{a^2x^2+1}\,ax+\text{arcsinh}(ax)^2+a^2x^2}{3a^3x^3}+\frac{\text{arcsinh}(ax)\ln\left(1+ax+\sqrt{a^2x^2+1}\right)}{3}+\frac{\text{polylog}\left(2,-ax-\sqrt{a^2}\right.}{3}\right.$
default	$a^3\left(-\frac{\text{arcsinh}(ax)\sqrt{a^2x^2+1}\,ax+\text{arcsinh}(ax)^2+a^2x^2}{3a^3x^3}+\frac{\text{arcsinh}(ax)\ln\left(1+ax+\sqrt{a^2x^2+1}\right)}{3}+\frac{\text{polylog}\left(2,-ax-\sqrt{a^2}\right.}{3}\right.$

```
[In] int(arcsinh(a*x)^2/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*(-1/3*(arcsinh(a*x)*(a^2*x^2+1)^(1/2)*a*x+arcsinh(a*x)^2+a^2*x^2)/a^3/x
^3+1/3*arcsinh(a*x)*ln(1+a*x+(a^2*x^2+1)^(1/2))+1/3*polylog(2,-a*x-(a^2*x^2
+1)^(1/2))-1/3*arcsinh(a*x)*ln(1-a*x-(a^2*x^2+1)^(1/2))-1/3*polylog(2,a*x+(
a^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{\text{arcsinh}(ax)^2}{x^4}\,dx = \int \frac{\text{arsinh}\left(ax\right)^2}{x^4}\,dx$$

```
[In] integrate(arcsinh(a*x)^2/x^4,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x)^2/x^4, x)
```

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^4}\,dx = \int \frac{\operatorname{asinh}^2(ax)}{x^4}\,dx$$

```
[In] integrate(asinh(a*x)**2/x**4,x)
```

```
[Out] Integral(asinh(a*x)**2/x**4, x)
```

Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^4}\,dx = \int \frac{\operatorname{arsinh}(ax)^2}{x^4}\,dx$$

```
[In] integrate(arcsinh(a*x)^2/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*log(a*x + sqrt(a^2*x^2 + 1))^2/x^3 + integrate(2/3*(a^3*x^2 + sqrt(a^2
*x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))/(a^3*x^6 + a*x^4 + (a^2*x
^5 + x^3)*sqrt(a^2*x^2 + 1)), x)
```

Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^4}\,dx = \int \frac{\operatorname{arsinh}(ax)^2}{x^4}\,dx$$

```
[In] integrate(arcsinh(a*x)^2/x^4,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^2/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^4}\,dx = \int \frac{\operatorname{asinh}(a\,x)^2}{x^4}\,dx$$

```
[In] int(asinh(a*x)^2/x^4,x)
```

```
[Out] int(asinh(a*x)^2/x^4, x)
```

3.21 $\int \frac{\operatorname{arcsinh}(ax)^2}{x^5}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 85

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^5}\,dx = -\frac{a^2}{12x^2} - \frac{a\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{6x^3} + \frac{a^3\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{3x} - \frac{\operatorname{arcsinh}(ax)^2}{4x^4} - \frac{1}{3}a^4\log(x)$$

```
[Out] -1/12*a^2/x^2-1/4*arcsinh(a*x)^2/x^4-1/3*a^4*ln(x)-1/6*a*arcsinh(a*x)*(a^2*
x^2+1)^(1/2)/x^3+1/3*a^3*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5776, 5809, 5800, 29, 30}

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^5}\,dx = -\frac{1}{3}a^4\log(x) - \frac{a\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)}{6x^3} - \frac{a^2}{12x^2} + \frac{a^3\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)}{3x} - \frac{\operatorname{arcsinh}(ax)^2}{4x^4}$$

```
[In] Int[ArcSinh[a*x]^2/x^5,x]
```

```
[Out] -1/12*a^2/x^2 - (a*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(6*x^3) + (a^3*Sqrt[1 +
a^2*x^2]*ArcSinh[a*x])/(3*x) - ArcSinh[a*x]^2/(4*x^4) - (a^4*Log[x])/3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5800

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*
ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[
e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\operatorname{arcsinh}(ax)^2}{4x^4} + \frac{1}{2}a\int \frac{\operatorname{arcsinh}(ax)}{x^4\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{a\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{6x^3} - \frac{\operatorname{arcsinh}(ax)^2}{4x^4} + \frac{1}{6}a^2\int \frac{1}{x^3}\,dx - \frac{1}{3}a^3\int \frac{\operatorname{arcsinh}(ax)}{x^2\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{a^2}{12x^2} - \frac{a\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{6x^3} + \frac{a^3\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{3x} - \frac{\operatorname{arcsinh}(ax)^2}{4x^4} - \frac{1}{3}a^4\int \frac{1}{x}\,dx \\
&= -\frac{a^2}{12x^2} - \frac{a\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{6x^3} + \frac{a^3\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{3x} - \frac{\operatorname{arcsinh}(ax)^2}{4x^4} - \frac{1}{3}a^4\log(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.75

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^5}\,dx = -\frac{a^2x^2-2ax\sqrt{1+a^2x^2}(-1+2a^2x^2)\operatorname{arcsinh}(ax)+3\operatorname{arcsinh}(ax)^2+4a^4x^4\log(x)}{12x^4}$$

```
[In] Integrate[ArcSinh[a*x]^2/x^5,x]
```

```
[Out] -1/12*(a^2*x^2 - 2*a*x*Sqrt[1 + a^2*x^2]*(-1 + 2*a^2*x^2)*ArcSinh[a*x] + 3*
ArcSinh[a*x]^2 + 4*a^4*x^4*Log[x])/x^4
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.32

method	result
derivativedivides	$a^4 \Bigg(\frac{2\ \operatorname{arcsinh}(ax)}{3} - \frac{-4a^3x^3\operatorname{arcsinh}(ax)\sqrt{a^2x^2+1}+4a^4x^4\operatorname{arcsinh}(ax)+2\ \operatorname{arcsinh}(ax)\sqrt{a^2x^2+1}\,ax+3\operatorname{arcsinh}(ax)^2}{12a^4x^4}$
default	$a^4 \Bigg(\frac{2\ \operatorname{arcsinh}(ax)}{3} - \frac{-4a^3x^3\operatorname{arcsinh}(ax)\sqrt{a^2x^2+1}+4a^4x^4\operatorname{arcsinh}(ax)+2\ \operatorname{arcsinh}(ax)\sqrt{a^2x^2+1}\,ax+3\operatorname{arcsinh}(ax)^2}{12a^4x^4}$

```
[In] int(arcsinh(a*x)^2/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] a^4*(2/3*arcsinh(a*x)-1/12*(-4*a^3*x^3*arcsinh(a*x)*(a^2*x^2+1)^(1/2)+4*a^4
*x^4*arcsinh(a*x)+2*arcsinh(a*x)*(a^2*x^2+1)^(1/2)*a*x+3*arcsinh(a*x)^2+a^2
*x^2)/a^4/x^4-1/3*ln((a*x+(a^2*x^2+1)^(1/2))^2-1))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^5}\,dx = -\frac{4\,a^4x^4\log(x)+a^2x^2-2\,(2\,a^3x^3-ax)\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)+3\,\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{12\,x^4}$$

```
[In] integrate(arcsinh(a*x)^2/x^5,x, algorithm="fricas")
```

```
[Out] -1/12*(4*a^4*x^4*log(x) + a^2*x^2 - 2*(2*a^3*x^3 - a*x)*sqrt(a^2*x^2 + 1)*l
og(a*x + sqrt(a^2*x^2 + 1)) + 3*log(a*x + sqrt(a^2*x^2 + 1))^2)/x^4
```

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^5}\,dx = \int \frac{\operatorname{asinh}^2(ax)}{x^5}\,dx$$

```
[In] integrate(asinh(a*x)**2/x**5,x)
```

```
[Out] Integral(asinh(a*x)**2/x**5, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.84

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^5}\,dx = -\frac{1}{12}\left(4\,a^2\log(x) + \frac{1}{x^2}\right)a^2 + \frac{1}{6}\left(\frac{2\sqrt{a^2x^2+1}a^2}{x} - \frac{\sqrt{a^2x^2+1}}{x^3}\right)a\,\operatorname{arsinh}(ax) - \frac{\operatorname{arsinh}(ax)^2}{4\,x^4}$$

```
[In] integrate(arcsinh(a*x)^2/x^5,x, algorithm="maxima")
```

```
[Out] -1/12*(4*a^2*log(x) + 1/x^2)*a^2 + 1/6*(2*sqrt(a^2*x^2 + 1)*a^2/x - sqrt(a^
2*x^2 + 1)/x^3)*a*arcsinh(a*x) - 1/4*arcsinh(a*x)^2/x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. 2(71) = 142.

Time = 0.34 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.74

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^5}\,dx =$$

$$-\frac{1}{12}\left(2\,a^3\log\left(x^2\right) - 4\,a^3\log\left(-x|a| + \sqrt{a^2x^2+1}\right) - \frac{8\left(3\left(x|a| - \sqrt{a^2x^2+1}\right)^2 - 1\right)a^2|a|\log\left(ax + \sqrt{a^2x\right.}{\left(\left(x|a| - \sqrt{a^2x^2+1}\right)^2 - 1\right)^3}\right. - \frac{\log\left(ax + \sqrt{a^2x^2+1}\right)^2}{4\,x^4}$$

```
[In] integrate(arcsinh(a*x)^2/x^5,x, algorithm="giac")
```

```
[Out] -1/12*(2*a^3*log(x^2) - 4*a^3*log(-x*abs(a) + sqrt(a^2*x^2 + 1)) - 8*(3*(x*
abs(a) - sqrt(a^2*x^2 + 1))^2 - 1)*a^2*abs(a)*log(a*x + sqrt(a^2*x^2 + 1))/
((x*abs(a) - sqrt(a^2*x^2 + 1))^2 - 1)^3 - (2*a^3*x^2 - a)/x^2)*a - 1/4*log
(a*x + sqrt(a^2*x^2 + 1))^2/x^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^2}{x^5}\,dx = \int \frac{{\mathrm{asinh}\left(a\,x\right)}^2}{x^5} \,d x$$

```
[In] int(asinh(a*x)^2/x^5,x)
```

```
[Out] int(asinh(a*x)^2/x^5, x)
```

3.22 $\int x^4\textbf{arcsinh}(ax)^3\,dx$

Optimal result

Integrand size = 10, antiderivative size = 195

$$\begin{aligned}\int x^4\text{arcsinh}(ax)^3\,dx = & -\frac{298\sqrt{1+a^2x^2}}{375a^5}+\frac{76(1+a^2x^2)^{3/2}}{1125a^5}-\frac{6(1+a^2x^2)^{5/2}}{625a^5}\\ & +\frac{16x\text{arcsinh}(ax)}{25a^4}-\frac{8x^3\text{arcsinh}(ax)}{75a^2}+\frac{6}{125}x^5\text{arcsinh}(ax)\\ & -\frac{8\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{25a^5}+\frac{4x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{25a^3}\\ & -\frac{3x^4\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{25a}+\frac{1}{5}x^5\text{arcsinh}(ax)^3\end{aligned}$$

```
[Out] 76/1125*(a^2*x^2+1)^(3/2)/a^5-6/625*(a^2*x^2+1)^(5/2)/a^5+16/25*x*arcsinh(a
*x)/a^4-8/75*x^3*arcsinh(a*x)/a^2+6/125*x^5*arcsinh(a*x)+1/5*x^5*arcsinh(a*
x)^3-298/375*(a^2*x^2+1)^(1/2)/a^5-8/25*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)/a^
5+4/25*x^2*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)/a^3-3/25*x^4*arcsinh(a*x)^2*(a^
2*x^2+1)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used

$= \{5776, 5812, 5798, 5772, 267, 272, 45\}$

$$\int x^4 \text{arcsinh}(ax)^3\,dx = \frac{16x\text{arcsinh}(ax)}{25a^4} - \frac{8x^3\text{arcsinh}(ax)}{75a^2} - \frac{3x^4\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{25a} - \frac{8\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{25a^5} - \frac{6(a^2x^2+1)^{5/2}}{625a^5} + \frac{76(a^2x^2+1)^{3/2}}{1125a^5} - \frac{298\sqrt{a^2x^2+1}}{375a^5} + \frac{4x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{25a^3} + \frac{1}{5}x^5\text{arcsinh}(ax)^3 + \frac{6}{125}x^5\text{arcsinh}(ax)$$

```
[In] Int[x^4*ArcSinh[a*x]^3,x]
```

```
[Out] (-298*Sqrt[1 + a^2*x^2])/(375*a^5) + (76*(1 + a^2*x^2)^(3/2))/(1125*a^5) -
(6*(1 + a^2*x^2)^(5/2))/(625*a^5) + (16*x*ArcSinh[a*x])/(25*a^4) - (8*x^3*A
rcSinh[a*x])/(75*a^2) + (6*x^5*ArcSinh[a*x])/125 - (8*Sqrt[1 + a^2*x^2]*Arc
Sinh[a*x]^2)/(25*a^5) + (4*x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(25*a^3) -
 (3*x^4*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(25*a) + (x^5*ArcSinh[a*x]^3)/5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
```

```
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{5}x^5\text{arcsinh}(ax)^3 - \frac{1}{5}(3a)\int \frac{x^5\text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{3x^4\sqrt{1+a^2x^2}\,\text{arcsinh}(ax)^2}{25a} + \frac{1}{5}x^5\text{arcsinh}(ax)^3 \\
&\quad + \frac{6}{25}\int x^4\text{arcsinh}(ax)\,dx + \frac{12\int \frac{x^3\text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx}{25a} \\
&= \frac{6}{125}x^5\text{arcsinh}(ax) + \frac{4x^2\sqrt{1+a^2x^2}\,\text{arcsinh}(ax)^2}{25a^3} \\
&\quad - \frac{3x^4\sqrt{1+a^2x^2}\,\text{arcsinh}(ax)^2}{25a} + \frac{1}{5}x^5\text{arcsinh}(ax)^3 - \frac{8\int \frac{x\,\text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx}{25a^3} \\
&\quad - \frac{8\int x^2\text{arcsinh}(ax)\,dx}{25a^2} - \frac{1}{125}(6a)\int \frac{x^5}{\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{8x^3\text{arcsinh}(ax)}{75a^2} + \frac{6}{125}x^5\text{arcsinh}(ax) - \frac{8\sqrt{1+a^2x^2}\,\text{arcsinh}(ax)^2}{25a^5} \\
&\quad + \frac{4x^2\sqrt{1+a^2x^2}\,\text{arcsinh}(ax)^2}{25a^3} - \frac{3x^4\sqrt{1+a^2x^2}\,\text{arcsinh}(ax)^2}{25a} + \frac{1}{5}x^5\text{arcsinh}(ax)^3 \\
&\quad + \frac{16\int \text{arcsinh}(ax)\,dx}{25a^4} + \frac{8\int \frac{x^3}{\sqrt{1+a^2x^2}}\,dx}{75a} - \frac{1}{125}(3a)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+a^2x}}\,dx, x, x^2\right)
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{16x\text{arcsinh}(ax)}{25a^4} - \frac{8x^3\text{arcsinh}(ax)}{75a^2} + \frac{6}{125}x^5\text{arcsinh}(ax) - \frac{8\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{25a^5} \\
&\quad + \frac{4x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{25a^3} - \frac{3x^4\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{25a} \\
&\quad + \frac{1}{5}x^5\text{arcsinh}(ax)^3 - \frac{16\int \frac{x}{\sqrt{1+a^2x^2}}\,dx}{25a^3} + \frac{4\text{Subst}\left(\int \frac{x}{\sqrt{1+a^2x}}\,dx, x, x^2\right)}{75a} \\
&\quad - \frac{1}{125}(3a)\text{Subst}\left(\int \left(\frac{1}{a^4\sqrt{1+a^2x}} - \frac{2\sqrt{1+a^2x}}{a^4} + \frac{(1+a^2x)^{3/2}}{a^4}\right)dx, x, x^2\right) \\
&= -\frac{86\sqrt{1+a^2x^2}}{125a^5} + \frac{4(1+a^2x^2)^{3/2}}{125a^5} - \frac{6(1+a^2x^2)^{5/2}}{625a^5} + \frac{16x\text{arcsinh}(ax)}{25a^4} \\
&\quad - \frac{8x^3\text{arcsinh}(ax)}{75a^2} + \frac{6}{125}x^5\text{arcsinh}(ax) - \frac{8\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{25a^5} \\
&\quad + \frac{4x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{25a^3} - \frac{3x^4\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{25a} \\
&\quad + \frac{1}{5}x^5\text{arcsinh}(ax)^3 + \frac{4\text{Subst}\left(\int \left(-\frac{1}{a^2\sqrt{1+a^2x}} + \frac{\sqrt{1+a^2x}}{a^2}\right)dx, x, x^2\right)}{75a} \\
&= -\frac{298\sqrt{1+a^2x^2}}{375a^5} + \frac{76(1+a^2x^2)^{3/2}}{1125a^5} - \frac{6(1+a^2x^2)^{5/2}}{625a^5} + \frac{16x\text{arcsinh}(ax)}{25a^4} \\
&\quad - \frac{8x^3\text{arcsinh}(ax)}{75a^2} + \frac{6}{125}x^5\text{arcsinh}(ax) - \frac{8\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{25a^5} \\
&\quad + \frac{4x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{25a^3} - \frac{3x^4\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{25a} + \frac{1}{5}x^5\text{arcsinh}(ax)^3
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.62

$$
\begin{aligned}
&\int x^4\text{arcsinh}(ax)^3\,dx \\
&= \frac{-2\sqrt{1+a^2x^2}(2072-136a^2x^2+27a^4x^4)+30ax(120-20a^2x^2+9a^4x^4)\,\text{arcsinh}(ax)-225\sqrt{1+a^2x^2}(8}{5625a^5}
\end{aligned}
$$

```
[In] Integrate[x^4*ArcSinh[a*x]^3,x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2]*(2072 - 136*a^2*x^2 + 27*a^4*x^4) + 30*a*x*(120 - 20*
a^2*x^2 + 9*a^4*x^4)*ArcSinh[a*x] - 225*Sqrt[1 + a^2*x^2]*(8 - 4*a^2*x^2 +
3*a^4*x^4)*ArcSinh[a*x]^2 + 1125*a^5*x^5*ArcSinh[a*x]^3)/(5625*a^5)
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{\frac{a^5x^5\operatorname{arcsinh}(ax)^3}{5}-\frac{8\operatorname{arcsinh}(ax)^2\sqrt{a^2x^2+1}}{25}-\frac{3a^4x^4\operatorname{arcsinh}(ax)^2\sqrt{a^2x^2+1}}{25}+\frac{4a^2x^2\operatorname{arcsinh}(ax)^2\sqrt{a^2x^2+1}}{25}+\frac{16ax\operatorname{arcsinh}(ax)}{25}-41}{a^5}$
default	$\frac{\frac{a^5x^5\operatorname{arcsinh}(ax)^3}{5}-\frac{8\operatorname{arcsinh}(ax)^2\sqrt{a^2x^2+1}}{25}-\frac{3a^4x^4\operatorname{arcsinh}(ax)^2\sqrt{a^2x^2+1}}{25}+\frac{4a^2x^2\operatorname{arcsinh}(ax)^2\sqrt{a^2x^2+1}}{25}+\frac{16ax\operatorname{arcsinh}(ax)}{25}-41}{a^5}$

```
[In] int(x^4*arcsinh(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^5*(1/5*a^5*x^5*arcsinh(a*x)^3-8/25*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)-3/2
5*a^4*x^4*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)+4/25*a^2*x^2*arcsinh(a*x)^2*(a^2
*x^2+1)^(1/2)+16/25*a*x*arcsinh(a*x)-4144/5625*(a^2*x^2+1)^(1/2)+6/125*a^5*
x^5*arcsinh(a*x)-6/625*a^4*x^4*(a^2*x^2+1)^(1/2)+272/5625*a^2*x^2*(a^2*x^2+
1)^(1/2)-8/75*a^3*x^3*arcsinh(a*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.77

$$\int x^4\operatorname{arcsinh}(ax)^3\,dx$$
$$=\frac{1125\,a^5x^5\log\left(ax+\sqrt{a^2x^2+1}\right)^3-225\,(3\,a^4x^4-4\,a^2x^2+8)\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)^2+30\,(9\,a^5x}{5625\,a^5}$$

```
[In] integrate(x^4*arcsinh(a*x)^3,x, algorithm="fricas")
```

```
[Out] 1/5625*(1125*a^5*x^5*log(a*x + sqrt(a^2*x^2 + 1))^3 - 225*(3*a^4*x^4 - 4*a^
2*x^2 + 8)*sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^2 + 30*(9*a^5*x^5
 - 20*a^3*x^3 + 120*a*x)*log(a*x + sqrt(a^2*x^2 + 1)) - 2*(27*a^4*x^4 - 136
*a^2*x^2 + 2072)*sqrt(a^2*x^2 + 1))/a^5
```

Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.01

$$\int x^4\operatorname{arcsinh}(ax)^3\,dx$$
$$=\begin{cases}\frac{x^5\operatorname{asinh}^3(ax)}{5}+\frac{6x^5\operatorname{asinh}(ax)}{125}-\frac{3x^4\sqrt{a^2x^2+1}\operatorname{asinh}^2(ax)}{25a}-\frac{6x^4\sqrt{a^2x^2+1}}{625a}-\frac{8x^3\operatorname{asinh}(ax)}{75a^2}+\frac{4x^2\sqrt{a^2x^2+1}\operatorname{asinh}^2(ax)}{25a^3}+\frac{272x^2\sqrt{a}}{5625}\\0\end{cases}$$

```
[In] integrate(x**4*asinh(a*x)**3,x)
```

```
[Out] Piecewise((x**5*asinh(a*x)**3/5 + 6*x**5*asinh(a*x)/125 - 3*x**4*sqrt(a**2*
x**2 + 1)*asinh(a*x)**2/(25*a) - 6*x**4*sqrt(a**2*x**2 + 1)/(625*a) - 8*x**
3*asinh(a*x)/(75*a**2) + 4*x**2*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/(25*a**3)
 + 272*x**2*sqrt(a**2*x**2 + 1)/(5625*a**3) + 16*x*asinh(a*x)/(25*a**4) - 8
*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/(25*a**5) - 4144*sqrt(a**2*x**2 + 1)/(56
25*a**5), Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.85

$$\int x^4 \text{arcsinh}(ax)^3\,dx$$
$$= \frac{1}{5}\,x^5\,\text{arsinh}\,(ax)^3 - \frac{1}{25}\left(\frac{3\,\sqrt{a^2x^2+1}x^4}{a^2} - \frac{4\,\sqrt{a^2x^2+1}x^2}{a^4} + \frac{8\,\sqrt{a^2x^2+1}}{a^6}\right)a\,\text{arsinh}\,(ax)^2$$
$$- \frac{2}{5625}\,a\Bigg(\frac{27\,\sqrt{a^2x^2+1}a^2x^4 - 136\,\sqrt{a^2x^2+1}x^2 + \frac{2072\,\sqrt{a^2x^2+1}}{a^2}}{a^4} - \frac{15\,(9\,a^4x^5 - 20\,a^2x^3 + 120\,x)\,\text{arsinh}\,(a}{a^5}$$

```
[In] integrate(x^4*arcsinh(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/5*x^5*arcsinh(a*x)^3 - 1/25*(3*sqrt(a^2*x^2 + 1)*x^4/a^2 - 4*sqrt(a^2*x^2
 + 1)*x^2/a^4 + 8*sqrt(a^2*x^2 + 1)/a^6)*a*arcsinh(a*x)^2 - 2/5625*a*((27*s
qrt(a^2*x^2 + 1)*a^2*x^4 - 136*sqrt(a^2*x^2 + 1)*x^2 + 2072*sqrt(a^2*x^2 +
1)/a^2)/a^4 - 15*(9*a^4*x^5 - 20*a^2*x^3 + 120*x)*arcsinh(a*x)/a^5)
```

Giac [F(-2)]

Exception generated.

$$\int x^4 \text{arcsinh}(ax)^3\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^4*arcsinh(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \text{arcsinh}(ax)^3 \, dx = \int x^4 \, \text{asinh}(a\,x)^3 \, dx$$

```
[In] int(x^4*asinh(a*x)^3,x)
```

```
[Out] int(x^4*asinh(a*x)^3, x)
```

3.23 $\int x^3 \text{arcsinh}(ax)^3\,dx$

Optimal result

Integrand size = 10, antiderivative size = 163

$$\int x^3\text{arcsinh}(ax)^3\,dx=\frac{45x\sqrt{1+a^2x^2}}{256a^3}-\frac{3x^3\sqrt{1+a^2x^2}}{128a}-\frac{45\text{arcsinh}(ax)}{256a^4}-\frac{9x^2\text{arcsinh}(ax)}{32a^2}+\frac{3}{32}x^4\text{arcsinh}(ax)+\frac{9x\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{32a^3}-\frac{3x^3\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{16a}-\frac{3\text{arcsinh}(ax)^3}{32a^4}+\frac{1}{4}x^4\text{arcsinh}(ax)^3$$

```
[Out] -45/256*arcsinh(a*x)/a^4-9/32*x^2*arcsinh(a*x)/a^2+3/32*x^4*arcsinh(a*x)-3/
32*arcsinh(a*x)^3/a^4+1/4*x^4*arcsinh(a*x)^3+45/256*x*(a^2*x^2+1)^(1/2)/a^3
-3/128*x^3*(a^2*x^2+1)^(1/2)/a+9/32*x*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)/a^3-
3/16*x^3*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5776, 5812, 5783, 327, 221}

$$\int x^3\text{arcsinh}(ax)^3\,dx=-\frac{3\text{arcsinh}(ax)^3}{32a^4}-\frac{45\text{arcsinh}(ax)}{256a^4}-\frac{9x^2\text{arcsinh}(ax)}{32a^2}-\frac{3x^3\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{16a}-\frac{3x^3\sqrt{a^2x^2+1}}{128a}+\frac{9x\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{32a^3}+\frac{45x\sqrt{a^2x^2+1}}{256a^3}+\frac{1}{4}x^4\text{arcsinh}(ax)^3+\frac{3}{32}x^4\text{arcsinh}(ax)$$

```
[In] Int[x^3*ArcSinh[a*x]^3,x]
```

```
[Out] (45*x*Sqrt[1 + a^2*x^2])/(256*a^3) - (3*x^3*Sqrt[1 + a^2*x^2])/(128*a) - (4
5*ArcSinh[a*x])/(256*a^4) - (9*x^2*ArcSinh[a*x])/(32*a^2) + (3*x^4*ArcSinh[
a*x])/32 + (9*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(32*a^3) - (3*x^3*Sqrt[1
+ a^2*x^2]*ArcSinh[a*x]^2)/(16*a) - (3*ArcSinh[a*x]^3)/(32*a^4) + (x^4*ArcS
inh[a*x]^3)/4
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{4}x^4\text{arcsinh}(ax)^3 - \frac{1}{4}(3a)\int \frac{x^4\text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{3x^3\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{16a} + \frac{1}{4}x^4\text{arcsinh}(ax)^3 \\
&\quad + \frac{3}{8}\int x^3\text{arcsinh}(ax)\,dx + \frac{9\int \frac{x^2\text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx}{16a} \\
&= \frac{3}{32}x^4\text{arcsinh}(ax) + \frac{9x\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{32a^3} - \frac{3x^3\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{16a} \\
&\quad + \frac{1}{4}x^4\text{arcsinh}(ax)^3 - \frac{9\int \frac{\text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx}{32a^3} - \frac{9\int x\text{arcsinh}(ax)\,dx}{16a^2} \\
&\quad - \frac{1}{32}(3a)\int \frac{x^4}{\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{3x^3\sqrt{1+a^2x^2}}{128a} - \frac{9x^2\text{arcsinh}(ax)}{32a^2} + \frac{3}{32}x^4\text{arcsinh}(ax) \\
&\quad + \frac{9x\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{32a^3} - \frac{3x^3\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{16a} \\
&\quad - \frac{3\text{arcsinh}(ax)^3}{32a^4} + \frac{1}{4}x^4\text{arcsinh}(ax)^3 + \frac{9\int \frac{x^2}{\sqrt{1+a^2x^2}}\,dx}{128a} + \frac{9\int \frac{x^2}{\sqrt{1+a^2x^2}}\,dx}{32a} \\
&= \frac{45x\sqrt{1+a^2x^2}}{256a^3} - \frac{3x^3\sqrt{1+a^2x^2}}{128a} - \frac{9x^2\text{arcsinh}(ax)}{32a^2} + \frac{3}{32}x^4\text{arcsinh}(ax) \\
&\quad + \frac{9x\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{32a^3} - \frac{3x^3\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{16a} \\
&\quad - \frac{3\text{arcsinh}(ax)^3}{32a^4} + \frac{1}{4}x^4\text{arcsinh}(ax)^3 - \frac{9\int \frac{1}{\sqrt{1+a^2x^2}}\,dx}{256a^3} - \frac{9\int \frac{1}{\sqrt{1+a^2x^2}}\,dx}{64a^3} \\
&= \frac{45x\sqrt{1+a^2x^2}}{256a^3} - \frac{3x^3\sqrt{1+a^2x^2}}{128a} - \frac{45\text{arcsinh}(ax)}{256a^4} \\
&\quad - \frac{9x^2\text{arcsinh}(ax)}{32a^2} + \frac{3}{32}x^4\text{arcsinh}(ax) + \frac{9x\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{32a^3} \\
&\quad - \frac{3x^3\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{16a} - \frac{3\text{arcsinh}(ax)^3}{32a^4} + \frac{1}{4}x^4\text{arcsinh}(ax)^3
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.67

$$\int x^3 \text{arcsinh}(ax)^3\,dx$$
$$= \frac{3ax(15-2a^2x^2)\sqrt{1+a^2x^2}+3(-15-24a^2x^2+8a^4x^4)\,\text{arcsinh}(ax)-24ax\sqrt{1+a^2x^2}(-3+2a^2x^2)\,\text{arcsin}}{256a^4}$$

```
[In] Integrate[x^3*ArcSinh[a*x]^3,x]
```

```
[Out] (3*a*x*(15 - 2*a^2*x^2)*Sqrt[1 + a^2*x^2] + 3*(-15 - 24*a^2*x^2 + 8*a^4*x^4
)*ArcSinh[a*x] - 24*a*x*Sqrt[1 + a^2*x^2]*(-3 + 2*a^2*x^2)*ArcSinh[a*x]^2 +
 8*(-3 + 8*a^4*x^4)*ArcSinh[a*x]^3)/(256*a^4)
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.87

method	result
derivativedivides	$\frac{\frac{a^4x^4\,\text{arcsinh}(ax)^3}{4}-\frac{3a^3x^3\,\text{arcsinh}(ax)^2\sqrt{a^2x^2+1}}{16}+\frac{9\,\text{arcsinh}(ax)^2\sqrt{a^2x^2+1}\,ax}{32}-\frac{3\,\text{arcsinh}(ax)^3}{32}+\frac{3a^4x^4\,\text{arcsinh}(ax)}{32}-\frac{3a^3x^3\sqrt{a^2x^2}}{128}}{a^4}$
default	$\frac{\frac{a^4x^4\,\text{arcsinh}(ax)^3}{4}-\frac{3a^3x^3\,\text{arcsinh}(ax)^2\sqrt{a^2x^2+1}}{16}+\frac{9\,\text{arcsinh}(ax)^2\sqrt{a^2x^2+1}\,ax}{32}-\frac{3\,\text{arcsinh}(ax)^3}{32}+\frac{3a^4x^4\,\text{arcsinh}(ax)}{32}-\frac{3a^3x^3\sqrt{a^2x^2}}{128}}{a^4}$

```
[In] int(x^3*arcsinh(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*(1/4*a^4*x^4*arcsinh(a*x)^3-3/16*a^3*x^3*arcsinh(a*x)^2*(a^2*x^2+1)^(
1/2)+9/32*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)*a*x-3/32*arcsinh(a*x)^3+3/32*a^4
*x^4*arcsinh(a*x)-3/128*a^3*x^3*(a^2*x^2+1)^(1/2)+45/256*a*x*(a^2*x^2+1)^(1
/2)+27/256*arcsinh(a*x)-9/32*(a^2*x^2+1)*arcsinh(a*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.87

$$\int x^3 \text{arcsinh}(ax)^3\,dx$$
$$= \frac{8\,(8\,a^4x^4-3)\log\left(ax+\sqrt{a^2x^2+1}\right)^3-24\,(2\,a^3x^3-3\,ax)\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)^2+3\,(8\,a^4x^4-2}{256\,a^4}$$

```
[In] integrate(x^3*arcsinh(a*x)^3,x, algorithm="fricas")
```

```
[Out] 1/256*(8*(8*a^4*x^4 - 3)*log(a*x + sqrt(a^2*x^2 + 1))^3 - 24*(2*a^3*x^3 - 3
*a*x)*sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^2 + 3*(8*a^4*x^4 - 24*
a^2*x^2 - 15)*log(a*x + sqrt(a^2*x^2 + 1)) - 3*(2*a^3*x^3 - 15*a*x)*sqrt(a^
2*x^2 + 1))/a^4
```

Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.98

$$\int x^3 \operatorname{arcsinh}(ax)^3\,dx$$

$$= \begin{cases} \frac{x^4 \operatorname{asinh}^3(ax)}{4} + \frac{3x^4 \operatorname{asinh}(ax)}{32} - \frac{3x^3\sqrt{a^2x^2+1}\operatorname{asinh}^2(ax)}{16a} - \frac{3x^3\sqrt{a^2x^2+1}}{128a} - \frac{9x^2\operatorname{asinh}(ax)}{32a^2} + \frac{9x\sqrt{a^2x^2+1}\operatorname{asinh}^2(ax)}{32a^3} + \frac{45x\sqrt{a^2}}{256} \\ 0 \end{cases}$$

```
[In] integrate(x**3*asinh(a*x)**3,x)
```

```
[Out] Piecewise((x**4*asinh(a*x)**3/4 + 3*x**4*asinh(a*x)/32 - 3*x**3*sqrt(a**2*x
**2 + 1)*asinh(a*x)**2/(16*a) - 3*x**3*sqrt(a**2*x**2 + 1)/(128*a) - 9*x**2
*asinh(a*x)/(32*a**2) + 9*x*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/(32*a**3) + 4
5*x*sqrt(a**2*x**2 + 1)/(256*a**3) - 3*asinh(a*x)**3/(32*a**4) - 45*asinh(a
*x)/(256*a**4), Ne(a, 0)), (0, True))
```

Maxima [F]

$$\int x^3 \operatorname{arcsinh}(ax)^3\,dx = \int x^3 \operatorname{arsinh}(ax)^3\,dx$$

```
[In] integrate(x^3*arcsinh(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/4*x^4*log(a*x + sqrt(a^2*x^2 + 1))^3 - integrate(3/4*(a^3*x^6 + sqrt(a^2*
x^2 + 1)*a^2*x^5 + a*x^4)*log(a*x + sqrt(a^2*x^2 + 1))^2/(a^3*x^3 + a*x + (
a^2*x^2 + 1)^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int x^3 \operatorname{arcsinh}(ax)^3\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3*arcsinh(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \text{arcsinh}(ax)^3 \, dx = \int x^3 \, \text{asinh}(a\,x)^3 \, dx$$

```
[In] int(x^3*asinh(a*x)^3,x)
```

```
[Out] int(x^3*asinh(a*x)^3, x)
```

3.24 $\int x^2 \operatorname{arcsinh}(ax)^3 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 132

$$\int x^2 \operatorname{arcsinh}(ax)^3 \, dx = \frac{14\sqrt{1+a^2x^2}}{9a^3} - \frac{2(1+a^2x^2)^{3/2}}{27a^3} - \frac{4x\operatorname{arcsinh}(ax)}{3a^2} + \frac{2}{9}x^3\operatorname{arcsinh}(ax) + \frac{2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2}{3a^3} - \frac{x^2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2}{3a} + \frac{1}{3}x^3\operatorname{arcsinh}(ax)^3$$

```
[Out] -2/27*(a^2*x^2+1)^(3/2)/a^3-4/3*x*arcsinh(a*x)/a^2+2/9*x^3*arcsinh(a*x)+1/3
*x^3*arcsinh(a*x)^3+14/9*(a^2*x^2+1)^(1/2)/a^3+2/3*arcsinh(a*x)^2*(a^2*x^2+
1)^(1/2)/a^3-1/3*x^2*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {5776, 5812, 5798, 5772, 267, 272, 45}

$$\int x^2 \operatorname{arcsinh}(ax)^3 \, dx = -\frac{x^2\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^2}{3a} - \frac{4x\operatorname{arcsinh}(ax)}{3a^2} + \frac{2\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^2}{3a^3} - \frac{2(a^2x^2+1)^{3/2}}{27a^3} + \frac{14\sqrt{a^2x^2+1}}{9a^3} + \frac{1}{3}x^3\operatorname{arcsinh}(ax)^3 + \frac{2}{9}x^3\operatorname{arcsinh}(ax)$$

```
[In] Int[x^2*ArcSinh[a*x]^3,x]
```

```
[Out] (14*Sqrt[1 + a^2*x^2])/(9*a^3) - (2*(1 + a^2*x^2)^(3/2))/(27*a^3) - (4*x*Ar
cSinh[a*x])/(3*a^2) + (2*x^3*ArcSinh[a*x])/9 + (2*Sqrt[1 + a^2*x^2]*ArcSinh
[a*x]^2)/(3*a^3) - (x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(3*a) + (x^3*ArcS
inh[a*x]^3)/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\text{arcsinh}(ax)^3 - a\int \frac{x^3\text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{x^2\sqrt{1+a^2x^2}\,\text{arcsinh}(ax)^2}{3a} + \frac{1}{3}x^3\text{arcsinh}(ax)^3 \\
&\quad + \frac{2}{3}\int x^2\text{arcsinh}(ax)\,dx + \frac{2\int \frac{x\text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx}{3a} \\
&= \frac{2}{9}x^3\text{arcsinh}(ax) + \frac{2\sqrt{1+a^2x^2}\,\text{arcsinh}(ax)^2}{3a^3} - \frac{x^2\sqrt{1+a^2x^2}\,\text{arcsinh}(ax)^2}{3a} \\
&\quad + \frac{1}{3}x^3\text{arcsinh}(ax)^3 - \frac{4\int \text{arcsinh}(ax)\,dx}{3a^2} - \frac{1}{9}(2a)\int \frac{x^3}{\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{4x\text{arcsinh}(ax)}{3a^2} + \frac{2}{9}x^3\text{arcsinh}(ax) + \frac{2\sqrt{1+a^2x^2}\,\text{arcsinh}(ax)^2}{3a^3} \\
&\quad - \frac{x^2\sqrt{1+a^2x^2}\,\text{arcsinh}(ax)^2}{3a} + \frac{1}{3}x^3\text{arcsinh}(ax)^3 \\
&\quad + \frac{4\int \frac{x}{\sqrt{1+a^2x^2}}\,dx}{3a} - \frac{1}{9}a\text{Subst}\left(\int \frac{x}{\sqrt{1+a^2x}}\,dx, x, x^2\right) \\
&= \frac{4\sqrt{1+a^2x^2}}{3a^3} - \frac{4x\text{arcsinh}(ax)}{3a^2} + \frac{2}{9}x^3\text{arcsinh}(ax) + \frac{2\sqrt{1+a^2x^2}\,\text{arcsinh}(ax)^2}{3a^3} \\
&\quad - \frac{x^2\sqrt{1+a^2x^2}\,\text{arcsinh}(ax)^2}{3a} + \frac{1}{3}x^3\text{arcsinh}(ax)^3 \\
&\quad - \frac{1}{9}a\text{Subst}\left(\int \left(-\frac{1}{a^2\sqrt{1+a^2x}} + \frac{\sqrt{1+a^2x}}{a^2}\right)dx, x, x^2\right) \\
&= \frac{14\sqrt{1+a^2x^2}}{9a^3} - \frac{2(1+a^2x^2)^{3/2}}{27a^3} - \frac{4x\text{arcsinh}(ax)}{3a^2} + \frac{2}{9}x^3\text{arcsinh}(ax) \\
&\qquad + \frac{2\sqrt{1+a^2x^2}\,\text{arcsinh}(ax)^2}{3a^3} - \frac{x^2\sqrt{1+a^2x^2}\,\text{arcsinh}(ax)^2}{3a} + \frac{1}{3}x^3\text{arcsinh}(ax)^3
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.70

$$\int x^2\text{arcsinh}(ax)^3\,dx$$
$$=\frac{-2(-20+a^2x^2)\sqrt{1+a^2x^2}+6ax(-6+a^2x^2)\,\text{arcsinh}(ax)-9(-2+a^2x^2)\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2+9a^3x}{27a^3}$$

```
[In] Integrate[x^2*ArcSinh[a*x]^3,x]
```

```
[Out] (-2*(-20 + a^2*x^2)*Sqrt[1 + a^2*x^2] + 6*a*x*(-6 + a^2*x^2)*ArcSinh[a*x] -
 9*(-2 + a^2*x^2)*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2 + 9*a^3*x^3*ArcSinh[a*x]
^3)/(27*a^3)
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{\frac{a^3x^3\,\text{arcsinh}(ax)^3}{3}+\frac{2\,\text{arcsinh}(ax)^2\sqrt{a^2x^2+1}}{3}-\frac{a^2x^2\,\text{arcsinh}(ax)^2\sqrt{a^2x^2+1}}{3}-\frac{4ax\,\text{arcsinh}(ax)}{3}+\frac{40\sqrt{a^2x^2+1}}{27}+\frac{2a^3x^3\,\text{arcsinh}(ax)}{9}-}{a^3}$
default	$\frac{\frac{a^3x^3\,\text{arcsinh}(ax)^3}{3}+\frac{2\,\text{arcsinh}(ax)^2\sqrt{a^2x^2+1}}{3}-\frac{a^2x^2\,\text{arcsinh}(ax)^2\sqrt{a^2x^2+1}}{3}-\frac{4ax\,\text{arcsinh}(ax)}{3}+\frac{40\sqrt{a^2x^2+1}}{27}+\frac{2a^3x^3\,\text{arcsinh}(ax)}{9}-}{a^3}$

```
[In] int(x^2*arcsinh(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(1/3*a^3*x^3*arcsinh(a*x)^3+2/3*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)-1/3*
a^2*x^2*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)-4/3*a*x*arcsinh(a*x)+40/27*(a^2*x^
2+1)^(1/2)+2/9*a^3*x^3*arcsinh(a*x)-2/27*a^2*x^2*(a^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.94

$$\int x^2\text{arcsinh}(ax)^3\,dx$$
$$=\frac{9\,a^3x^3\log\left(ax+\sqrt{a^2x^2+1}\right)^3-9\,\sqrt{a^2x^2+1}(a^2x^2-2)\log\left(ax+\sqrt{a^2x^2+1}\right)^2+6\,(a^3x^3-6\,ax)\log\left(ax+\right.}{27\,a^3}$$

```
[In] integrate(x^2*arcsinh(a*x)^3,x, algorithm="fricas")
```

```
[Out] 1/27*(9*a^3*x^3*log(a*x + sqrt(a^2*x^2 + 1))^3 - 9*sqrt(a^2*x^2 + 1)*(a^2*x
^2 - 2)*log(a*x + sqrt(a^2*x^2 + 1))^2 + 6*(a^3*x^3 - 6*a*x)*log(a*x + sqrt
(a^2*x^2 + 1)) - 2*sqrt(a^2*x^2 + 1)*(a^2*x^2 - 20))/a^3
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.97

$$\int x^2\text{arcsinh}(ax)^3\,dx$$
$$= \begin{cases} \frac{x^3\operatorname{asinh}^3(ax)}{3} + \frac{2x^3\operatorname{asinh}(ax)}{9} - \frac{x^2\sqrt{a^2x^2+1}\operatorname{asinh}^2(ax)}{3a} - \frac{2x^2\sqrt{a^2x^2+1}}{27a} - \frac{4x\operatorname{asinh}(ax)}{3a^2} + \frac{2\sqrt{a^2x^2+1}\operatorname{asinh}^2(ax)}{3a^3} + \frac{40\sqrt{a^2x^2+1}}{27a^3} \\ 0 \end{cases}$$

```
[In] integrate(x**2*asinh(a*x)**3,x)
```

```
[Out] Piecewise((x**3*asinh(a*x)**3/3 + 2*x**3*asinh(a*x)/9 - x**2*sqrt(a**2*x**2
 + 1)*asinh(a*x)**2/(3*a) - 2*x**2*sqrt(a**2*x**2 + 1)/(27*a) - 4*x*asinh(a
*x)/(3*a**2) + 2*sqrt(a**2*x**2 + 1)*asinh(a*x)**2/(3*a**3) + 40*sqrt(a**2*
x**2 + 1)/(27*a**3), Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.88

$$\int x^2\text{arcsinh}(ax)^3\,dx = \frac{1}{3}\,x^3\,\text{arsinh}\,(ax)^3 - \frac{1}{3}\,a\left(\frac{\sqrt{a^2x^2+1}x^2}{a^2} - \frac{2\sqrt{a^2x^2+1}}{a^4}\right)\text{arsinh}\,(ax)^2 - \frac{2}{27}\,a\left(\frac{\sqrt{a^2x^2+1}x^2 - \frac{20\sqrt{a^2x^2+1}}{a^2}}{a^2} - \frac{3\,(a^2x^3-6\,x)\,\text{arsinh}\,(ax)}{a^3}\right)$$

```
[In] integrate(x^2*arcsinh(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arcsinh(a*x)^3 - 1/3*a*(sqrt(a^2*x^2 + 1)*x^2/a^2 - 2*sqrt(a^2*x^2
+ 1)/a^4)*arcsinh(a*x)^2 - 2/27*a*((sqrt(a^2*x^2 + 1)*x^2 - 20*sqrt(a^2*x^2
 + 1)/a^2)/a^2 - 3*(a^2*x^3 - 6*x)*arcsinh(a*x)/a^3)
```

Giac [F(-2)]

Exception generated.

$$\int x^2\text{arcsinh}(ax)^3\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2*arcsinh(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^2\text{arcsinh}(ax)^3\,dx=\int x^2\,\text{asinh}(a\,x)^3\,dx$$

```
[In] int(x^2*asinh(a*x)^3,x)
[Out] int(x^2*asinh(a*x)^3, x)
```

3.25 $\int x\text{arcsinh}(ax)^3\,dx$

Optimal result

Integrand size = 8, antiderivative size = 97

$$\int x\text{arcsinh}(ax)^3\,dx=-\frac{3x\sqrt{1+a^2x^2}}{8a}+\frac{3\text{arcsinh}(ax)}{8a^2}+\frac{3}{4}x^2\text{arcsinh}(ax)-\frac{3x\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{4a}+\frac{\text{arcsinh}(ax)^3}{4a^2}+\frac{1}{2}x^2\text{arcsinh}(ax)^3$$

```
[Out] 3/8*arcsinh(a*x)/a^2+3/4*x^2*arcsinh(a*x)+1/4*arcsinh(a*x)^3/a^2+1/2*x^2*ar
csinh(a*x)^3-3/8*x*(a^2*x^2+1)^(1/2)/a-3/4*x*arcsinh(a*x)^2*(a^2*x^2+1)^(1/
2)/a
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5776, 5812, 5783, 327, 221}

$$\int x\text{arcsinh}(ax)^3\,dx=-\frac{3x\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{4a}+\frac{\text{arcsinh}(ax)^3}{4a^2}+\frac{3\text{arcsinh}(ax)}{8a^2}-\frac{3x\sqrt{a^2x^2+1}}{8a}+\frac{1}{2}x^2\text{arcsinh}(ax)^3+\frac{3}{4}x^2\text{arcsinh}(ax)$$

```
[In] Int[x*ArcSinh[a*x]^3,x]
```

```
[Out] (-3*x*Sqrt[1 + a^2*x^2])/(8*a) + (3*ArcSinh[a*x])/(8*a^2) + (3*x^2*ArcSinh[
a*x])/4 - (3*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(4*a) + ArcSinh[a*x]^3/(4*
a^2) + (x^2*ArcSinh[a*x]^3)/2
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{2}x^2\text{arcsinh}(ax)^3 - \frac{1}{2}(3a)\int \frac{x^2\text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx$$

$$= -\frac{3x\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{4a} + \frac{1}{2}x^2\text{arcsinh}(ax)^3 + \frac{3}{2}\int x\text{arcsinh}(ax)\,dx + \frac{3\int \frac{\text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx}{4a}$$

$$
\begin{aligned}
&= \frac{3}{4}x^2\text{arcsinh}(ax) - \frac{3x\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{4a} + \frac{\text{arcsinh}(ax)^3}{4a^2} \\
&\quad + \frac{1}{2}x^2\text{arcsinh}(ax)^3 - \frac{1}{4}(3a)\int \frac{x^2}{\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{3x\sqrt{1+a^2x^2}}{8a} + \frac{3}{4}x^2\text{arcsinh}(ax) - \frac{3x\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{4a} \\
&\quad + \frac{\text{arcsinh}(ax)^3}{4a^2} + \frac{1}{2}x^2\text{arcsinh}(ax)^3 + \frac{3\int \frac{1}{\sqrt{1+a^2x^2}}\,dx}{8a} \\
&= -\frac{3x\sqrt{1+a^2x^2}}{8a} + \frac{3\text{arcsinh}(ax)}{8a^2} + \frac{3}{4}x^2\text{arcsinh}(ax) \\
&\quad - \frac{3x\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{4a} + \frac{\text{arcsinh}(ax)^3}{4a^2} + \frac{1}{2}x^2\text{arcsinh}(ax)^3
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$
\int x\text{arcsinh}(ax)^3\,dx
= \frac{-3ax\sqrt{1+a^2x^2} + (3+6a^2x^2)\,\text{arcsinh}(ax) - 6ax\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2 + (2+4a^2x^2)\,\text{arcsinh}(ax)^3}{8a^2}
$$

```
[In] Integrate[x*ArcSinh[a*x]^3,x]
```

```
[Out] (-3*a*x*Sqrt[1 + a^2*x^2] + (3 + 6*a^2*x^2)*ArcSinh[a*x] - 6*a*x*Sqrt[1 + a
^2*x^2]*ArcSinh[a*x]^2 + (2 + 4*a^2*x^2)*ArcSinh[a*x]^3)/(8*a^2)
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{\frac{\operatorname{arcsinh}(ax)^3\left(a^2x^2+1\right)}{2}-\frac{3\operatorname{arcsinh}(ax)^2\sqrt{a^2x^2+1}\,ax}{4}-\frac{\operatorname{arcsinh}(ax)^3}{4}+\frac{3\left(a^2x^2+1\right)\operatorname{arcsinh}(ax)}{4}-\frac{3ax\sqrt{a^2x^2+1}}{8}-\frac{3\ \operatorname{arcsinh}(ax)}{8}}{a^2}$
default	$\frac{\frac{\operatorname{arcsinh}(ax)^3\left(a^2x^2+1\right)}{2}-\frac{3\operatorname{arcsinh}(ax)^2\sqrt{a^2x^2+1}\,ax}{4}-\frac{\operatorname{arcsinh}(ax)^3}{4}+\frac{3\left(a^2x^2+1\right)\operatorname{arcsinh}(ax)}{4}-\frac{3ax\sqrt{a^2x^2+1}}{8}-\frac{3\ \operatorname{arcsinh}(ax)}{8}}{a^2}$

```
[In] int(x*arcsinh(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(1/2*arcsinh(a*x)^3*(a^2*x^2+1)-3/4*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)*
a*x-1/4*arcsinh(a*x)^3+3/4*(a^2*x^2+1)*arcsinh(a*x)-3/8*a*x*(a^2*x^2+1)^(1/
2)-3/8*arcsinh(a*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.15

$$\int x\text{arcsinh}(ax)^3\,dx =$$

$$-\frac{6\,\sqrt{a^2x^2+1}ax\log\left(ax+\sqrt{a^2x^2+1}\right)^2-2\left(2\,a^2x^2+1\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^3+3\,\sqrt{a^2x^2+1}ax-3\left(2\,a^2\right.}{8\,a^2}$$

```
[In] integrate(x*arcsinh(a*x)^3,x, algorithm="fricas")
```

```
[Out] -1/8*(6*sqrt(a^2*x^2 + 1)*a*x*log(a*x + sqrt(a^2*x^2 + 1))^2 - 2*(2*a^2*x^2
 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^3 + 3*sqrt(a^2*x^2 + 1)*a*x - 3*(2*a^2*x
^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1)))/a^2
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.95

$$\int x\text{arcsinh}(ax)^3\,dx$$

$$= \begin{cases} \frac{x^2\operatorname{asinh}^3(ax)}{2}+\frac{3x^2\operatorname{asinh}(ax)}{4}-\frac{3x\sqrt{a^2x^2+1}\operatorname{asinh}^2(ax)}{4a}-\frac{3x\sqrt{a^2x^2+1}}{8a}+\frac{\operatorname{asinh}^3(ax)}{4a^2}+\frac{3\operatorname{asinh}(ax)}{8a^2} & \text{for } a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

```
[In] integrate(x*asinh(a*x)**3,x)
```

```
[Out] Piecewise((x**2*asinh(a*x)**3/2 + 3*x**2*asinh(a*x)/4 - 3*x*sqrt(a**2*x**2
+ 1)*asinh(a*x)**2/(4*a) - 3*x*sqrt(a**2*x**2 + 1)/(8*a) + asinh(a*x)**3/(4
*a**2) + 3*asinh(a*x)/(8*a**2), Ne(a, 0)), (0, True))
```

Maxima [F]

$$\int x\text{arcsinh}(ax)^3\,dx = \int x\,\text{arsinh}\left(ax\right)^3\,dx$$

```
[In] integrate(x*arcsinh(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*x^2*log(a*x + sqrt(a^2*x^2 + 1))^3 - integrate(3/2*(a^3*x^4 + sqrt(a^2*
x^2 + 1)*a^2*x^3 + a*x^2)*log(a*x + sqrt(a^2*x^2 + 1))^2/(a^3*x^3 + a*x + (
a^2*x^2 + 1)^(3/2)), x)
```

Giac [F]

$$\int x\mathrm{arcsinh}(ax)^3\,dx = \int x\,\operatorname{arsinh}\left(ax\right)^3\,dx$$

```
[In] integrate(x*arcsinh(a*x)^3,x, algorithm="giac")
[Out] integrate(x*arcsinh(a*x)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\mathrm{arcsinh}(ax)^3\,dx = \int x\,\mathrm{asinh}(a\,x)^3\,dx$$

```
[In] int(x*asinh(a*x)^3,x)
[Out] int(x*asinh(a*x)^3, x)
```

3.26 $\int \operatorname{arcsinh}(ax)^3 \, dx$

Optimal result

Integrand size = 6, antiderivative size = 58

$$\int \operatorname{arcsinh}(ax)^3 \, dx = -\frac{6\sqrt{1+a^2x^2}}{a} + 6x\operatorname{arcsinh}(ax) - \frac{3\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2}{a} + x\operatorname{arcsinh}(ax)^3$$

```
[Out] 6*x*arcsinh(a*x)+x*arcsinh(a*x)^3-6*(a^2*x^2+1)^(1/2)/a-3*arcsinh(a*x)^2*(a
^2*x^2+1)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5772, 5798, 267}

$$\int \operatorname{arcsinh}(ax)^3 \, dx = -\frac{3\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^2}{a} - \frac{6\sqrt{a^2x^2+1}}{a} + x\operatorname{arcsinh}(ax)^3 + 6x\operatorname{arcsinh}(ax)$$

```
[In] Int[ArcSinh[a*x]^3,x]
```

```
[Out] (-6*Sqrt[1 + a^2*x^2])/a + 6*x*ArcSinh[a*x] - (3*Sqrt[1 + a^2*x^2]*ArcSinh[
a*x]^2)/a + x*ArcSinh[a*x]^3
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= x\text{arcsinh}(ax)^3-(3a)\int\frac{x\text{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx\\ &= -\frac{3\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{a}+x\text{arcsinh}(ax)^3+6\int\text{arcsinh}(ax)\,dx\\ &= 6x\text{arcsinh}(ax)-\frac{3\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{a}+x\text{arcsinh}(ax)^3-(6a)\int\frac{x}{\sqrt{1+a^2x^2}}\,dx\\ &= -\frac{6\sqrt{1+a^2x^2}}{a}+6x\text{arcsinh}(ax)-\frac{3\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{a}+x\text{arcsinh}(ax)^3\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int\text{arcsinh}(ax)^3\,dx=-\frac{6\sqrt{1+a^2x^2}}{a}+6x\text{arcsinh}(ax)-\frac{3\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{a}+x\text{arcsinh}(ax)^3$$

```
[In] Integrate[ArcSinh[a*x]^3,x]
```

```
[Out] (-6*Sqrt[1 + a^2*x^2])/a + 6*x*ArcSinh[a*x] - (3*Sqrt[1 + a^2*x^2]*ArcSinh[
a*x]^2)/a + x*ArcSinh[a*x]^3
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$\frac{ax\operatorname{arcsinh}(ax)^3-3\operatorname{arcsinh}(ax)^2\sqrt{a^2x^2+1}+6ax\ \operatorname{arcsinh}(ax)-6\sqrt{a^2x^2+1}}{a}$	55
default	$\frac{ax\operatorname{arcsinh}(ax)^3-3\operatorname{arcsinh}(ax)^2\sqrt{a^2x^2+1}+6ax\ \operatorname{arcsinh}(ax)-6\sqrt{a^2x^2+1}}{a}$	55

```
[In] int(arcsinh(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(a*x*arcsinh(a*x)^3-3*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)+6*a*x*arcsinh(a*
x)-6*(a^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.55

$$\int \operatorname{arcsinh}(ax)^3\,dx = \frac{ax\log\left(ax+\sqrt{a^2x^2+1}\right)^3+6\,ax\log\left(ax+\sqrt{a^2x^2+1}\right)-3\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)^2-6\sqrt{a^2x^2+}}{a}$$

```
[In] integrate(arcsinh(a*x)^3,x, algorithm="fricas")
```

```
[Out] (a*x*log(a*x + sqrt(a^2*x^2 + 1))^3 + 6*a*x*log(a*x + sqrt(a^2*x^2 + 1)) -
3*sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^2 - 6*sqrt(a^2*x^2 + 1))/a
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.93

$$\int \operatorname{arcsinh}(ax)^3\,dx = \begin{cases} x\operatorname{asinh}^3(ax)+6x\operatorname{asinh}(ax)-\frac{3\sqrt{a^2x^2+1}\operatorname{asinh}^2(ax)}{a}-\frac{6\sqrt{a^2x^2+1}}{a} & \text{for } a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

```
[In] integrate(asinh(a*x)**3,x)
```

```
[Out] Piecewise((x*asinh(a*x)**3 + 6*x*asinh(a*x) - 3*sqrt(a**2*x**2 + 1)*asinh(a
*x)**2/a - 6*sqrt(a**2*x**2 + 1)/a, Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98

$$\int \operatorname{arcsinh}(ax)^3\,dx = x\operatorname{arsinh}(ax)^3 - \frac{3\sqrt{a^2x^2+1}\operatorname{arsinh}(ax)^2}{a} + \frac{6\left(ax\operatorname{arsinh}(ax) - \sqrt{a^2x^2+1}\right)}{a}$$

```
[In] integrate(arcsinh(a*x)^3,x, algorithm="maxima")
```

```
[Out] x*arcsinh(a*x)^3 - 3*sqrt(a^2*x^2 + 1)*arcsinh(a*x)^2/a + 6*(a*x*arcsinh(a*
x) - sqrt(a^2*x^2 + 1))/a
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.69

$$\begin{aligned}\int \operatorname{arcsinh}(ax)^3\,dx &= x\log\left(ax+\sqrt{a^2x^2+1}\right)^3 \\ &\quad - 3a\left(\frac{\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)^2}{a^2} - \frac{2\left(x\log\left(ax+\sqrt{a^2x^2+1}\right) - \frac{\sqrt{a^2x^2+1}}{a}\right)}{a}\right)\end{aligned}$$

```
[In] integrate(arcsinh(a*x)^3,x, algorithm="giac")
```

```
[Out] x*log(a*x + sqrt(a^2*x^2 + 1))^3 - 3*a*(sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^
2*x^2 + 1))^2/a^2 - 2*(x*log(a*x + sqrt(a^2*x^2 + 1)) - sqrt(a^2*x^2 + 1)/a
)/a)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{arcsinh}(ax)^3\,dx = \int \operatorname{asinh}(a\,x)^3\,dx$$

```
[In] int(asinh(a*x)^3,x)
```

```
[Out] int(asinh(a*x)^3, x)
```

3.27 $\int \frac{\operatorname{arcsinh}(ax)^3}{x}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 83

$$\begin{aligned}\int \frac{\operatorname{arcsinh}(ax)^3}{x}\,dx = &-\frac{1}{4}\operatorname{arcsinh}(ax)^4 + \operatorname{arcsinh}(ax)^3\log\left(1-e^{2\operatorname{arcsinh}(ax)}\right)\\ &+\frac{3}{2}\operatorname{arcsinh}(ax)^2\operatorname{PolyLog}\left(2,e^{2\operatorname{arcsinh}(ax)}\right)\\ &-\frac{3}{2}\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(3,e^{2\operatorname{arcsinh}(ax)}\right)+\frac{3}{4}\operatorname{PolyLog}\left(4,e^{2\operatorname{arcsinh}(ax)}\right)\end{aligned}$$

```
[Out] -1/4*arcsinh(a*x)^4+arcsinh(a*x)^3*ln(1-(a*x+(a^2*x^2+1)^(1/2))^2)+3/2*arcs
inh(a*x)^2*polylog(2,(a*x+(a^2*x^2+1)^(1/2))^2)-3/2*arcsinh(a*x)*polylog(3,
(a*x+(a^2*x^2+1)^(1/2))^2)+3/4*polylog(4,(a*x+(a^2*x^2+1)^(1/2))^2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {5775, 3797, 2221, 2611, 6744, 2320, 6724}

$$\begin{aligned}\int \frac{\operatorname{arcsinh}(ax)^3}{x}\,dx = &\frac{3}{2}\operatorname{arcsinh}(ax)^2\operatorname{PolyLog}\left(2,e^{2\operatorname{arcsinh}(ax)}\right)\\ &-\frac{3}{2}\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(3,e^{2\operatorname{arcsinh}(ax)}\right)+\frac{3}{4}\operatorname{PolyLog}\left(4,e^{2\operatorname{arcsinh}(ax)}\right)\\ &-\frac{1}{4}\operatorname{arcsinh}(ax)^4+\operatorname{arcsinh}(ax)^3\log\left(1-e^{2\operatorname{arcsinh}(ax)}\right)\end{aligned}$$

```
[In] Int[ArcSinh[a*x]^3/x,x]
```

```
[Out] -1/4*ArcSinh[a*x]^4 + ArcSinh[a*x]^3*Log[1 - E^(2*ArcSinh[a*x])] + (3*ArcSi
nh[a*x]^2*PolyLog[2, E^(2*ArcSinh[a*x])])/2 - (3*ArcSinh[a*x]*PolyLog[3, E^
(2*ArcSinh[a*x])])/2 + (3*PolyLog[4, E^(2*ArcSinh[a*x])])/4
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int x^3\coth(x)\,dx, x, \text{arcsinh}(ax)\right)\\
&= -\frac{1}{4}\text{arcsinh}(ax)^4 - 2\text{Subst}\left(\int \frac{e^{2x}x^3}{1-e^{2x}}\,dx, x, \text{arcsinh}(ax)\right)\\
&= -\frac{1}{4}\text{arcsinh}(ax)^4 + \text{arcsinh}(ax)^3\log\left(1-e^{2\text{arcsinh}(ax)}\right)\\
&\quad - 3\text{Subst}\left(\int x^2\log\left(1-e^{2x}\right)\,dx, x, \text{arcsinh}(ax)\right)\\
&= -\frac{1}{4}\text{arcsinh}(ax)^4 + \text{arcsinh}(ax)^3\log\left(1-e^{2\text{arcsinh}(ax)}\right)\\
&\quad + \frac{3}{2}\text{arcsinh}(ax)^2\,\text{PolyLog}\left(2, e^{2\text{arcsinh}(ax)}\right)\\
&\quad - 3\text{Subst}\left(\int x\,\text{PolyLog}\left(2, e^{2x}\right)\,dx, x, \text{arcsinh}(ax)\right)\\
&= -\frac{1}{4}\text{arcsinh}(ax)^4 + \text{arcsinh}(ax)^3\log\left(1-e^{2\text{arcsinh}(ax)}\right)\\
&\quad + \frac{3}{2}\text{arcsinh}(ax)^2\,\text{PolyLog}\left(2, e^{2\text{arcsinh}(ax)}\right) - \frac{3}{2}\text{arcsinh}(ax)\,\text{PolyLog}\left(3, e^{2\text{arcsinh}(ax)}\right)\\
&\quad + \frac{3}{2}\text{Subst}\left(\int \text{PolyLog}\left(3, e^{2x}\right)\,dx, x, \text{arcsinh}(ax)\right)\\
&= -\frac{1}{4}\text{arcsinh}(ax)^4 + \text{arcsinh}(ax)^3\log\left(1-e^{2\text{arcsinh}(ax)}\right)\\
&\quad + \frac{3}{2}\text{arcsinh}(ax)^2\,\text{PolyLog}\left(2, e^{2\text{arcsinh}(ax)}\right) - \frac{3}{2}\text{arcsinh}(ax)\,\text{PolyLog}\left(3, e^{2\text{arcsinh}(ax)}\right)\\
&\quad + \frac{3}{4}\text{Subst}\left(\int \frac{\text{PolyLog}(3,x)}{x}\,dx, x, e^{2\text{arcsinh}(ax)}\right)\\
&= -\frac{1}{4}\text{arcsinh}(ax)^4 + \text{arcsinh}(ax)^3\log\left(1-e^{2\text{arcsinh}(ax)}\right)\\
&\quad + \frac{3}{2}\text{arcsinh}(ax)^2\,\text{PolyLog}\left(2, e^{2\text{arcsinh}(ax)}\right)\\
&\quad - \frac{3}{2}\text{arcsinh}(ax)\,\text{PolyLog}\left(3, e^{2\text{arcsinh}(ax)}\right) + \frac{3}{4}\,\text{PolyLog}\left(4, e^{2\text{arcsinh}(ax)}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00

$$\begin{aligned}\int \frac{\operatorname{arcsinh}(ax)^3}{x}\,dx = &-\frac{1}{4}\operatorname{arcsinh}(ax)^4+\operatorname{arcsinh}(ax)^3\log\left(1-e^{2\operatorname{arcsinh}(ax)}\right)\\&+\frac{3}{2}\operatorname{arcsinh}(ax)^2\operatorname{PolyLog}\left(2,e^{2\operatorname{arcsinh}(ax)}\right)\\&-\frac{3}{2}\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(3,e^{2\operatorname{arcsinh}(ax)}\right)+\frac{3}{4}\operatorname{PolyLog}\left(4,e^{2\operatorname{arcsinh}(ax)}\right)\end{aligned}$$

```
[In] Integrate[ArcSinh[a*x]^3/x,x]
```

```
[Out] -1/4*ArcSinh[a*x]^4 + ArcSinh[a*x]^3*Log[1 - E^(2*ArcSinh[a*x])] + (3*ArcSi
nh[a*x]^2*PolyLog[2, E^(2*ArcSinh[a*x])])/2 - (3*ArcSinh[a*x]*PolyLog[3, E^
(2*ArcSinh[a*x])])/2 + (3*PolyLog[4, E^(2*ArcSinh[a*x])])/4
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.46

method	result
derivativedivides	$-\frac{\operatorname{arcsinh}(ax)^4}{4}+\operatorname{arcsinh}(ax)^3\ln\left(1+ax+\sqrt{a^2x^2+1}\right)+3\operatorname{arcsinh}(ax)^2\operatorname{polylog}(2,-ax$
default	$-\frac{\operatorname{arcsinh}(ax)^4}{4}+\operatorname{arcsinh}(ax)^3\ln\left(1+ax+\sqrt{a^2x^2+1}\right)+3\operatorname{arcsinh}(ax)^2\operatorname{polylog}(2,-ax$

```
[In] int(arcsinh(a*x)^3/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*arcsinh(a*x)^4+arcsinh(a*x)^3*ln(1+a*x+(a^2*x^2+1)^(1/2))+3*arcsinh(a*
x)^2*polylog(2,-a*x-(a^2*x^2+1)^(1/2))-6*arcsinh(a*x)*polylog(3,-a*x-(a^2*x
^2+1)^(1/2))+6*polylog(4,-a*x-(a^2*x^2+1)^(1/2))+arcsinh(a*x)^3*ln(1-a*x-(a
^2*x^2+1)^(1/2))+3*arcsinh(a*x)^2*polylog(2,a*x+(a^2*x^2+1)^(1/2))-6*arcsin
h(a*x)*polylog(3,a*x+(a^2*x^2+1)^(1/2))+6*polylog(4,a*x+(a^2*x^2+1)^(1/2))
```

Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^3}{x}\,dx$$

```
[In] integrate(arcsinh(a*x)^3/x,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x)^3/x, x)
```

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x}\,dx = \int \frac{\operatorname{asinh}^3(ax)}{x}\,dx$$

```
[In] integrate(asinh(a*x)**3/x,x)
[Out] Integral(asinh(a*x)**3/x, x)
```

Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x}\,dx = \int \frac{\operatorname{arsinh}(ax)^3}{x}\,dx$$

```
[In] integrate(arcsinh(a*x)^3/x,x, algorithm="maxima")
[Out] integrate(arcsinh(a*x)^3/x, x)
```

Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x}\,dx = \int \frac{\operatorname{arsinh}(ax)^3}{x}\,dx$$

```
[In] integrate(arcsinh(a*x)^3/x,x, algorithm="giac")
[Out] integrate(arcsinh(a*x)^3/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x}\,dx = \int \frac{\operatorname{asinh}(a\,x)^3}{x}\,dx$$

```
[In] int(asinh(a*x)^3/x,x)
[Out] int(asinh(a*x)^3/x, x)
```

3.28 $\int \frac{\operatorname{arcsinh}(ax)^3}{x^2} dx$

Optimal result

Integrand size = 10, antiderivative size = 84

$$\begin{aligned}\int \frac{\operatorname{arcsinh}(ax)^3}{x^2} dx = &-\frac{\operatorname{arcsinh}(ax)^3}{x} - 6a\operatorname{arcsinh}(ax)^2\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(ax)}\right) \\ &- 6a\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(ax)}\right) \\ &+ 6a\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(ax)}\right) \\ &+ 6a\operatorname{PolyLog}\left(3,-e^{\operatorname{arcsinh}(ax)}\right) - 6a\operatorname{PolyLog}\left(3,e^{\operatorname{arcsinh}(ax)}\right)\end{aligned}$$

```
[Out] -arcsinh(a*x)^3/x-6*a*arcsinh(a*x)^2*arctanh(a*x+(a^2*x^2+1)^(1/2))-6*a*arc
sinh(a*x)*polylog(2,-a*x-(a^2*x^2+1)^(1/2))+6*a*arcsinh(a*x)*polylog(2,a*x+
(a^2*x^2+1)^(1/2))+6*a*polylog(3,-a*x-(a^2*x^2+1)^(1/2))-6*a*polylog(3,a*x+
(a^2*x^2+1)^(1/2))
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5776, 5816, 4267, 2611, 2320, 6724}

$$\begin{aligned}\int \frac{\operatorname{arcsinh}(ax)^3}{x^2} dx = &-6a\operatorname{arcsinh}(ax)^2\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(ax)}\right) \\ &- 6a\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(ax)}\right) \\ &+ 6a\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(ax)}\right) + 6a\operatorname{PolyLog}\left(3,-e^{\operatorname{arcsinh}(ax)}\right) \\ &- 6a\operatorname{PolyLog}\left(3,e^{\operatorname{arcsinh}(ax)}\right) - \frac{\operatorname{arcsinh}(ax)^3}{x}\end{aligned}$$

```
[In] Int[ArcSinh[a*x]^3/x^2,x]
```

```
[Out] -(ArcSinh[a*x]^3/x) - 6*a*ArcSinh[a*x]^2*ArcTanh[E^ArcSinh[a*x]] - 6*a*ArcS
inh[a*x]*PolyLog[2, -E^ArcSinh[a*x]] + 6*a*ArcSinh[a*x]*PolyLog[2, E^ArcSin
h[a*x]] + 6*a*PolyLog[3, -E^ArcSinh[a*x]] - 6*a*PolyLog[3, E^ArcSinh[a*x]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\operatorname{arcsinh}(ax)^3}{x} + (3a)\int \frac{\operatorname{arcsinh}(ax)^2}{x\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{\operatorname{arcsinh}(ax)^3}{x} + (3a)\text{Subst}\left(\int x^2\operatorname{csch}(x)\,dx, x, \operatorname{arcsinh}(ax)\right) \\
&= -\frac{\operatorname{arcsinh}(ax)^3}{x} - 6a\operatorname{arcsinh}(ax)^2\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(ax)}\right) \\
&\quad - (6a)\text{Subst}\left(\int x\log\left(1-e^x\right)\,dx, x, \operatorname{arcsinh}(ax)\right) \\
&\quad + (6a)\text{Subst}\left(\int x\log\left(1+e^x\right)\,dx, x, \operatorname{arcsinh}(ax)\right) \\
&= -\frac{\operatorname{arcsinh}(ax)^3}{x} - 6a\operatorname{arcsinh}(ax)^2\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(ax)}\right) \\
&\quad - 6a\operatorname{arcsinh}(ax)\,\text{PolyLog}\left(2,-e^{\operatorname{arcsinh}(ax)}\right) + 6a\operatorname{arcsinh}(ax)\,\text{PolyLog}\left(2,e^{\operatorname{arcsinh}(ax)}\right) \\
&\quad + (6a)\text{Subst}\left(\int \text{PolyLog}\left(2,-e^x\right)\,dx, x, \operatorname{arcsinh}(ax)\right) \\
&\quad - (6a)\text{Subst}\left(\int \text{PolyLog}\left(2,e^x\right)\,dx, x, \operatorname{arcsinh}(ax)\right) \\
&= -\frac{\operatorname{arcsinh}(ax)^3}{x} - 6a\operatorname{arcsinh}(ax)^2\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(ax)}\right) \\
&\quad - 6a\operatorname{arcsinh}(ax)\,\text{PolyLog}\left(2,-e^{\operatorname{arcsinh}(ax)}\right) + 6a\operatorname{arcsinh}(ax)\,\text{PolyLog}\left(2,e^{\operatorname{arcsinh}(ax)}\right) \\
&\quad + (6a)\text{Subst}\left(\int \frac{\text{PolyLog}(2,-x)}{x}\,dx, x, e^{\operatorname{arcsinh}(ax)}\right) \\
&\quad - (6a)\text{Subst}\left(\int \frac{\text{PolyLog}(2,x)}{x}\,dx, x, e^{\operatorname{arcsinh}(ax)}\right) \\
&= -\frac{\operatorname{arcsinh}(ax)^3}{x} - 6a\operatorname{arcsinh}(ax)^2\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(ax)}\right) \\
&\quad - 6a\operatorname{arcsinh}(ax)\,\text{PolyLog}\left(2,-e^{\operatorname{arcsinh}(ax)}\right) + 6a\operatorname{arcsinh}(ax)\,\text{PolyLog}\left(2,e^{\operatorname{arcsinh}(ax)}\right) \\
&\quad + 6a\,\text{PolyLog}\left(3,-e^{\operatorname{arcsinh}(ax)}\right) - 6a\,\text{PolyLog}\left(3,e^{\operatorname{arcsinh}(ax)}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.39

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^2}\,dx = a\bigg(-\frac{\operatorname{arcsinh}(ax)^3}{ax} + 3\operatorname{arcsinh}(ax)^2\log\left(1-e^{-\operatorname{arcsinh}(ax)}\right) - 3\operatorname{arcsinh}(ax)^2\log\left(1+e^{-\operatorname{arcsinh}(ax)}\right) + 6\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(2,-e^{-\operatorname{arcsinh}(ax)}\right) - 6\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(2,e^{-\operatorname{arcsinh}(ax)}\right) + 6\operatorname{PolyLog}\left(3,-e^{-\operatorname{arcsinh}(ax)}\right) - 6\operatorname{PolyLog}\left(3,e^{-\operatorname{arcsinh}(ax)}\right)\bigg)$$

```
[In] Integrate[ArcSinh[a*x]^3/x^2,x]
```

```
[Out] a*(-(ArcSinh[a*x]^3/(a*x)) + 3*ArcSinh[a*x]^2*Log[1 - E^(-ArcSinh[a*x])] -
3*ArcSinh[a*x]^2*Log[1 + E^(-ArcSinh[a*x])] + 6*ArcSinh[a*x]*PolyLog[2, -E^
(-ArcSinh[a*x])] - 6*ArcSinh[a*x]*PolyLog[2, E^(-ArcSinh[a*x])] + 6*PolyLog
[3, -E^(-ArcSinh[a*x])] - 6*PolyLog[3, E^(-ArcSinh[a*x])])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.92

method	result
derivativedivides	$a\left(-\frac{\operatorname{arcsinh}(ax)^3}{ax} - 3\operatorname{arcsinh}(ax)^2\ln\left(1+ax+\sqrt{a^2x^2+1}\right) - 6\operatorname{arcsinh}(ax)\operatorname{polylog}(2,-\right.$
default	$a\left(-\frac{\operatorname{arcsinh}(ax)^3}{ax} - 3\operatorname{arcsinh}(ax)^2\ln\left(1+ax+\sqrt{a^2x^2+1}\right) - 6\operatorname{arcsinh}(ax)\operatorname{polylog}(2,-\right.$

```
[In] int(arcsinh(a*x)^3/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] a*(-arcsinh(a*x)^3/a/x-3*arcsinh(a*x)^2*ln(1+a*x+(a^2*x^2+1)^(1/2))-6*arcsi
nh(a*x)*polylog(2,-a*x-(a^2*x^2+1)^(1/2))+6*polylog(3,-a*x-(a^2*x^2+1)^(1/2
))+3*arcsinh(a*x)^2*ln(1-a*x-(a^2*x^2+1)^(1/2))+6*arcsinh(a*x)*polylog(2,a*
x+(a^2*x^2+1)^(1/2))-6*polylog(3,a*x+(a^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^2}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^3}{x^2}\,dx$$

```
[In] integrate(arcsinh(a*x)^3/x^2,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x)^3/x^2, x)
```

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^2}\,dx = \int \frac{\operatorname{asinh}^3\left(ax\right)}{x^2}\,dx$$

```
[In] integrate(asinh(a*x)**3/x**2,x)
```

```
[Out] Integral(asinh(a*x)**3/x**2, x)
```

Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^2}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^3}{x^2}\,dx$$

```
[In] integrate(arcsinh(a*x)^3/x^2,x, algorithm="maxima")
```

```
[Out] -log(a*x + sqrt(a^2*x^2 + 1))^3/x + integrate(3*(a^3*x^2 + sqrt(a^2*x^2 + 1
)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))^2/(a^3*x^4 + a*x^2 + (a^2*x^3 + x
)*sqrt(a^2*x^2 + 1)), x)
```

Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^2}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^3}{x^2}\,dx$$

```
[In] integrate(arcsinh(a*x)^3/x^2,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^3/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^2}\,dx=\int \frac{\mathrm{asinh}(a\,x)^3}{x^2}\,dx$$

```
[In] int(asinh(a*x)^3/x^2,x)
[Out] int(asinh(a*x)^3/x^2, x)
```

3.29 $\int \frac{\textbf{arcsinh}(ax)^3}{x^3}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 93

$$\int \frac{\text{arcsinh}(ax)^3}{x^3}\,dx = -\frac{3}{2}a^2\text{arcsinh}(ax)^2 - \frac{3a\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{2x} - \frac{\text{arcsinh}(ax)^3}{2x^2} + 3a^2\text{arcsinh}(ax)\log\left(1-e^{2\text{arcsinh}(ax)}\right) + \frac{3}{2}a^2\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(ax)}\right)$$

```
[Out] -3/2*a^2*arcsinh(a*x)^2-1/2*arcsinh(a*x)^3/x^2+3*a^2*arcsinh(a*x)*ln(1-(a*x
+(a^2*x^2+1)^(1/2))^2)+3/2*a^2*polylog(2,(a*x+(a^2*x^2+1)^(1/2))^2)-3/2*a*a
rcsinh(a*x)^2*(a^2*x^2+1)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {5776, 5800, 5775, 3797, 2221, 2317, 2438}

$$\int \frac{\text{arcsinh}(ax)^3}{x^3}\,dx = \frac{3}{2}a^2\,\text{PolyLog}\left(2,e^{2\text{arcsinh}(ax)}\right) - \frac{3a\sqrt{a^2x^2+1}\text{arcsinh}(ax)^2}{2x} - \frac{3}{2}a^2\text{arcsinh}(ax)^2 + 3a^2\text{arcsinh}(ax)\log\left(1-e^{2\text{arcsinh}(ax)}\right) - \frac{\text{arcsinh}(ax)^3}{2x^2}$$

```
[In] Int[ArcSinh[a*x]^3/x^3,x]
```

```
[Out] (-3*a^2*ArcSinh[a*x]^2)/2 - (3*a*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(2*x) -
ArcSinh[a*x]^3/(2*x^2) + 3*a^2*ArcSinh[a*x]*Log[1 - E^(2*ArcSinh[a*x])] + (
3*a^2*PolyLog[2, E^(2*ArcSinh[a*x])])/2
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5800

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*
ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[
```

```
e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\operatorname{arcsinh}(ax)^3}{2x^2}+\frac{1}{2}(3a)\int \frac{\operatorname{arcsinh}(ax)^2}{x^2\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{3a\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2}{2x}-\frac{\operatorname{arcsinh}(ax)^3}{2x^2}+\left(3a^2\right)\int \frac{\operatorname{arcsinh}(ax)}{x}\,dx\\
&= -\frac{3a\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2}{2x}-\frac{\operatorname{arcsinh}(ax)^3}{2x^2}+\left(3a^2\right)\operatorname{Subst}(\int x\coth(x)\,dx,x,\operatorname{arcsinh}(ax))\\
&= -\frac{3}{2}a^2\operatorname{arcsinh}(ax)^2-\frac{3a\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2}{2x}\\
&\quad -\frac{\operatorname{arcsinh}(ax)^3}{2x^2}-\left(6a^2\right)\operatorname{Subst}\left(\int \frac{e^{2x}x}{1-e^{2x}}\,dx,x,\operatorname{arcsinh}(ax)\right)\\
&= -\frac{3}{2}a^2\operatorname{arcsinh}(ax)^2-\frac{3a\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2}{2x}\\
&\quad -\frac{\operatorname{arcsinh}(ax)^3}{2x^2}+3a^2\operatorname{arcsinh}(ax)\log\left(1-e^{2\operatorname{arcsinh}(ax)}\right)\\
&\quad -\left(3a^2\right)\operatorname{Subst}\left(\int \log\left(1-e^{2x}\right)\,dx,x,\operatorname{arcsinh}(ax)\right)\\
&= -\frac{3}{2}a^2\operatorname{arcsinh}(ax)^2-\frac{3a\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2}{2x}\\
&\quad -\frac{\operatorname{arcsinh}(ax)^3}{2x^2}+3a^2\operatorname{arcsinh}(ax)\log\left(1-e^{2\operatorname{arcsinh}(ax)}\right)\\
&\quad -\frac{1}{2}\left(3a^2\right)\operatorname{Subst}\left(\int \frac{\log(1-x)}{x}\,dx,x,e^{2\operatorname{arcsinh}(ax)}\right)\\
&= -\frac{3}{2}a^2\operatorname{arcsinh}(ax)^2-\frac{3a\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2}{2x}-\frac{\operatorname{arcsinh}(ax)^3}{2x^2}\\
&\quad +3a^2\operatorname{arcsinh}(ax)\log\left(1-e^{2\operatorname{arcsinh}(ax)}\right)+\frac{3}{2}a^2\operatorname{PolyLog}\left(2,e^{2\operatorname{arcsinh}(ax)}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.86

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^3}\,dx =$$
$$-\frac{\operatorname{arcsinh}(ax)^3-3ax\left(\operatorname{arcsinh}(ax)\left(\left(ax-\sqrt{1+a^2x^2}\right)\operatorname{arcsinh}(ax)+2ax\log\left(1-e^{-2\operatorname{arcsinh}(ax)}\right)\right)-ax\,\mathrm{Poly}}{2x^2}$$

```
[In] Integrate[ArcSinh[a*x]^3/x^3,x]
```

```
[Out] -1/2*(ArcSinh[a*x]^3 - 3*a*x*(ArcSinh[a*x]*((a*x - Sqrt[1 + a^2*x^2])*ArcSi
nh[a*x] + 2*a*x*Log[1 - E^(-2*ArcSinh[a*x])]) - a*x*PolyLog[2, E^(-2*ArcSin
h[a*x])]))/x^2
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.57

method	result
derivativedivides	$a^2\left(-\frac{\operatorname{arcsinh}(ax)^2\left(3ax\sqrt{a^2x^2+1}-3a^2x^2+\operatorname{arcsinh}(ax)\right)}{2a^2x^2}-3\operatorname{arcsinh}(ax)^2+3\operatorname{arcsinh}(ax)\ln\left(1+a\right.\right.$
default	$a^2\left(-\frac{\operatorname{arcsinh}(ax)^2\left(3ax\sqrt{a^2x^2+1}-3a^2x^2+\operatorname{arcsinh}(ax)\right)}{2a^2x^2}-3\operatorname{arcsinh}(ax)^2+3\operatorname{arcsinh}(ax)\ln\left(1+a\right.\right.$

```
[In] int(arcsinh(a*x)^3/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(-1/2*arcsinh(a*x)^2*(3*a*x*(a^2*x^2+1)^(1/2)-3*a^2*x^2+arcsinh(a*x))/a
^2/x^2-3*arcsinh(a*x)^2+3*arcsinh(a*x)*ln(1+a*x+(a^2*x^2+1)^(1/2))+3*polylo
g(2,-a*x-(a^2*x^2+1)^(1/2))+3*arcsinh(a*x)*ln(1-a*x-(a^2*x^2+1)^(1/2))+3*po
lylog(2,a*x+(a^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^3}\,dx = \int \frac{\operatorname{arsinh}(ax)^3}{x^3}\,dx$$

```
[In] integrate(arcsinh(a*x)^3/x^3,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x)^3/x^3, x)
```

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^3}\,dx = \int \frac{\operatorname{asinh}^3(ax)}{x^3}\,dx$$

```
[In] integrate(asinh(a*x)**3/x**3,x)
```

```
[Out] Integral(asinh(a*x)**3/x**3, x)
```

Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^3}\,dx = \int \frac{\operatorname{arsinh}(ax)^3}{x^3}\,dx$$

```
[In] integrate(arcsinh(a*x)^3/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*log(a*x + sqrt(a^2*x^2 + 1))^3/x^2 + integrate(3/2*(a^3*x^2 + sqrt(a^2
*x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))^2/(a^3*x^5 + a*x^3 + (a^2
*x^4 + x^2)*sqrt(a^2*x^2 + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^3}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(arcsinh(a*x)^3/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^3}\,dx = \int \frac{\operatorname{asinh}(a\,x)^3}{x^3}\,dx$$

```
[In] int(asinh(a*x)^3/x^3,x)
```

```
[Out] int(asinh(a*x)^3/x^3, x)
```

3.30 $\int \frac{\operatorname{arcsinh}(ax)^3}{x^4}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 151

$$
\begin{aligned}
\int \frac{\operatorname{arcsinh}(ax)^3}{x^4}\,dx = {} & -\frac{a^2\operatorname{arcsinh}(ax)}{x} - \frac{a\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2}{2x^2} \\
& - \frac{\operatorname{arcsinh}(ax)^3}{3x^3} + a^3\operatorname{arcsinh}(ax)^2\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(ax)}\right) \\
& - a^3\operatorname{arctanh}\left(\sqrt{1+a^2x^2}\right) + a^3\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(ax)}\right) \\
& - a^3\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(ax)}\right) \\
& - a^3\operatorname{PolyLog}\left(3,-e^{\operatorname{arcsinh}(ax)}\right) + a^3\operatorname{PolyLog}\left(3,e^{\operatorname{arcsinh}(ax)}\right)
\end{aligned}
$$

```
[Out] -a^2*arcsinh(a*x)/x-1/3*arcsinh(a*x)^3/x^3+a^3*arcsinh(a*x)^2*arctanh(a*x+(
a^2*x^2+1)^(1/2))-a^3*arctanh((a^2*x^2+1)^(1/2))+a^3*arcsinh(a*x)*polylog(2
,-a*x-(a^2*x^2+1)^(1/2))-a^3*arcsinh(a*x)*polylog(2,a*x+(a^2*x^2+1)^(1/2))-
a^3*polylog(3,-a*x-(a^2*x^2+1)^(1/2))+a^3*polylog(3,a*x+(a^2*x^2+1)^(1/2))-
1/2*a*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules

used = {5776, 5809, 5816, 4267, 2611, 2320, 6724, 272, 65, 214}

$$\begin{aligned}\int \frac{\operatorname{arcsinh}(ax)^3}{x^4}\,dx = {} & a^3\operatorname{arcsinh}(ax)^2\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(ax)}\right) \\ & + a^3\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(2, -e^{\operatorname{arcsinh}(ax)}\right) \\ & - a^3\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(2, e^{\operatorname{arcsinh}(ax)}\right) - a^3\operatorname{PolyLog}\left(3, -e^{\operatorname{arcsinh}(ax)}\right) \\ & + a^3\operatorname{PolyLog}\left(3, e^{\operatorname{arcsinh}(ax)}\right) - \frac{a\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^2}{2x^2} \\ & - \frac{a^2\operatorname{arcsinh}(ax)}{x} - a^3\operatorname{arctanh}\left(\sqrt{a^2x^2+1}\right) - \frac{\operatorname{arcsinh}(ax)^3}{3x^3}\end{aligned}$$

```
[In] Int[ArcSinh[a*x]^3/x^4,x]
```

```
[Out] -((a^2*ArcSinh[a*x])/x) - (a*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(2*x^2) - Ar
cSinh[a*x]^3/(3*x^3) + a^3*ArcSinh[a*x]^2*ArcTanh[E^ArcSinh[a*x]] - a^3*Arc
Tanh[Sqrt[1 + a^2*x^2]] + a^3*ArcSinh[a*x]*PolyLog[2, -E^ArcSinh[a*x]] - a^
3*ArcSinh[a*x]*PolyLog[2, E^ArcSinh[a*x]] - a^3*PolyLog[3, -E^ArcSinh[a*x]]
 + a^3*PolyLog[3, E^ArcSinh[a*x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = -\frac{\operatorname{arcsinh}(ax)^3}{3x^3} + a\int \frac{\operatorname{arcsinh}(ax)^2}{x^3\sqrt{1+a^2x^2}}\,dx$$

$$= -\frac{a\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2}{2x^2} - \frac{\operatorname{arcsinh}(ax)^3}{3x^3} + a^2\int \frac{\operatorname{arcsinh}(ax)}{x^2}\,dx - \frac{1}{2}a^3\int \frac{\operatorname{arcsinh}(ax)^2}{x\sqrt{1+a^2x^2}}\,dx$$

$$= -\frac{a^2\operatorname{arcsinh}(ax)}{x} - \frac{a\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2}{2x^2} - \frac{\operatorname{arcsinh}(ax)^3}{3x^3} - \frac{1}{2}a^3\text{Subst}\left(\int x^2\operatorname{csch}(x)\,dx, x, \operatorname{arcsinh}(ax)\right) + a^3\int \frac{1}{x\sqrt{1+a^2x^2}}\,dx$$

$$= -\frac{a^2\operatorname{arcsinh}(ax)}{x} - \frac{a\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2}{2x^2} - \frac{\operatorname{arcsinh}(ax)^3}{3x^3} + a^3\operatorname{arcsinh}(ax)^2\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(ax)}\right) + \frac{1}{2}a^3\text{Subst}\left(\int \frac{1}{x\sqrt{1+a^2x}}\,dx, x, x^2\right) + a^3\text{Subst}\left(\int x\log\left(1-e^x\right)\,dx, x, \operatorname{arcsinh}(ax)\right) - a^3\text{Subst}\left(\int x\log\left(1+e^x\right)\,dx, x, \operatorname{arcsinh}(ax)\right)$$

$$= -\frac{a^2\operatorname{arcsinh}(ax)}{x} - \frac{a\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2}{2x^2} - \frac{\operatorname{arcsinh}(ax)^3}{3x^3} + a^3\operatorname{arcsinh}(ax)^2\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(ax)}\right) + a^3\operatorname{arcsinh}(ax)\,\text{PolyLog}\left(2, -e^{\operatorname{arcsinh}(ax)}\right) - a^3\operatorname{arcsinh}(ax)\,\text{PolyLog}\left(2, e^{\operatorname{arcsinh}(ax)}\right) + a\text{Subst}\left(\int \frac{1}{-\frac{1}{a^2}+\frac{x^2}{a^2}}\,dx, x, \sqrt{1+a^2x^2}\right) - a^3\text{Subst}\left(\int \text{PolyLog}\left(2, -e^x\right)\,dx, x, \operatorname{arcsinh}(ax)\right) + a^3\text{Subst}\left(\int \text{PolyLog}\left(2, e^x\right)\,dx, x, \operatorname{arcsinh}(ax)\right)$$

$$= -\frac{a^2\operatorname{arcsinh}(ax)}{x} - \frac{a\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2}{2x^2} - \frac{\operatorname{arcsinh}(ax)^3}{3x^3} + a^3\operatorname{arcsinh}(ax)^2\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(ax)}\right) - a^3\operatorname{arctanh}\left(\sqrt{1+a^2x^2}\right) + a^3\operatorname{arcsinh}(ax)\,\text{PolyLog}\left(2, -e^{\operatorname{arcsinh}(ax)}\right) - a^3\operatorname{arcsinh}(ax)\,\text{PolyLog}\left(2, e^{\operatorname{arcsinh}(ax)}\right) - a^3\text{Subst}\left(\int \frac{\text{PolyLog}(2,-x)}{x}\,dx, x, e^{\operatorname{arcsinh}(ax)}\right) + a^3\text{Subst}\left(\int \frac{\text{PolyLog}(2,x)}{x}\,dx, x, e^{\operatorname{arcsinh}(ax)}\right)$$

$$
\begin{aligned}
&= -\frac{a^2\text{arcsinh}(ax)}{x} - \frac{a\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{2x^2} - \frac{\text{arcsinh}(ax)^3}{3x^3} \\
&\quad + a^3\text{arcsinh}(ax)^2\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right) - a^3\text{arctanh}\left(\sqrt{1+a^2x^2}\right) \\
&\quad + a^3\text{arcsinh}(ax)\,\text{PolyLog}\left(2,-e^{\text{arcsinh}(ax)}\right) - a^3\text{arcsinh}(ax)\,\text{PolyLog}\left(2,e^{\text{arcsinh}(ax)}\right) \\
&\quad - a^3\,\text{PolyLog}\left(3,-e^{\text{arcsinh}(ax)}\right) + a^3\,\text{PolyLog}\left(3,e^{\text{arcsinh}(ax)}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.69 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.77

$$
\begin{aligned}
\int \frac{\text{arcsinh}(ax)^3}{x^4}\,dx = \frac{1}{48}a^3\Bigg(&-24\text{arcsinh}(ax)\coth\left(\frac{1}{2}\text{arcsinh}(ax)\right) \\
&+ 4\text{arcsinh}(ax)^3\coth\left(\frac{1}{2}\text{arcsinh}(ax)\right) \\
&- 6\text{arcsinh}(ax)^2\text{csch}^2\left(\frac{1}{2}\text{arcsinh}(ax)\right) \\
&- ax\text{arcsinh}(ax)^3\text{csch}^4\left(\frac{1}{2}\text{arcsinh}(ax)\right) \\
&- 24\text{arcsinh}(ax)^2\log\left(1-e^{-\text{arcsinh}(ax)}\right) \\
&+ 24\text{arcsinh}(ax)^2\log\left(1+e^{-\text{arcsinh}(ax)}\right) \\
&+ 48\log\left(\tanh\left(\frac{1}{2}\text{arcsinh}(ax)\right)\right) \\
&- 48\text{arcsinh}(ax)\,\text{PolyLog}\left(2,-e^{-\text{arcsinh}(ax)}\right) \\
&+ 48\text{arcsinh}(ax)\,\text{PolyLog}\left(2,e^{-\text{arcsinh}(ax)}\right) \\
&- 48\,\text{PolyLog}\left(3,-e^{-\text{arcsinh}(ax)}\right) + 48\,\text{PolyLog}\left(3,e^{-\text{arcsinh}(ax)}\right) \\
&- 6\text{arcsinh}(ax)^2\text{sech}^2\left(\frac{1}{2}\text{arcsinh}(ax)\right) \\
&- \frac{16\text{arcsinh}(ax)^3\sinh^4\left(\frac{1}{2}\text{arcsinh}(ax)\right)}{a^3x^3} \\
&+ 24\text{arcsinh}(ax)\tanh\left(\frac{1}{2}\text{arcsinh}(ax)\right) \\
&- 4\text{arcsinh}(ax)^3\tanh\left(\frac{1}{2}\text{arcsinh}(ax)\right)\Bigg)
\end{aligned}
$$

```
[In] Integrate[ArcSinh[a*x]^3/x^4,x]
```

```
[Out] (a^3*(-24*ArcSinh[a*x]*Coth[ArcSinh[a*x]/2] + 4*ArcSinh[a*x]^3*Coth[ArcSinh
[a*x]/2] - 6*ArcSinh[a*x]^2*Csch[ArcSinh[a*x]/2]^2 - a*x*ArcSinh[a*x]^3*Csc
h[ArcSinh[a*x]/2]^4 - 24*ArcSinh[a*x]^2*Log[1 - E^(-ArcSinh[a*x])] + 24*Arc
Sinh[a*x]^2*Log[1 + E^(-ArcSinh[a*x])] + 48*Log[Tanh[ArcSinh[a*x]/2]] - 48*
```

```
ArcSinh[a*x]*PolyLog[2, -E^(-ArcSinh[a*x])] + 48*ArcSinh[a*x]*PolyLog[2, E^
(-ArcSinh[a*x])] - 48*PolyLog[3, -E^(-ArcSinh[a*x])] + 48*PolyLog[3, E^(-Ar
cSinh[a*x])] - 6*ArcSinh[a*x]^2*Sech[ArcSinh[a*x]/2]^2 - (16*ArcSinh[a*x]^3
*Sinh[ArcSinh[a*x]/2]^4)/(a^3*x^3) + 24*ArcSinh[a*x]*Tanh[ArcSinh[a*x]/2] -
 4*ArcSinh[a*x]^3*Tanh[ArcSinh[a*x]/2]))/48
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.40

method	result
derivativedivides	$a^{3}\left(-\frac{\operatorname{arcsinh}(ax)\left(3\operatorname{arcsinh}(ax)\sqrt{a^{2}x^{2}+1}\,ax+2\operatorname{arcsinh}(ax)^{2}+6a^{2}x^{2}\right)}{6a^{3}x^{3}}+\frac{\operatorname{arcsinh}(ax)^{2}\ln\left(1+ax+\sqrt{a^{2}x^{2}+1}\right)}{2}+\mathrm{a}\right.$
default	$a^{3}\left(-\frac{\operatorname{arcsinh}(ax)\left(3\operatorname{arcsinh}(ax)\sqrt{a^{2}x^{2}+1}\,ax+2\operatorname{arcsinh}(ax)^{2}+6a^{2}x^{2}\right)}{6a^{3}x^{3}}+\frac{\operatorname{arcsinh}(ax)^{2}\ln\left(1+ax+\sqrt{a^{2}x^{2}+1}\right)}{2}+\mathrm{a}\right.$

```
[In] int(arcsinh(a*x)^3/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*(-1/6/a^3/x^3*arcsinh(a*x)*(3*arcsinh(a*x)*(a^2*x^2+1)^(1/2)*a*x+2*arcs
inh(a*x)^2+6*a^2*x^2)+1/2*arcsinh(a*x)^2*ln(1+a*x+(a^2*x^2+1)^(1/2))+arcsin
h(a*x)*polylog(2,-a*x-(a^2*x^2+1)^(1/2))-polylog(3,-a*x-(a^2*x^2+1)^(1/2))-
1/2*arcsinh(a*x)^2*ln(1-a*x-(a^2*x^2+1)^(1/2))-arcsinh(a*x)*polylog(2,a*x+(
a^2*x^2+1)^(1/2))+polylog(3,a*x+(a^2*x^2+1)^(1/2))-2*arctanh(a*x+(a^2*x^2+1
)^(1/2)))
```

Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^4}\,dx=\int \frac{\operatorname{arsinh}\left(ax\right)^3}{x^4}\,dx$$

```
[In] integrate(arcsinh(a*x)^3/x^4,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x)^3/x^4, x)
```

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^4}\,dx = \int \frac{\operatorname{asinh}^3\left(ax\right)}{x^4}\,dx$$

```
[In] integrate(asinh(a*x)**3/x**4,x)
```

```
[Out] Integral(asinh(a*x)**3/x**4, x)
```

Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^4}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^3}{x^4}\,dx$$

```
[In] integrate(arcsinh(a*x)^3/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*log(a*x + sqrt(a^2*x^2 + 1))^3/x^3 + integrate((a^3*x^2 + sqrt(a^2*x^2
 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))^2/(a^3*x^6 + a*x^4 + (a^2*x^5
 + x^3)*sqrt(a^2*x^2 + 1)), x)
```

Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^4}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^3}{x^4}\,dx$$

```
[In] integrate(arcsinh(a*x)^3/x^4,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^3/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^4}\,dx = \int \frac{\operatorname{asinh}(a\,x)^3}{x^4}\,dx$$

```
[In] int(asinh(a*x)^3/x^4,x)
```

```
[Out] int(asinh(a*x)^3/x^4, x)
```

3.31 $\int \frac{\operatorname{arcsinh}(ax)^3}{x^5}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 159

$$\begin{aligned}\int \frac{\operatorname{arcsinh}(ax)^3}{x^5}\,dx = &-\frac{a^3\sqrt{1+a^2x^2}}{4x}-\frac{a^2\operatorname{arcsinh}(ax)}{4x^2}+\frac{1}{2}a^4\operatorname{arcsinh}(ax)^2\\&-\frac{a\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2}{4x^3}+\frac{a^3\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2}{2x}-\frac{\operatorname{arcsinh}(ax)^3}{4x^4}\\&-a^4\operatorname{arcsinh}(ax)\log\left(1-e^{2\operatorname{arcsinh}(ax)}\right)-\frac{1}{2}a^4\operatorname{PolyLog}\left(2,e^{2\operatorname{arcsinh}(ax)}\right)\end{aligned}$$

```
[Out] -1/4*a^2*arcsinh(a*x)/x^2+1/2*a^4*arcsinh(a*x)^2-1/4*arcsinh(a*x)^3/x^4-a^4
*arcsinh(a*x)*ln(1-(a*x+(a^2*x^2+1)^(1/2))^2)-1/2*a^4*polylog(2,(a*x+(a^2*x
^2+1)^(1/2))^2)-1/4*a^3*(a^2*x^2+1)^(1/2)/x-1/4*a*arcsinh(a*x)^2*(a^2*x^2+1
)^(1/2)/x^3+1/2*a^3*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {5776, 5809, 5800, 5775, 3797, 2221, 2317, 2438, 270}

$$\begin{aligned}\int \frac{\operatorname{arcsinh}(ax)^3}{x^5}\,dx = &-\frac{1}{2}a^4\operatorname{PolyLog}\left(2,e^{2\operatorname{arcsinh}(ax)}\right)+\frac{1}{2}a^4\operatorname{arcsinh}(ax)^2\\&-a^4\operatorname{arcsinh}(ax)\log\left(1-e^{2\operatorname{arcsinh}(ax)}\right)\\&-\frac{a^2\operatorname{arcsinh}(ax)}{4x^2}-\frac{a\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^2}{4x^3}\\&+\frac{a^3\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^2}{2x}-\frac{a^3\sqrt{a^2x^2+1}}{4x}-\frac{\operatorname{arcsinh}(ax)^3}{4x^4}\end{aligned}$$

```
[In] Int[ArcSinh[a*x]^3/x^5,x]
```

```
[Out] -1/4*(a^3*Sqrt[1 + a^2*x^2])/x - (a^2*ArcSinh[a*x])/(4*x^2) + (a^4*ArcSinh[
a*x]^2)/2 - (a*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2)/(4*x^3) + (a^3*Sqrt[1 + a^
2*x^2]*ArcSinh[a*x]^2)/(2*x) - ArcSinh[a*x]^3/(4*x^4) - a^4*ArcSinh[a*x]*Lo
g[1 - E^(2*ArcSinh[a*x])] - (a^4*PolyLog[2, E^(2*ArcSinh[a*x])])/2
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
```

```
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5800

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*
ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[
e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\operatorname{arcsinh}(ax)^3}{4x^4}+\frac{1}{4}(3a)\int\frac{\operatorname{arcsinh}(ax)^2}{x^4\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{a\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2}{4x^3}-\frac{\operatorname{arcsinh}(ax)^3}{4x^4}+\frac{1}{2}a^2\int\frac{\operatorname{arcsinh}(ax)}{x^3}\,dx-\frac{1}{2}a^3\int\frac{\operatorname{arcsinh}(ax)^2}{x^2\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{a^2\operatorname{arcsinh}(ax)}{4x^2}-\frac{a\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2}{4x^3}+\frac{a^3\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2}{2x}\\
&\quad -\frac{\operatorname{arcsinh}(ax)^3}{4x^4}+\frac{1}{4}a^3\int\frac{1}{x^2\sqrt{1+a^2x^2}}\,dx-a^4\int\frac{\operatorname{arcsinh}(ax)}{x}\,dx\\
&= -\frac{a^3\sqrt{1+a^2x^2}}{4x}-\frac{a^2\operatorname{arcsinh}(ax)}{4x^2}-\frac{a\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2}{4x^3}\\
&\quad +\frac{a^3\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2}{2x}-\frac{\operatorname{arcsinh}(ax)^3}{4x^4}\\
&\quad -a^4\operatorname{Subst}\left(\int x\coth(x)\,dx,x,\operatorname{arcsinh}(ax)\right)\\
&= -\frac{a^3\sqrt{1+a^2x^2}}{4x}-\frac{a^2\operatorname{arcsinh}(ax)}{4x^2}+\frac{1}{2}a^4\operatorname{arcsinh}(ax)^2-\frac{a\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2}{4x^3}\\
&\quad +\frac{a^3\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2}{2x}-\frac{\operatorname{arcsinh}(ax)^3}{4x^4}+\left(2a^4\right)\operatorname{Subst}\left(\int\frac{e^{2x}x}{1-e^{2x}}\,dx,x,\operatorname{arcsinh}(ax)\right)
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{a^3\sqrt{1+a^2x^2}}{4x} - \frac{a^2\text{arcsinh}(ax)}{4x^2} + \frac{1}{2}a^4\text{arcsinh}(ax)^2 - \frac{a\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{4x^3} \\
&\quad + \frac{a^3\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{2x} - \frac{\text{arcsinh}(ax)^3}{4x^4} - a^4\text{arcsinh}(ax)\log\left(1-e^{2\text{arcsinh}(ax)}\right) \\
&\quad + a^4\text{Subst}\left(\int \log\left(1-e^{2x}\right)\, dx, x, \text{arcsinh}(ax)\right) \\
&= -\frac{a^3\sqrt{1+a^2x^2}}{4x} - \frac{a^2\text{arcsinh}(ax)}{4x^2} + \frac{1}{2}a^4\text{arcsinh}(ax)^2 \\
&\quad - \frac{a\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{4x^3} + \frac{a^3\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{2x} - \frac{\text{arcsinh}(ax)^3}{4x^4} \\
&\quad - a^4\text{arcsinh}(ax)\log\left(1-e^{2\text{arcsinh}(ax)}\right) + \frac{1}{2}a^4\text{Subst}\left(\int \frac{\log(1-x)}{x}\, dx, x, e^{2\text{arcsinh}(ax)}\right) \\
&= -\frac{a^3\sqrt{1+a^2x^2}}{4x} - \frac{a^2\text{arcsinh}(ax)}{4x^2} + \frac{1}{2}a^4\text{arcsinh}(ax)^2 \\
&\quad - \frac{a\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{4x^3} + \frac{a^3\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}{2x} - \frac{\text{arcsinh}(ax)^3}{4x^4} \\
&\quad - a^4\text{arcsinh}(ax)\log\left(1-e^{2\text{arcsinh}(ax)}\right) - \frac{1}{2}a^4\,\text{PolyLog}\left(2, e^{2\text{arcsinh}(ax)}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.67

$$
\begin{aligned}
\int \frac{\text{arcsinh}(ax)^3}{x^5}\, dx = \frac{1}{4}\Bigg(&-\frac{\text{arcsinh}(ax)^3}{x^4} + a^4\Bigg(-\frac{\sqrt{1+a^2x^2}\left(1+\left(-2+\frac{1}{a^2x^2}\right)\text{arcsinh}(ax)^2\right)}{ax} \\
&- \text{arcsinh}(ax)\left(\frac{1}{a^2x^2} + 2\text{arcsinh}(ax) + 4\log\left(1-e^{-2\text{arcsinh}(ax)}\right)\right) \\
&+ 2\,\text{PolyLog}\left(2, e^{-2\text{arcsinh}(ax)}\right)\Bigg)\Bigg)
\end{aligned}
$$

```
[In] Integrate[ArcSinh[a*x]^3/x^5,x]
```

```
[Out] (-(ArcSinh[a*x]^3/x^4) + a^4*(-((Sqrt[1 + a^2*x^2]*(1 + (-2 + 1/(a^2*x^2))*
ArcSinh[a*x]^2))/(a*x)) - ArcSinh[a*x]*(1/(a^2*x^2) + 2*ArcSinh[a*x] + 4*Lo
g[1 - E^(-2*ArcSinh[a*x])]) + 2*PolyLog[2, E^(-2*ArcSinh[a*x])]))/4
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.34

method	result
derivativedivides	$a^{4}\left(-\frac{-2a^{3}x^{3}\operatorname{arcsinh}(ax)^{2}\sqrt{a^{2}x^{2}+1}+2a^{4}x^{4}\operatorname{arcsinh}(ax)^{2}+\operatorname{arcsinh}(ax)^{2}\sqrt{a^{2}x^{2}+1}\,ax+a^{3}x^{3}\sqrt{a^{2}x^{2}+1}-a^{4}x^{4}+\mathrm{arcsin}}{4a^{4}x^{4}}\right.$
default	$a^{4}\left(-\frac{-2a^{3}x^{3}\operatorname{arcsinh}(ax)^{2}\sqrt{a^{2}x^{2}+1}+2a^{4}x^{4}\operatorname{arcsinh}(ax)^{2}+\operatorname{arcsinh}(ax)^{2}\sqrt{a^{2}x^{2}+1}\,ax+a^{3}x^{3}\sqrt{a^{2}x^{2}+1}-a^{4}x^{4}+\mathrm{arcsin}}{4a^{4}x^{4}}\right.$

```
[In] int(arcsinh(a*x)^3/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] a^4*(-1/4*(-2*a^3*x^3*arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)+2*a^4*x^4*arcsinh(a*
x)^2+arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)*a*x+a^3*x^3*(a^2*x^2+1)^(1/2)-a^4*x^4
+arcsinh(a*x)^3+a^2*x^2*arcsinh(a*x))/a^4/x^4+arcsinh(a*x)^2-arcsinh(a*x)*l
n(1+a*x+(a^2*x^2+1)^(1/2))-polylog(2,-a*x-(a^2*x^2+1)^(1/2))-arcsinh(a*x)*l
n(1-a*x-(a^2*x^2+1)^(1/2))-polylog(2,a*x+(a^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax)^{3}}{x^{5}}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^{3}}{x^{5}}\,dx$$

```
[In] integrate(arcsinh(a*x)^3/x^5,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x)^3/x^5, x)
```

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)^{3}}{x^{5}}\,dx = \int \frac{\operatorname{asinh}^{3}\left(ax\right)}{x^{5}}\,dx$$

```
[In] integrate(asinh(a*x)**3/x**5,x)
```

```
[Out] Integral(asinh(a*x)**3/x**5, x)
```

Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^5}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^3}{x^5}\,dx$$

```
[In] integrate(arcsinh(a*x)^3/x^5,x, algorithm="maxima")
```

```
[Out] -1/4*log(a*x + sqrt(a^2*x^2 + 1))^3/x^4 + integrate(3/4*(a^3*x^2 + sqrt(a^2
*x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))^2/(a^3*x^7 + a*x^5 + (a^2
*x^6 + x^4)*sqrt(a^2*x^2 + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^5}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(arcsinh(a*x)^3/x^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^3}{x^5}\,dx = \int \frac{\operatorname{asinh}(a\,x)^3}{x^5}\,dx$$

```
[In] int(asinh(a*x)^3/x^5,x)
```

```
[Out] int(asinh(a*x)^3/x^5, x)
```

3.32 $\int x^5 \text{arcsinh}(ax)^4 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 276

$$
\begin{aligned}
\int x^5 \text{arcsinh}(ax)^4 \, dx &= \frac{245x^2}{1152a^4} - \frac{65x^4}{3456a^2} + \frac{x^6}{324} - \frac{245x\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{576a^5} \\
&+ \frac{65x^3\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{864a^3} - \frac{x^5\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{54a} \\
&+ \frac{245\text{arcsinh}(ax)^2}{1152a^6} + \frac{5x^2\text{arcsinh}(ax)^2}{16a^4} - \frac{5x^4\text{arcsinh}(ax)^2}{48a^2} \\
&+ \frac{1}{18}x^6\text{arcsinh}(ax)^2 - \frac{5x\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{24a^5} \\
&+ \frac{5x^3\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{36a^3} - \frac{x^5\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{9a} \\
&+ \frac{5\text{arcsinh}(ax)^4}{96a^6} + \frac{1}{6}x^6\text{arcsinh}(ax)^4
\end{aligned}
$$

```
[Out] 245/1152*x^2/a^4-65/3456*x^4/a^2+1/324*x^6+245/1152*arcsinh(a*x)^2/a^6+5/16
*x^2*arcsinh(a*x)^2/a^4-5/48*x^4*arcsinh(a*x)^2/a^2+1/18*x^6*arcsinh(a*x)^2
+5/96*arcsinh(a*x)^4/a^6+1/6*x^6*arcsinh(a*x)^4-245/576*x*arcsinh(a*x)*(a^2
*x^2+1)^(1/2)/a^5+65/864*x^3*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a^3-1/54*x^5*ar
csinh(a*x)*(a^2*x^2+1)^(1/2)/a-5/24*x*arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)/a^5+
5/36*x^3*arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)/a^3-1/9*x^5*arcsinh(a*x)^3*(a^2*x
^2+1)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5776, 5812, 5783, 30}

$$\begin{aligned}\int x^5\text{arcsinh}(ax)^4\,dx=&\frac{5\text{arcsinh}(ax)^4}{96a^6}+\frac{245\text{arcsinh}(ax)^2}{1152a^6}+\frac{5x^2\text{arcsinh}(ax)^2}{16a^4}+\frac{245x^2}{1152a^4}\\&-\frac{5x^4\text{arcsinh}(ax)^2}{48a^2}-\frac{x^5\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{9a}\\&-\frac{x^5\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{54a}-\frac{65x^4}{3456a^2}\\&-\frac{5x\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{24a^5}-\frac{245x\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{576a^5}\\&+\frac{5x^3\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{36a^3}+\frac{65x^3\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{864a^3}\\&+\frac{1}{6}x^6\text{arcsinh}(ax)^4+\frac{1}{18}x^6\text{arcsinh}(ax)^2+\frac{x^6}{324}\end{aligned}$$

```
[In] Int[x^5*ArcSinh[a*x]^4,x]
```

```
[Out] (245*x^2)/(1152*a^4) - (65*x^4)/(3456*a^2) + x^6/324 - (245*x*Sqrt[1 + a^2*
x^2]*ArcSinh[a*x])/(576*a^5) + (65*x^3*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(864
*a^3) - (x^5*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(54*a) + (245*ArcSinh[a*x]^2)/
(1152*a^6) + (5*x^2*ArcSinh[a*x]^2)/(16*a^4) - (5*x^4*ArcSinh[a*x]^2)/(48*a
^2) + (x^6*ArcSinh[a*x]^2)/18 - (5*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(24*
a^5) + (5*x^3*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(36*a^3) - (x^5*Sqrt[1 + a^
2*x^2]*ArcSinh[a*x]^3)/(9*a) + (5*ArcSinh[a*x]^4)/(96*a^6) + (x^6*ArcSinh[a
*x]^4)/6
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
```

```
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{6}x^6\text{arcsinh}(ax)^4-\frac{1}{3}(2a)\int\frac{x^6\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{x^5\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{9a}+\frac{1}{6}x^6\text{arcsinh}(ax)^4\\
&\quad+\frac{1}{3}\int x^5\text{arcsinh}(ax)^2\,dx+\frac{5\int\frac{x^4\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx}{9a}\\
&= \frac{1}{18}x^6\text{arcsinh}(ax)^2+\frac{5x^3\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{36a^3}\\
&\quad-\frac{x^5\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{9a}+\frac{1}{6}x^6\text{arcsinh}(ax)^4-\frac{5\int\frac{x^2\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx}{12a^3}\\
&\quad-\frac{5\int x^3\text{arcsinh}(ax)^2\,dx}{12a^2}-\frac{1}{9}a\int\frac{x^6\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{x^5\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{54a}-\frac{5x^4\text{arcsinh}(ax)^2}{48a^2}+\frac{1}{18}x^6\text{arcsinh}(ax)^2\\
&\quad-\frac{5x\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{24a^5}+\frac{5x^3\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{36a^3}\\
&\quad-\frac{x^5\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{9a}+\frac{1}{6}x^6\text{arcsinh}(ax)^4+\frac{\int x^5\,dx}{54}+\frac{5\int\frac{\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx}{24a^5}\\
&\quad+\frac{5\int x\text{arcsinh}(ax)^2\,dx}{8a^4}+\frac{5\int\frac{x^4\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx}{54a}+\frac{5\int\frac{x^4\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx}{24a}
\end{aligned}
$$

$$\begin{aligned}
&= \frac{x^6}{324} + \frac{65x^3\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{864a^3} - \frac{x^5\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{54a} + \frac{5x^2\operatorname{arcsinh}(ax)^2}{16a^4} \\
&\quad - \frac{5x^4\operatorname{arcsinh}(ax)^2}{48a^2} + \frac{1}{18}x^6\operatorname{arcsinh}(ax)^2 - \frac{5x\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}{24a^5} \\
&\quad + \frac{5x^3\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}{36a^3} - \frac{x^5\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}{9a} \\
&\quad + \frac{5\operatorname{arcsinh}(ax)^4}{96a^6} + \frac{1}{6}x^6\operatorname{arcsinh}(ax)^4 - \frac{5\int \frac{x^2\operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx}{72a^3} \\
&\quad - \frac{5\int \frac{x^2\operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx}{32a^3} - \frac{5\int \frac{x^2\operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx}{8a^3} - \frac{5\int x^3\,dx}{216a^2} - \frac{5\int x^3\,dx}{96a^2} \\
&= -\frac{65x^4}{3456a^2} + \frac{x^6}{324} - \frac{245x\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{576a^5} + \frac{65x^3\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{864a^3} \\
&\quad - \frac{x^5\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{54a} + \frac{5x^2\operatorname{arcsinh}(ax)^2}{16a^4} - \frac{5x^4\operatorname{arcsinh}(ax)^2}{48a^2} \\
&\quad + \frac{1}{18}x^6\operatorname{arcsinh}(ax)^2 - \frac{5x\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}{24a^5} \\
&\quad + \frac{5x^3\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}{36a^3} - \frac{x^5\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}{9a} \\
&\quad + \frac{5\operatorname{arcsinh}(ax)^4}{96a^6} + \frac{1}{6}x^6\operatorname{arcsinh}(ax)^4 + \frac{5\int \frac{\operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx}{144a^5} \\
&\quad + \frac{5\int \frac{\operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx}{64a^5} + \frac{5\int \frac{\operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx}{16a^5} + \frac{5\int x\,dx}{144a^4} + \frac{5\int x\,dx}{64a^4} + \frac{5\int x\,dx}{16a^4} \\
&= \frac{245x^2}{1152a^4} - \frac{65x^4}{3456a^2} + \frac{x^6}{324} - \frac{245x\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{576a^5} \\
&\quad + \frac{65x^3\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{864a^3} - \frac{x^5\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{54a} + \frac{245\operatorname{arcsinh}(ax)^2}{1152a^6} \\
&\quad + \frac{5x^2\operatorname{arcsinh}(ax)^2}{16a^4} - \frac{5x^4\operatorname{arcsinh}(ax)^2}{48a^2} + \frac{1}{18}x^6\operatorname{arcsinh}(ax)^2 \\
&\quad - \frac{5x\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}{24a^5} + \frac{5x^3\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}{36a^3} \\
&\quad - \frac{x^5\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}{9a} + \frac{5\operatorname{arcsinh}(ax)^4}{96a^6} + \frac{1}{6}x^6\operatorname{arcsinh}(ax)^4
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.60

$$\int x^5\operatorname{arcsinh}(ax)^4\,dx$$
$$= \frac{a^2x^2(2205-195a^2x^2+32a^4x^4)-6ax\sqrt{1+a^2x^2}(735-130a^2x^2+32a^4x^4)\operatorname{arcsinh}(ax)+9(245+360a^2}{}$$

```
[In] Integrate[x^5*ArcSinh[a*x]^4,x]
```

```
[Out] (a^2*x^2*(2205 - 195*a^2*x^2 + 32*a^4*x^4) - 6*a*x*Sqrt[1 + a^2*x^2]*(735 -
 130*a^2*x^2 + 32*a^4*x^4)*ArcSinh[a*x] + 9*(245 + 360*a^2*x^2 - 120*a^4*x^
4 + 64*a^6*x^6)*ArcSinh[a*x]^2 - 144*a*x*Sqrt[1 + a^2*x^2]*(15 - 10*a^2*x^2
 + 8*a^4*x^4)*ArcSinh[a*x]^3 + 108*(5 + 16*a^6*x^6)*ArcSinh[a*x]^4)/(10368*
a^6)
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{a^6x^6\operatorname{arcsinh}(ax)^4}{6}-\frac{a^5x^5\operatorname{arcsinh}(ax)^3\sqrt{a^2x^2+1}}{9}+\frac{5a^3x^3\operatorname{arcsinh}(ax)^3\sqrt{a^2x^2+1}}{36}-\frac{5\operatorname{arcsinh}(ax)^3\sqrt{a^2x^2+1}\,ax}{24}+\frac{5\operatorname{arcsinh}(ax)^4}{96}+$
default	$\frac{a^6x^6\operatorname{arcsinh}(ax)^4}{6}-\frac{a^5x^5\operatorname{arcsinh}(ax)^3\sqrt{a^2x^2+1}}{9}+\frac{5a^3x^3\operatorname{arcsinh}(ax)^3\sqrt{a^2x^2+1}}{36}-\frac{5\operatorname{arcsinh}(ax)^3\sqrt{a^2x^2+1}\,ax}{24}+\frac{5\operatorname{arcsinh}(ax)^4}{96}+$

```
[In] int(x^5*arcsinh(a*x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^6*(1/6*a^6*x^6*arcsinh(a*x)^4-1/9*a^5*x^5*arcsinh(a*x)^3*(a^2*x^2+1)^(1
/2)+5/36*a^3*x^3*arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)-5/24*arcsinh(a*x)^3*(a^2*
x^2+1)^(1/2)*a*x+5/96*arcsinh(a*x)^4+1/18*arcsinh(a*x)^2*a^6*x^6-1/54*arcsi
nh(a*x)*(a^2*x^2+1)^(1/2)*a^5*x^5+65/864*a^3*x^3*arcsinh(a*x)*(a^2*x^2+1)^(
1/2)-245/576*arcsinh(a*x)*(a^2*x^2+1)^(1/2)*a*x-115/1152*arcsinh(a*x)^2+1/3
24*a^6*x^6-65/3456*a^4*x^4+245/1152*a^2*x^2+245/1152-5/48*a^4*x^4*arcsinh(a
*x)^2+5/16*arcsinh(a*x)^2*(a^2*x^2+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.75

$$\int x^5 \text{arcsinh}(ax)^4 \, dx$$

$$= \frac{32\,a^6x^6 - 195\,a^4x^4 + 108\,(16\,a^6x^6 + 5)\log\left(ax + \sqrt{a^2x^2 + 1}\right)^4 - 144\,(8\,a^5x^5 - 10\,a^3x^3 + 15\,ax)\sqrt{a^2x^2 + 1}}{}$$

```
[In] integrate(x^5*arcsinh(a*x)^4,x, algorithm="fricas")
```

```
[Out] 1/10368*(32*a^6*x^6 - 195*a^4*x^4 + 108*(16*a^6*x^6 + 5)*log(a*x + sqrt(a^2
*x^2 + 1))^4 - 144*(8*a^5*x^5 - 10*a^3*x^3 + 15*a*x)*sqrt(a^2*x^2 + 1)*log(
a*x + sqrt(a^2*x^2 + 1))^3 + 2205*a^2*x^2 + 9*(64*a^6*x^6 - 120*a^4*x^4 + 3
60*a^2*x^2 + 245)*log(a*x + sqrt(a^2*x^2 + 1))^2 - 6*(32*a^5*x^5 - 130*a^3*
x^3 + 735*a*x)*sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1)))/a^6
```

Sympy [A] (verification not implemented)

Time = 1.32 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.97

$$\int x^5 \text{arcsinh}(ax)^4 \, dx$$

$$= \begin{cases} \frac{x^6 \operatorname{asinh}^4(ax)}{6} + \frac{x^6 \operatorname{asinh}^2(ax)}{18} + \frac{x^6}{324} - \frac{x^5\sqrt{a^2x^2+1}\operatorname{asinh}^3(ax)}{9a} - \frac{x^5\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{54a} - \frac{5x^4\operatorname{asinh}^2(ax)}{48a^2} - \frac{65x^4}{3456a^2} + \frac{5x^3\sqrt{a^2x}}{} \\ 0 \end{cases}$$

```
[In] integrate(x**5*asinh(a*x)**4,x)
```

```
[Out] Piecewise((x**6*asinh(a*x)**4/6 + x**6*asinh(a*x)**2/18 + x**6/324 - x**5*s
qrt(a**2*x**2 + 1)*asinh(a*x)**3/(9*a) - x**5*sqrt(a**2*x**2 + 1)*asinh(a*x
)/(54*a) - 5*x**4*asinh(a*x)**2/(48*a**2) - 65*x**4/(3456*a**2) + 5*x**3*sq
rt(a**2*x**2 + 1)*asinh(a*x)**3/(36*a**3) + 65*x**3*sqrt(a**2*x**2 + 1)*asi
nh(a*x)/(864*a**3) + 5*x**2*asinh(a*x)**2/(16*a**4) + 245*x**2/(1152*a**4)
- 5*x*sqrt(a**2*x**2 + 1)*asinh(a*x)**3/(24*a**5) - 245*x*sqrt(a**2*x**2 +
1)*asinh(a*x)/(576*a**5) + 5*asinh(a*x)**4/(96*a**6) + 245*asinh(a*x)**2/(1
152*a**6), Ne(a, 0)), (0, True))
```

Maxima [F]

$$\int x^5 \text{arcsinh}(ax)^4 \, dx = \int x^5 \operatorname{arsinh}(ax)^4 \, dx$$

```
[In] integrate(x^5*arcsinh(a*x)^4,x, algorithm="maxima")
```

```
[Out] 1/6*x^6*log(a*x + sqrt(a^2*x^2 + 1))^4 - integrate(2/3*(a^3*x^8 + sqrt(a^2*
x^2 + 1)*a^2*x^7 + a*x^6)*log(a*x + sqrt(a^2*x^2 + 1))^3/(a^3*x^3 + a*x + (
a^2*x^2 + 1)^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int x^5 \text{arcsinh}(ax)^4 \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^5*arcsinh(a*x)^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^5 \text{arcsinh}(ax)^4 \, dx = \int x^5 \operatorname{asinh}(a\,x)^4 \, dx$$

```
[In] int(x^5*asinh(a*x)^4,x)
```

```
[Out] int(x^5*asinh(a*x)^4, x)
```

3.33 $\int x^4 \text{arcsinh}(ax)^4 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 244

$$\int x^4 \text{arcsinh}(ax)^4 \, dx = \frac{16576x}{5625a^4} - \frac{1088x^3}{16875a^2} + \frac{24x^5}{3125} - \frac{16576\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{5625a^5} + \frac{1088x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{5625a^3} - \frac{24x^4\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{625a} + \frac{32x\text{arcsinh}(ax)^2}{25a^4} - \frac{16x^3\text{arcsinh}(ax)^2}{75a^2} + \frac{12}{125}x^5\text{arcsinh}(ax)^2 - \frac{32\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{75a^5} + \frac{16x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{75a^3} - \frac{4x^4\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{25a} + \frac{1}{5}x^5\text{arcsinh}(ax)^4$$

```
[Out] 16576/5625*x/a^4-1088/16875*x^3/a^2+24/3125*x^5+32/25*x*arcsinh(a*x)^2/a^4-
16/75*x^3*arcsinh(a*x)^2/a^2+12/125*x^5*arcsinh(a*x)^2+1/5*x^5*arcsinh(a*x)
^4-16576/5625*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a^5+1088/5625*x^2*arcsinh(a*x)
*(a^2*x^2+1)^(1/2)/a^3-24/625*x^4*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a-32/75*ar
csinh(a*x)^3*(a^2*x^2+1)^(1/2)/a^5+16/75*x^2*arcsinh(a*x)^3*(a^2*x^2+1)^(1/
2)/a^3-4/25*x^4*arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5776, 5812, 5798, 5772, 8, 30}

$$\begin{aligned}\int x^4\text{arcsinh}(ax)^4\,dx = {} & \frac{32x\text{arcsinh}(ax)^2}{25a^4}+\frac{16576x}{5625a^4}-\frac{16x^3\text{arcsinh}(ax)^2}{75a^2} \\ & -\frac{4x^4\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{25a}-\frac{24x^4\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{625a} \\ & -\frac{1088x^3}{16875a^2}-\frac{32\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{75a^5} \\ & -\frac{16576\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{5625a^5}+\frac{16x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{75a^3} \\ & +\frac{1088x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{5625a^3} \\ & +\frac{1}{5}x^5\text{arcsinh}(ax)^4+\frac{12}{125}x^5\text{arcsinh}(ax)^2+\frac{24x^5}{3125}\end{aligned}$$

```
[In] Int[x^4*ArcSinh[a*x]^4,x]
```

```
[Out] (16576*x)/(5625*a^4) - (1088*x^3)/(16875*a^2) + (24*x^5)/3125 - (16576*Sqrt
[1 + a^2*x^2]*ArcSinh[a*x])/(5625*a^5) + (1088*x^2*Sqrt[1 + a^2*x^2]*ArcSin
h[a*x])/(5625*a^3) - (24*x^4*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(625*a) + (32*
x*ArcSinh[a*x]^2)/(25*a^4) - (16*x^3*ArcSinh[a*x]^2)/(75*a^2) + (12*x^5*Arc
Sinh[a*x]^2)/125 - (32*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(75*a^5) + (16*x^2
*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(75*a^3) - (4*x^4*Sqrt[1 + a^2*x^2]*ArcS
inh[a*x]^3)/(25*a) + (x^5*ArcSinh[a*x]^4)/5
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{5}x^5\operatorname{arcsinh}(ax)^4-\frac{1}{5}(4a)\int\frac{x^5\operatorname{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{4x^4\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}{25a}+\frac{1}{5}x^5\operatorname{arcsinh}(ax)^4\\
&\quad+\frac{12}{25}\int x^4\operatorname{arcsinh}(ax)^2\,dx+\frac{16\int\frac{x^3\operatorname{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx}{25a}\\
&= \frac{12}{125}x^5\operatorname{arcsinh}(ax)^2+\frac{16x^2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}{75a^3}\\
&\quad-\frac{4x^4\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}{25a}+\frac{1}{5}x^5\operatorname{arcsinh}(ax)^4-\frac{32\int\frac{x\operatorname{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx}{75a^3}\\
&\quad-\frac{16\int x^2\operatorname{arcsinh}(ax)^2\,dx}{25a^2}-\frac{1}{125}(24a)\int\frac{x^5\operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{24x^4\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{625a} - \frac{16x^3\operatorname{arcsinh}(ax)^2}{75a^2} + \frac{12}{125}x^5\operatorname{arcsinh}(ax)^2 \\
&\quad - \frac{32\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}{75a^5} + \frac{16x^2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}{75a^3} \\
&\quad - \frac{4x^4\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}{25a} + \frac{1}{5}x^5\operatorname{arcsinh}(ax)^4 + \frac{24\int x^4\,dx}{625} \\
&\quad + \frac{32\int \operatorname{arcsinh}(ax)^2\,dx}{25a^4} + \frac{96\int \frac{x^3\operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx}{625a} + \frac{32\int \frac{x^3\operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx}{75a}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{24x^5}{3125} + \frac{1088x^2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{5625a^3} - \frac{24x^4\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{625a} \\
&\quad + \frac{32x\operatorname{arcsinh}(ax)^2}{25a^4} - \frac{16x^3\operatorname{arcsinh}(ax)^2}{75a^2} + \frac{12}{125}x^5\operatorname{arcsinh}(ax)^2 \\
&\quad - \frac{32\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}{75a^5} + \frac{16x^2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}{75a^3} \\
&\quad - \frac{4x^4\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}{25a} + \frac{1}{5}x^5\operatorname{arcsinh}(ax)^4 - \frac{64\int \frac{x\operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx}{625a^3} \\
&\quad - \frac{64\int \frac{x\operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx}{225a^3} - \frac{64\int \frac{x\operatorname{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx}{25a^3} - \frac{32\int x^2\,dx}{625a^2} - \frac{32\int x^2\,dx}{225a^2}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{1088x^3}{16875a^2} + \frac{24x^5}{3125} - \frac{16576\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{5625a^5} + \frac{1088x^2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{5625a^3} \\
&\quad - \frac{24x^4\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{625a} + \frac{32x\operatorname{arcsinh}(ax)^2}{25a^4} - \frac{16x^3\operatorname{arcsinh}(ax)^2}{75a^2} \\
&\quad + \frac{12}{125}x^5\operatorname{arcsinh}(ax)^2 - \frac{32\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}{75a^5} + \frac{16x^2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}{75a^3} \\
&\quad - \frac{4x^4\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}{25a} + \frac{1}{5}x^5\operatorname{arcsinh}(ax)^4 + \frac{64\int 1\,dx}{625a^4} + \frac{64\int 1\,dx}{225a^4} + \frac{64\int 1\,dx}{25a^4}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{16576x}{5625a^4} - \frac{1088x^3}{16875a^2} + \frac{24x^5}{3125} - \frac{16576\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{5625a^5} \\
&\quad + \frac{1088x^2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{5625a^3} - \frac{24x^4\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{625a} + \frac{32x\operatorname{arcsinh}(ax)^2}{25a^4} \\
&\quad - \frac{16x^3\operatorname{arcsinh}(ax)^2}{75a^2} + \frac{12}{125}x^5\operatorname{arcsinh}(ax)^2 - \frac{32\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}{75a^5} \\
&\quad + \frac{16x^2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}{75a^3} - \frac{4x^4\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}{25a} + \frac{1}{5}x^5\operatorname{arcsinh}(ax)^4
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.61

$$\int x^4 \operatorname{arcsinh}(ax)^4\,dx$$
$$= \frac{8ax(31080-680a^2x^2+81a^4x^4)-120\sqrt{1+a^2x^2}(2072-136a^2x^2+27a^4x^4)\operatorname{arcsinh}(ax)+900ax(120-2}{8437}$$

```
[In] Integrate[x^4*ArcSinh[a*x]^4,x]
```

```
[Out] (8*a*x*(31080 - 680*a^2*x^2 + 81*a^4*x^4) - 120*Sqrt[1 + a^2*x^2]*(2072 - 1
36*a^2*x^2 + 27*a^4*x^4)*ArcSinh[a*x] + 900*a*x*(120 - 20*a^2*x^2 + 9*a^4*x
^4)*ArcSinh[a*x]^2 - 4500*Sqrt[1 + a^2*x^2]*(8 - 4*a^2*x^2 + 3*a^4*x^4)*Arc
Sinh[a*x]^3 + 16875*a^5*x^5*ArcSinh[a*x]^4)/(84375*a^5)
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{a^5x^5\operatorname{arcsinh}(ax)^4}{5}-\frac{32\operatorname{arcsinh}(ax)^3\sqrt{a^2x^2+1}}{75}-\frac{4a^4x^4\operatorname{arcsinh}(ax)^3\sqrt{a^2x^2+1}}{25}+\frac{16a^2x^2\operatorname{arcsinh}(ax)^3\sqrt{a^2x^2+1}}{75}+\frac{32ax\operatorname{arcsinh}(ax)^2}{25}-$
default	$\frac{a^5x^5\operatorname{arcsinh}(ax)^4}{5}-\frac{32\operatorname{arcsinh}(ax)^3\sqrt{a^2x^2+1}}{75}-\frac{4a^4x^4\operatorname{arcsinh}(ax)^3\sqrt{a^2x^2+1}}{25}+\frac{16a^2x^2\operatorname{arcsinh}(ax)^3\sqrt{a^2x^2+1}}{75}+\frac{32ax\operatorname{arcsinh}(ax)^2}{25}-$

```
[In] int(x^4*arcsinh(a*x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^5*(1/5*a^5*x^5*arcsinh(a*x)^4-32/75*arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)-4/
25*a^4*x^4*arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)+16/75*a^2*x^2*arcsinh(a*x)^3*(a
^2*x^2+1)^(1/2)+32/25*a*x*arcsinh(a*x)^2-16576/5625*arcsinh(a*x)*(a^2*x^2+1
)^(1/2)+16576/5625*a*x+12/125*a^5*x^5*arcsinh(a*x)^2-24/625*a^4*x^4*arcsinh
(a*x)*(a^2*x^2+1)^(1/2)+1088/5625*arcsinh(a*x)*(a^2*x^2+1)^(1/2)*a^2*x^2+24
/3125*a^5*x^5-1088/16875*a^3*x^3-16/75*a^3*x^3*arcsinh(a*x)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.77

$$\int x^4 \operatorname{arcsinh}(ax)^4\,dx$$
$$= \frac{16875\,a^5x^5\log\left(ax+\sqrt{a^2x^2+1}\right)^4+648\,a^5x^5-5440\,a^3x^3-4500\,(3\,a^4x^4-4\,a^2x^2+8)\sqrt{a^2x^2+1}\log\left(ax+\right.}{}$$

```
[In] integrate(x^4*arcsinh(a*x)^4,x, algorithm="fricas")
```

```
[Out] 1/84375*(16875*a^5*x^5*log(a*x + sqrt(a^2*x^2 + 1))^4 + 648*a^5*x^5 - 5440*
a^3*x^3 - 4500*(3*a^4*x^4 - 4*a^2*x^2 + 8)*sqrt(a^2*x^2 + 1)*log(a*x + sqrt
(a^2*x^2 + 1))^3 + 900*(9*a^5*x^5 - 20*a^3*x^3 + 120*a*x)*log(a*x + sqrt(a^
2*x^2 + 1))^2 - 120*(27*a^4*x^4 - 136*a^2*x^2 + 2072)*sqrt(a^2*x^2 + 1)*log
(a*x + sqrt(a^2*x^2 + 1)) + 248640*a*x)/a^5
```

Sympy [A] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.99

$$\int x^4 \operatorname{arcsinh}(ax)^4 \, dx$$

$$= \begin{cases} \frac{x^5 \operatorname{asinh}^4(ax)}{5} + \frac{12x^5 \operatorname{asinh}^2(ax)}{125} + \frac{24x^5}{3125} - \frac{4x^4\sqrt{a^2x^2+1}\operatorname{asinh}^3(ax)}{25a} - \frac{24x^4\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{625a} - \frac{16x^3\operatorname{asinh}^2(ax)}{75a^2} - \frac{1088x^3}{16875a^2} + \\ 0 \end{cases}$$

```
[In] integrate(x**4*asinh(a*x)**4,x)
```

```
[Out] Piecewise((x**5*asinh(a*x)**4/5 + 12*x**5*asinh(a*x)**2/125 + 24*x**5/3125
- 4*x**4*sqrt(a**2*x**2 + 1)*asinh(a*x)**3/(25*a) - 24*x**4*sqrt(a**2*x**2
+ 1)*asinh(a*x)/(625*a) - 16*x**3*asinh(a*x)**2/(75*a**2) - 1088*x**3/(1687
5*a**2) + 16*x**2*sqrt(a**2*x**2 + 1)*asinh(a*x)**3/(75*a**3) + 1088*x**2*s
qrt(a**2*x**2 + 1)*asinh(a*x)/(5625*a**3) + 32*x*asinh(a*x)**2/(25*a**4) +
16576*x/(5625*a**4) - 32*sqrt(a**2*x**2 + 1)*asinh(a*x)**3/(75*a**5) - 1657
6*sqrt(a**2*x**2 + 1)*asinh(a*x)/(5625*a**5), Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.82

$$\int x^4 \operatorname{arcsinh}(ax)^4 \, dx$$

$$= \frac{1}{5}x^5 \operatorname{arsinh}(ax)^4 - \frac{4}{75}\left(\frac{3\sqrt{a^2x^2+1}x^4}{a^2} - \frac{4\sqrt{a^2x^2+1}x^2}{a^4} + \frac{8\sqrt{a^2x^2+1}}{a^6}\right)a \operatorname{arsinh}(ax)^3$$
$$- \frac{4}{84375}\left(2a\left(\frac{15\left(27\sqrt{a^2x^2+1}a^2x^4 - 136\sqrt{a^2x^2+1}x^2 + \frac{2072\sqrt{a^2x^2+1}}{a^2}\right)\operatorname{arsinh}(ax)}{a^5} - \frac{81a^4x^5 - 680a^2}{a^6}\right.\right.$$

```
[In] integrate(x^4*arcsinh(a*x)^4,x, algorithm="maxima")
```

```
[Out] 1/5*x^5*arcsinh(a*x)^4 - 4/75*(3*sqrt(a^2*x^2 + 1)*x^4/a^2 - 4*sqrt(a^2*x^2
 + 1)*x^2/a^4 + 8*sqrt(a^2*x^2 + 1)/a^6)*a*arcsinh(a*x)^3 - 4/84375*(2*a*(1
```

```
5*(27*sqrt(a^2*x^2 + 1)*a^2*x^4 - 136*sqrt(a^2*x^2 + 1)*x^2 + 2072*sqrt(a^2
*x^2 + 1)/a^2)*arcsinh(a*x)/a^5 - (81*a^4*x^5 - 680*a^2*x^3 + 31080*x)/a^6)
 - 225*(9*a^4*x^5 - 20*a^2*x^3 + 120*x)*arcsinh(a*x)^2/a^5)*a
```

Giac [F(-2)]

Exception generated.

$$\int x^4 \operatorname{arcsinh}(ax)^4 \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^4*arcsinh(a*x)^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \operatorname{arcsinh}(ax)^4 \, dx = \int x^4 \operatorname{asinh}(a\,x)^4 \, dx$$

```
[In] int(x^4*asinh(a*x)^4,x)
```

```
[Out] int(x^4*asinh(a*x)^4, x)
```

3.34 $\int x^3 \operatorname{arcsinh}(ax)^4 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 194

$$\begin{aligned}\int x^3 \operatorname{arcsinh}(ax)^4 \, dx = &-\frac{45x^2}{128a^2}+\frac{3x^4}{128}+\frac{45x\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{64a^3}\\&-\frac{3x^3\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{32a}-\frac{45\operatorname{arcsinh}(ax)^2}{128a^4}-\frac{9x^2\operatorname{arcsinh}(ax)^2}{16a^2}\\&+\frac{3}{16}x^4\operatorname{arcsinh}(ax)^2+\frac{3x\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}{8a^3}\\&-\frac{x^3\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}{4a}-\frac{3\operatorname{arcsinh}(ax)^4}{32a^4}+\frac{1}{4}x^4\operatorname{arcsinh}(ax)^4\end{aligned}$$

```
[Out] -45/128*x^2/a^2+3/128*x^4-45/128*arcsinh(a*x)^2/a^4-9/16*x^2*arcsinh(a*x)^2
/a^2+3/16*x^4*arcsinh(a*x)^2-3/32*arcsinh(a*x)^4/a^4+1/4*x^4*arcsinh(a*x)^4
+45/64*x*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a^3-3/32*x^3*arcsinh(a*x)*(a^2*x^2+
1)^(1/2)/a+3/8*x*arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)/a^3-1/4*x^3*arcsinh(a*x)^
3*(a^2*x^2+1)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used

$= \{5776, 5812, 5783, 30\}$

$$\int x^3\text{arcsinh}(ax)^4\,dx = -\frac{3\text{arcsinh}(ax)^4}{32a^4} - \frac{45\text{arcsinh}(ax)^2}{128a^4} - \frac{9x^2\text{arcsinh}(ax)^2}{16a^2} - \frac{x^3\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{4a} - \frac{3x^3\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{32a} - \frac{45x^2}{128a^2} + \frac{3x\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{8a^3} + \frac{45x\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{64a^3} + \frac{1}{4}x^4\text{arcsinh}(ax)^4 + \frac{3}{16}x^4\text{arcsinh}(ax)^2 + \frac{3x^4}{128}$$

```
[In] Int[x^3*ArcSinh[a*x]^4,x]
```

```
[Out] (-45*x^2)/(128*a^2) + (3*x^4)/128 + (45*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(
64*a^3) - (3*x^3*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(32*a) - (45*ArcSinh[a*x]^
2)/(128*a^4) - (9*x^2*ArcSinh[a*x]^2)/(16*a^2) + (3*x^4*ArcSinh[a*x]^2)/16
+ (3*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(8*a^3) - (x^3*Sqrt[1 + a^2*x^2]*A
rcSinh[a*x]^3)/(4*a) - (3*ArcSinh[a*x]^4)/(32*a^4) + (x^4*ArcSinh[a*x]^4)/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
```

```
1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{4}x^4\text{arcsinh}(ax)^4 - a\int \frac{x^4\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{x^3\sqrt{1+a^2x^2}\,\text{arcsinh}(ax)^3}{4a} + \frac{1}{4}x^4\text{arcsinh}(ax)^4 \\
&\quad + \frac{3}{4}\int x^3\text{arcsinh}(ax)^2\,dx + \frac{3\int \frac{x^2\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx}{4a} \\
&= \frac{3}{16}x^4\text{arcsinh}(ax)^2 + \frac{3x\sqrt{1+a^2x^2}\,\text{arcsinh}(ax)^3}{8a^3} \\
&\quad - \frac{x^3\sqrt{1+a^2x^2}\,\text{arcsinh}(ax)^3}{4a} + \frac{1}{4}x^4\text{arcsinh}(ax)^4 - \frac{3\int \frac{\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx}{8a^3} \\
&\quad - \frac{9\int x\text{arcsinh}(ax)^2\,dx}{8a^2} - \frac{1}{8}(3a)\int \frac{x^4\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{3x^3\sqrt{1+a^2x^2}\,\text{arcsinh}(ax)}{32a} - \frac{9x^2\text{arcsinh}(ax)^2}{16a^2} + \frac{3}{16}x^4\text{arcsinh}(ax)^2 \\
&\quad + \frac{3x\sqrt{1+a^2x^2}\,\text{arcsinh}(ax)^3}{8a^3} - \frac{x^3\sqrt{1+a^2x^2}\,\text{arcsinh}(ax)^3}{4a} - \frac{3\text{arcsinh}(ax)^4}{32a^4} \\
&\quad + \frac{1}{4}x^4\text{arcsinh}(ax)^4 + \frac{3\int x^3\,dx}{32} + \frac{9\int \frac{x^2\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx}{32a} + \frac{9\int \frac{x^2\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx}{8a} \\
&= \frac{3x^4}{128} + \frac{45x\sqrt{1+a^2x^2}\,\text{arcsinh}(ax)}{64a^3} - \frac{3x^3\sqrt{1+a^2x^2}\,\text{arcsinh}(ax)}{32a} \\
&\quad - \frac{9x^2\text{arcsinh}(ax)^2}{16a^2} + \frac{3}{16}x^4\text{arcsinh}(ax)^2 + \frac{3x\sqrt{1+a^2x^2}\,\text{arcsinh}(ax)^3}{8a^3} \\
&\quad - \frac{x^3\sqrt{1+a^2x^2}\,\text{arcsinh}(ax)^3}{4a} - \frac{3\text{arcsinh}(ax)^4}{32a^4} + \frac{1}{4}x^4\text{arcsinh}(ax)^4 \\
&\quad - \frac{9\int \frac{\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx}{64a^3} - \frac{9\int \frac{\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx}{16a^3} - \frac{9\int x\,dx}{64a^2} - \frac{9\int x\,dx}{16a^2} \\
&= -\frac{45x^2}{128a^2} + \frac{3x^4}{128} + \frac{45x\sqrt{1+a^2x^2}\,\text{arcsinh}(ax)}{64a^3} \\
&\quad - \frac{3x^3\sqrt{1+a^2x^2}\,\text{arcsinh}(ax)}{32a} - \frac{45\text{arcsinh}(ax)^2}{128a^4} - \frac{9x^2\text{arcsinh}(ax)^2}{16a^2} \\
&\quad + \frac{3}{16}x^4\text{arcsinh}(ax)^2 + \frac{3x\sqrt{1+a^2x^2}\,\text{arcsinh}(ax)^3}{8a^3} \\
&\quad - \frac{x^3\sqrt{1+a^2x^2}\,\text{arcsinh}(ax)^3}{4a} - \frac{3\text{arcsinh}(ax)^4}{32a^4} + \frac{1}{4}x^4\text{arcsinh}(ax)^4
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.69

$$\int x^3 \operatorname{arcsinh}(ax)^4\,dx$$
$$= \frac{3a^2x^2(-15+a^2x^2)-6ax\sqrt{1+a^2x^2}(-15+2a^2x^2)\operatorname{arcsinh}(ax)+3(-15-24a^2x^2+8a^4x^4)\operatorname{arcsinh}(ax)^2 -}{128a^4}$$

```
[In] Integrate[x^3*ArcSinh[a*x]^4,x]
```

```
[Out] (3*a^2*x^2*(-15 + a^2*x^2) - 6*a*x*Sqrt[1 + a^2*x^2]*(-15 + 2*a^2*x^2)*ArcS
inh[a*x] + 3*(-15 - 24*a^2*x^2 + 8*a^4*x^4)*ArcSinh[a*x]^2 - 16*a*x*Sqrt[1
+ a^2*x^2]*(-3 + 2*a^2*x^2)*ArcSinh[a*x]^3 + 4*(-3 + 8*a^4*x^4)*ArcSinh[a*x
]^4)/(128*a^4)
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{\frac{a^4x^4\operatorname{arcsinh}(ax)^4}{4}-\frac{a^3x^3\operatorname{arcsinh}(ax)^3\sqrt{a^2x^2+1}}{4}+\frac{3\operatorname{arcsinh}(ax)^3\sqrt{a^2x^2+1}\,ax}{8}-\frac{3\operatorname{arcsinh}(ax)^4}{32}+\frac{3a^4x^4\operatorname{arcsinh}(ax)^2}{16}-\frac{3a^3x^3\operatorname{arcsinh}}{}}{a^4}$
default	$\frac{\frac{a^4x^4\operatorname{arcsinh}(ax)^4}{4}-\frac{a^3x^3\operatorname{arcsinh}(ax)^3\sqrt{a^2x^2+1}}{4}+\frac{3\operatorname{arcsinh}(ax)^3\sqrt{a^2x^2+1}\,ax}{8}-\frac{3\operatorname{arcsinh}(ax)^4}{32}+\frac{3a^4x^4\operatorname{arcsinh}(ax)^2}{16}-\frac{3a^3x^3\operatorname{arcsinh}}{}}{a^4}$

```
[In] int(x^3*arcsinh(a*x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*(1/4*a^4*x^4*arcsinh(a*x)^4-1/4*a^3*x^3*arcsinh(a*x)^3*(a^2*x^2+1)^(1
/2)+3/8*arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)*a*x-3/32*arcsinh(a*x)^4+3/16*a^4*x
^4*arcsinh(a*x)^2-3/32*a^3*x^3*arcsinh(a*x)*(a^2*x^2+1)^(1/2)+45/64*arcsinh
(a*x)*(a^2*x^2+1)^(1/2)*a*x+27/128*arcsinh(a*x)^2+3/128*a^4*x^4-45/128*a^2*
x^2-45/128-9/16*arcsinh(a*x)^2*(a^2*x^2+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.91

$$\int x^3 \operatorname{arcsinh}(ax)^4\,dx$$
$$= \frac{3\,a^4x^4+4\,(8\,a^4x^4-3)\log\left(ax+\sqrt{a^2x^2+1}\right)^4-16\,(2\,a^3x^3-3\,ax)\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)^3-45}{12}$$

```
[In] integrate(x^3*arcsinh(a*x)^4,x, algorithm="fricas")
```

```
[Out] 1/128*(3*a^4*x^4 + 4*(8*a^4*x^4 - 3)*log(a*x + sqrt(a^2*x^2 + 1))^4 - 16*(2
*a^3*x^3 - 3*a*x)*sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^3 - 45*a^2
*x^2 + 3*(8*a^4*x^4 - 24*a^2*x^2 - 15)*log(a*x + sqrt(a^2*x^2 + 1))^2 - 6*(
2*a^3*x^3 - 15*a*x)*sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1)))/a^4
```

Sympy [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.98

$$\int x^3 \operatorname{arcsinh}(ax)^4\,dx$$
$$= \begin{cases} \frac{x^4 \operatorname{asinh}^4(ax)}{4} + \frac{3x^4 \operatorname{asinh}^2(ax)}{16} + \frac{3x^4}{128} - \frac{x^3\sqrt{a^2x^2+1}\operatorname{asinh}^3(ax)}{4a} - \frac{3x^3\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{32a} - \frac{9x^2\operatorname{asinh}^2(ax)}{16a^2} - \frac{45x^2}{128a^2} + \\ 0 \end{cases}$$

```
[In] integrate(x**3*asinh(a*x)**4,x)
```

```
[Out] Piecewise((x**4*asinh(a*x)**4/4 + 3*x**4*asinh(a*x)**2/16 + 3*x**4/128 - x*
*3*sqrt(a**2*x**2 + 1)*asinh(a*x)**3/(4*a) - 3*x**3*sqrt(a**2*x**2 + 1)*asi
nh(a*x)/(32*a) - 9*x**2*asinh(a*x)**2/(16*a**2) - 45*x**2/(128*a**2) + 3*x*
sqrt(a**2*x**2 + 1)*asinh(a*x)**3/(8*a**3) + 45*x*sqrt(a**2*x**2 + 1)*asinh
(a*x)/(64*a**3) - 3*asinh(a*x)**4/(32*a**4) - 45*asinh(a*x)**2/(128*a**4),
Ne(a, 0)), (0, True))
```

Maxima [F]

$$\int x^3 \operatorname{arcsinh}(ax)^4\,dx = \int x^3 \operatorname{arsinh}(ax)^4\,dx$$

```
[In] integrate(x^3*arcsinh(a*x)^4,x, algorithm="maxima")
```

```
[Out] 1/4*x^4*log(a*x + sqrt(a^2*x^2 + 1))^4 - integrate((a^3*x^6 + sqrt(a^2*x^2
+ 1)*a^2*x^5 + a*x^4)*log(a*x + sqrt(a^2*x^2 + 1))^3/(a^3*x^3 + a*x + (a^2*
x^2 + 1)^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int x^3 \operatorname{arcsinh}(ax)^4\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3*arcsinh(a*x)^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \operatorname{arcsinh}(ax)^4 \, dx = \int x^3 \operatorname{asinh}(a\,x)^4 \, dx$$

```
[In] int(x^3*asinh(a*x)^4,x)
```

```
[Out] int(x^3*asinh(a*x)^4, x)
```

3.35 $\int x^2 \mathbf{arcsinh}(ax)^4 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 162

$$\begin{aligned}\int x^2 \operatorname{arcsinh}(ax)^4 \, dx = & -\frac{160x}{27a^2} + \frac{8x^3}{81} + \frac{160\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{27a^3} \\ & - \frac{8x^2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{27a} - \frac{8x\operatorname{arcsinh}(ax)^2}{3a^2} \\ & + \frac{4}{9}x^3\operatorname{arcsinh}(ax)^2 + \frac{8\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}{9a^3} \\ & - \frac{4x^2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}{9a} + \frac{1}{3}x^3\operatorname{arcsinh}(ax)^4\end{aligned}$$

```
[Out] -160/27*x/a^2+8/81*x^3-8/3*x*arcsinh(a*x)^2/a^2+4/9*x^3*arcsinh(a*x)^2+1/3*
x^3*arcsinh(a*x)^4+160/27*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a^3-8/27*x^2*arcsi
nh(a*x)*(a^2*x^2+1)^(1/2)/a+8/9*arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)/a^3-4/9*x^
2*arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5776, 5812, 5798, 5772, 8, 30}

$$\begin{aligned}\int x^2 \operatorname{arcsinh}(ax)^4 \, dx = & -\frac{4x^2\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^3}{9a} \\ & - \frac{8x^2\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)}{27a} - \frac{8x\operatorname{arcsinh}(ax)^2}{3a^2} - \frac{160x}{27a^2} \\ & + \frac{8\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^3}{9a^3} + \frac{160\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)}{27a^3} \\ & + \frac{1}{3}x^3\operatorname{arcsinh}(ax)^4 + \frac{4}{9}x^3\operatorname{arcsinh}(ax)^2 + \frac{8x^3}{81}\end{aligned}$$

```
[In] Int[x^2*ArcSinh[a*x]^4,x]
```

```
[Out] (-160*x)/(27*a^2) + (8*x^3)/81 + (160*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(27*a
^3) - (8*x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(27*a) - (8*x*ArcSinh[a*x]^2)/
(3*a^2) + (4*x^3*ArcSinh[a*x]^2)/9 + (8*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(
9*a^3) - (4*x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(9*a) + (x^3*ArcSinh[a*x]
^4)/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
```

```
1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\text{arcsinh}(ax)^4 - \frac{1}{3}(4a)\int \frac{x^3\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{4x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{9a} + \frac{1}{3}x^3\text{arcsinh}(ax)^4 \\
&\quad + \frac{4}{3}\int x^2\text{arcsinh}(ax)^2\,dx + \frac{8\int \frac{x\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx}{9a} \\
&= \frac{4}{9}x^3\text{arcsinh}(ax)^2 + \frac{8\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{9a^3} - \frac{4x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{9a} \\
&\quad + \frac{1}{3}x^3\text{arcsinh}(ax)^4 - \frac{8\int \text{arcsinh}(ax)^2\,dx}{3a^2} - \frac{1}{9}(8a)\int \frac{x^3\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{8x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{27a} - \frac{8x\text{arcsinh}(ax)^2}{3a^2} + \frac{4}{9}x^3\text{arcsinh}(ax)^2 \\
&\quad + \frac{8\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{9a^3} - \frac{4x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{9a} + \frac{1}{3}x^3\text{arcsinh}(ax)^4 \\
&\quad + \frac{8\int x^2\,dx}{27} + \frac{16\int \frac{x\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx}{27a} + \frac{16\int \frac{x\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx}{3a} \\
&= \frac{8x^3}{81} + \frac{160\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{27a^3} - \frac{8x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{27a} \\
&\quad - \frac{8x\text{arcsinh}(ax)^2}{3a^2} + \frac{4}{9}x^3\text{arcsinh}(ax)^2 + \frac{8\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{9a^3} \\
&\quad - \frac{4x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{9a} + \frac{1}{3}x^3\text{arcsinh}(ax)^4 - \frac{16\int 1\,dx}{27a^2} - \frac{16\int 1\,dx}{3a^2} \\
&= -\frac{160x}{27a^2} + \frac{8x^3}{81} + \frac{160\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{27a^3} - \frac{8x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{27a} \\
&\quad - \frac{8x\text{arcsinh}(ax)^2}{3a^2} + \frac{4}{9}x^3\text{arcsinh}(ax)^2 + \frac{8\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{9a^3} \\
&\quad - \frac{4x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{9a} + \frac{1}{3}x^3\text{arcsinh}(ax)^4
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.69

$$\int x^2 \operatorname{arcsinh}(ax)^4\,dx$$
$$= \frac{8ax(-60+a^2x^2)-24(-20+a^2x^2)\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)+36ax(-6+a^2x^2)\operatorname{arcsinh}(ax)^2-36(-2+a^2}{81a^3}$$

```
[In] Integrate[x^2*ArcSinh[a*x]^4,x]
```

```
[Out] (8*a*x*(-60 + a^2*x^2) - 24*(-20 + a^2*x^2)*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]
+ 36*a*x*(-6 + a^2*x^2)*ArcSinh[a*x]^2 - 36*(-2 + a^2*x^2)*Sqrt[1 + a^2*x^2
]*ArcSinh[a*x]^3 + 27*a^3*x^3*ArcSinh[a*x]^4)/(81*a^3)
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{\frac{a^3x^3\operatorname{arcsinh}(ax)^4}{3}+\frac{8\operatorname{arcsinh}(ax)^3\sqrt{a^2x^2+1}}{9}-\frac{4a^2x^2\operatorname{arcsinh}(ax)^3\sqrt{a^2x^2+1}}{9}-\frac{8ax\operatorname{arcsinh}(ax)^2}{3}+\frac{160\operatorname{arcsinh}(ax)\sqrt{a^2x^2+1}}{27}-\frac{160ax}{27}}{a^3}$
default	$\frac{\frac{a^3x^3\operatorname{arcsinh}(ax)^4}{3}+\frac{8\operatorname{arcsinh}(ax)^3\sqrt{a^2x^2+1}}{9}-\frac{4a^2x^2\operatorname{arcsinh}(ax)^3\sqrt{a^2x^2+1}}{9}-\frac{8ax\operatorname{arcsinh}(ax)^2}{3}+\frac{160\operatorname{arcsinh}(ax)\sqrt{a^2x^2+1}}{27}-\frac{160ax}{27}}{a^3}$

```
[In] int(x^2*arcsinh(a*x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(1/3*a^3*x^3*arcsinh(a*x)^4+8/9*arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)-4/9*
a^2*x^2*arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)-8/3*a*x*arcsinh(a*x)^2+160/27*arcs
inh(a*x)*(a^2*x^2+1)^(1/2)-160/27*a*x+4/9*a^3*x^3*arcsinh(a*x)^2-8/27*arcsi
nh(a*x)*(a^2*x^2+1)^(1/2)*a^2*x^2+8/81*a^3*x^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.95

$$\int x^2 \operatorname{arcsinh}(ax)^4\,dx$$
$$= \frac{27\,a^3x^3\log\left(ax+\sqrt{a^2x^2+1}\right)^4+8\,a^3x^3-36\,\sqrt{a^2x^2+1}(a^2x^2-2)\log\left(ax+\sqrt{a^2x^2+1}\right)^3+36\,(a^3x^3-6\,a}{81\,a^3}$$

```
[In] integrate(x^2*arcsinh(a*x)^4,x, algorithm="fricas")
```

```
[Out] 1/81*(27*a^3*x^3*log(a*x + sqrt(a^2*x^2 + 1))^4 + 8*a^3*x^3 - 36*sqrt(a^2*x
^2 + 1)*(a^2*x^2 - 2)*log(a*x + sqrt(a^2*x^2 + 1))^3 + 36*(a^3*x^3 - 6*a*x)
```

```
*log(a*x + sqrt(a^2*x^2 + 1))^2 - 24*sqrt(a^2*x^2 + 1)*(a^2*x^2 - 20)*log(a
*x + sqrt(a^2*x^2 + 1)) - 480*a*x)/a^3
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.98

$$\int x^2 \text{arcsinh}(ax)^4\,dx$$

$$= \begin{cases} \frac{x^3\operatorname{asinh}^4(ax)}{3} + \frac{4x^3\operatorname{asinh}^2(ax)}{9} + \frac{8x^3}{81} - \frac{4x^2\sqrt{a^2x^2+1}\operatorname{asinh}^3(ax)}{9a} - \frac{8x^2\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{27a} - \frac{8x\operatorname{asinh}^2(ax)}{3a^2} - \frac{160x}{27a^2} + \\ 0 \end{cases}$$

```
[In] integrate(x**2*asinh(a*x)**4,x)
```

```
[Out] Piecewise((x**3*asinh(a*x)**4/3 + 4*x**3*asinh(a*x)**2/9 + 8*x**3/81 - 4*x*
*2*sqrt(a**2*x**2 + 1)*asinh(a*x)**3/(9*a) - 8*x**2*sqrt(a**2*x**2 + 1)*asi
nh(a*x)/(27*a) - 8*x*asinh(a*x)**2/(3*a**2) - 160*x/(27*a**2) + 8*sqrt(a**2
*x**2 + 1)*asinh(a*x)**3/(9*a**3) + 160*sqrt(a**2*x**2 + 1)*asinh(a*x)/(27*
a**3), Ne(a, 0)), (0, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.88

$$\int x^2 \text{arcsinh}(ax)^4\,dx = \frac{1}{3}\,x^3\,\operatorname{arsinh}(ax)^4 - \frac{4}{9}\,a\left(\frac{\sqrt{a^2x^2+1}x^2}{a^2} - \frac{2\,\sqrt{a^2x^2+1}}{a^4}\right)\operatorname{arsinh}(ax)^3 - \frac{4}{81}\left(2\,a\left(\frac{3\left(\sqrt{a^2x^2+1}x^2 - \frac{20\,\sqrt{a^2x^2+1}}{a^2}\right)\operatorname{arsinh}(ax)}{a^3} - \frac{a^2x^3 - 60\,x}{a^4}\right) - \frac{9\,(a^2x^3 - 6\,x)\operatorname{arsinh}(ax)^2}{a^3}\right)a$$

```
[In] integrate(x^2*arcsinh(a*x)^4,x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arcsinh(a*x)^4 - 4/9*a*(sqrt(a^2*x^2 + 1)*x^2/a^2 - 2*sqrt(a^2*x^2
+ 1)/a^4)*arcsinh(a*x)^3 - 4/81*(2*a*(3*(sqrt(a^2*x^2 + 1)*x^2 - 20*sqrt(a^
2*x^2 + 1)/a^2)*arcsinh(a*x)/a^3 - (a^2*x^3 - 60*x)/a^4) - 9*(a^2*x^3 - 6*x
)*arcsinh(a*x)^2/a^3)*a
```

Giac [F(-2)]

Exception generated.

$$\int x^2 \text{arcsinh}(ax)^4 \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2*arcsinh(a*x)^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \text{arcsinh}(ax)^4 \, dx = \int x^2 \operatorname{asinh}(a\,x)^4 \, dx$$

```
[In] int(x^2*asinh(a*x)^4,x)
```

```
[Out] int(x^2*asinh(a*x)^4, x)
```

3.36 $\int x\text{arcsinh}(ax)^4\,dx$

Optimal result

Integrand size = 8, antiderivative size = 110

$$\int x\text{arcsinh}(ax)^4\,dx = \frac{3x^2}{4} - \frac{3x\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{2a} + \frac{3\text{arcsinh}(ax)^2}{4a^2} + \frac{3}{2}x^2\text{arcsinh}(ax)^2 - \frac{x\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{a} + \frac{\text{arcsinh}(ax)^4}{4a^2} + \frac{1}{2}x^2\text{arcsinh}(ax)^4$$

```
[Out] 3/4*x^2+3/4*arcsinh(a*x)^2/a^2+3/2*x^2*arcsinh(a*x)^2+1/4*arcsinh(a*x)^4/a^
2+1/2*x^2*arcsinh(a*x)^4-3/2*x*arcsinh(a*x)*(a^2*x^2+1)^(1/2)/a-x*arcsinh(a
*x)^3*(a^2*x^2+1)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5776, 5812, 5783, 30}

$$\int x\text{arcsinh}(ax)^4\,dx = -\frac{x\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{a} - \frac{3x\sqrt{a^2x^2+1}\text{arcsinh}(ax)}{2a} + \frac{\text{arcsinh}(ax)^4}{4a^2} + \frac{3\text{arcsinh}(ax)^2}{4a^2} + \frac{1}{2}x^2\text{arcsinh}(ax)^4 + \frac{3}{2}x^2\text{arcsinh}(ax)^2 + \frac{3x^2}{4}$$

```
[In] Int[x*ArcSinh[a*x]^4,x]
```

```
[Out] (3*x^2)/4 - (3*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/(2*a) + (3*ArcSinh[a*x]^2)
/(4*a^2) + (3*x^2*ArcSinh[a*x]^2)/2 - (x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/
a + ArcSinh[a*x]^4/(4*a^2) + (x^2*ArcSinh[a*x]^4)/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}x^2\text{arcsinh}(ax)^4 - (2a)\int \frac{x^2\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{x\sqrt{1+a^2x^2}\,\text{arcsinh}(ax)^3}{a} + \frac{1}{2}x^2\text{arcsinh}(ax)^4 + 3\int x\,\text{arcsinh}(ax)^2\,dx + \frac{\int \frac{\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx}{a} \\
&= \frac{3}{2}x^2\text{arcsinh}(ax)^2 - \frac{x\sqrt{1+a^2x^2}\,\text{arcsinh}(ax)^3}{a} + \frac{\text{arcsinh}(ax)^4}{4a^2} \\
&\quad + \frac{1}{2}x^2\text{arcsinh}(ax)^4 - (3a)\int \frac{x^2\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{3x\sqrt{1+a^2x^2}\,\text{arcsinh}(ax)}{2a} + \frac{3}{2}x^2\text{arcsinh}(ax)^2 - \frac{x\sqrt{1+a^2x^2}\,\text{arcsinh}(ax)^3}{a} \\
&\quad + \frac{\text{arcsinh}(ax)^4}{4a^2} + \frac{1}{2}x^2\text{arcsinh}(ax)^4 + \frac{3\int x\,dx}{2} + \frac{3\int \frac{\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx}{2a}
\end{aligned}
$$

$$= \frac{3x^2}{4} - \frac{3x\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{2a} + \frac{3\operatorname{arcsinh}(ax)^2}{4a^2} + \frac{3}{2}x^2\operatorname{arcsinh}(ax)^2 - \frac{x\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}{a} + \frac{\operatorname{arcsinh}(ax)^4}{4a^2} + \frac{1}{2}x^2\operatorname{arcsinh}(ax)^4$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.85

$$\int x\operatorname{arcsinh}(ax)^4\,dx$$

$$= \frac{3a^2x^2 - 6ax\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax) + (3+6a^2x^2)\operatorname{arcsinh}(ax)^2 - 4ax\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3 + (1+2a^2x}{4a^2}$$

```
[In] Integrate[x*ArcSinh[a*x]^4,x]
```

```
[Out] (3*a^2*x^2 - 6*a*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x] + (3 + 6*a^2*x^2)*ArcSinh
[a*x]^2 - 4*a*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3 + (1 + 2*a^2*x^2)*ArcSinh[
a*x]^4)/(4*a^2)
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{\frac{\operatorname{arcsinh}(ax)^4\left(a^2x^2+1\right)}{2} - \operatorname{arcsinh}(ax)^3\sqrt{a^2x^2+1}\,ax - \frac{\operatorname{arcsinh}(ax)^4}{4} + \frac{3\operatorname{arcsinh}(ax)^2\left(a^2x^2+1\right)}{2} - \frac{3\operatorname{arcsinh}(ax)\sqrt{a^2x^2+1}\,ax}{2} - 3\,\mathrm{ar}}{a^2}$
default	$\frac{\frac{\operatorname{arcsinh}(ax)^4\left(a^2x^2+1\right)}{2} - \operatorname{arcsinh}(ax)^3\sqrt{a^2x^2+1}\,ax - \frac{\operatorname{arcsinh}(ax)^4}{4} + \frac{3\operatorname{arcsinh}(ax)^2\left(a^2x^2+1\right)}{2} - \frac{3\operatorname{arcsinh}(ax)\sqrt{a^2x^2+1}\,ax}{2} - 3\,\mathrm{ar}}{a^2}$

```
[In] int(x*arcsinh(a*x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(1/2*arcsinh(a*x)^4*(a^2*x^2+1)-arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)*a*x-
1/4*arcsinh(a*x)^4+3/2*arcsinh(a*x)^2*(a^2*x^2+1)-3/2*arcsinh(a*x)*(a^2*x^2
+1)^(1/2)*a*x-3/4*arcsinh(a*x)^2+3/4*a^2*x^2+3/4)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.25

$$\int x \operatorname{arcsinh}(ax)^4 \, dx =$$
$$-\frac{4\sqrt{a^2x^2+1}ax\log\left(ax+\sqrt{a^2x^2+1}\right)^3 - \left(2a^2x^2+1\right)\log\left(ax+\sqrt{a^2x^2+1}\right)^4 - 3a^2x^2 + 6\sqrt{a^2x^2+1}ax\,\mathrm{lo}}{4a^2}$$

```
[In] integrate(x*arcsinh(a*x)^4,x, algorithm="fricas")
```

```
[Out] -1/4*(4*sqrt(a^2*x^2 + 1)*a*x*log(a*x + sqrt(a^2*x^2 + 1))^3 - (2*a^2*x^2 +
 1)*log(a*x + sqrt(a^2*x^2 + 1))^4 - 3*a^2*x^2 + 6*sqrt(a^2*x^2 + 1)*a*x*lo
g(a*x + sqrt(a^2*x^2 + 1)) - 3*(2*a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))
^2)/a^2
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.95

$$\int x \operatorname{arcsinh}(ax)^4 \, dx$$
$$= \begin{cases} \frac{x^2 \operatorname{asinh}^4(ax)}{2} + \frac{3x^2 \operatorname{asinh}^2(ax)}{2} + \frac{3x^2}{4} - \frac{x\sqrt{a^2x^2+1}\operatorname{asinh}^3(ax)}{a} - \frac{3x\sqrt{a^2x^2+1}\operatorname{asinh}(ax)}{2a} + \frac{\operatorname{asinh}^4(ax)}{4a^2} + \frac{3\operatorname{asinh}^2(ax)}{4a^2} & \text{for } a \neq \\ 0 & \text{otherw} \end{cases}$$

```
[In] integrate(x*asinh(a*x)**4,x)
```

```
[Out] Piecewise((x**2*asinh(a*x)**4/2 + 3*x**2*asinh(a*x)**2/2 + 3*x**2/4 - x*sqr
t(a**2*x**2 + 1)*asinh(a*x)**3/a - 3*x*sqrt(a**2*x**2 + 1)*asinh(a*x)/(2*a)
 + asinh(a*x)**4/(4*a**2) + 3*asinh(a*x)**2/(4*a**2), Ne(a, 0)), (0, True))
```

Maxima [F]

$$\int x \operatorname{arcsinh}(ax)^4 \, dx = \int x \operatorname{arsinh}(ax)^4 \, dx$$

```
[In] integrate(x*arcsinh(a*x)^4,x, algorithm="maxima")
```

```
[Out] 1/2*x^2*log(a*x + sqrt(a^2*x^2 + 1))^4 - integrate(2*(a^3*x^4 + sqrt(a^2*x^
2 + 1)*a^2*x^3 + a*x^2)*log(a*x + sqrt(a^2*x^2 + 1))^3/(a^3*x^3 + a*x + (a^
2*x^2 + 1)^(3/2)), x)
```

Giac [F(-2)]

Exception generated.

$$\int x \operatorname{arcsinh}(ax)^4 \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*arcsinh(a*x)^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x \operatorname{arcsinh}(ax)^4 \, dx = \int x \operatorname{asinh}(a\,x)^4 \, dx$$

```
[In] int(x*asinh(a*x)^4,x)
```

```
[Out] int(x*asinh(a*x)^4, x)
```

3.37 $\int \operatorname{arcsinh}(ax)^4 \, dx$

Optimal result

Integrand size = 6, antiderivative size = 67

$$\int \operatorname{arcsinh}(ax)^4 \, dx = 24x - \frac{24\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)}{a} + 12x\operatorname{arcsinh}(ax)^2 - \frac{4\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}{a} + x\operatorname{arcsinh}(ax)^4$$

```
[Out] 24*x+12*x*arcsinh(a*x)^2+x*arcsinh(a*x)^4-24*arcsinh(a*x)*(a^2*x^2+1)^(1/2)
/a-4*arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5772, 5798, 8}

$$\int \operatorname{arcsinh}(ax)^4 \, dx = -\frac{4\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^3}{a} - \frac{24\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)}{a} + x\operatorname{arcsinh}(ax)^4 + 12x\operatorname{arcsinh}(ax)^2 + 24x$$

```
[In] Int[ArcSinh[a*x]^4,x]
```

```
[Out] 24*x - (24*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/a + 12*x*ArcSinh[a*x]^2 - (4*Sqr
t[1 + a^2*x^2]*ArcSinh[a*x]^3)/a + x*ArcSinh[a*x]^4
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\text{arcsinh}(ax)^4-(4a)\int\frac{x\text{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}\,dx\\
&=-\frac{4\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{a}+x\text{arcsinh}(ax)^4+12\int\text{arcsinh}(ax)^2\,dx\\
&=12x\text{arcsinh}(ax)^2-\frac{4\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{a}+x\text{arcsinh}(ax)^4-(24a)\int\frac{x\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx\\
&=-\frac{24\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{a}+12x\text{arcsinh}(ax)^2\\
&\quad-\frac{4\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{a}+x\text{arcsinh}(ax)^4+24\int 1\,dx\\
&=24x-\frac{24\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{a}+12x\text{arcsinh}(ax)^2\\
&\quad-\frac{4\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{a}+x\text{arcsinh}(ax)^4
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\begin{aligned}\int\text{arcsinh}(ax)^4\,dx=24x-\frac{24\sqrt{1+a^2x^2}\text{arcsinh}(ax)}{a}+12x\text{arcsinh}(ax)^2\\
-\frac{4\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{a}+x\text{arcsinh}(ax)^4\end{aligned}$$

```
[In] Integrate[ArcSinh[a*x]^4,x]
```

```
[Out] 24*x - (24*Sqrt[1 + a^2*x^2]*ArcSinh[a*x])/a + 12*x*ArcSinh[a*x]^2 - (4*Sqr
t[1 + a^2*x^2]*ArcSinh[a*x]^3)/a + x*ArcSinh[a*x]^4
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.97

method	result	size
derivativedivides	$\frac{ax\operatorname{arcsinh}(ax)^{4}-4\operatorname{arcsinh}(ax)^{3}\sqrt{a^{2}x^{2}+1}+12ax\operatorname{arcsinh}(ax)^{2}-24\operatorname{arcsinh}(ax)\sqrt{a^{2}x^{2}+1}+24ax}{a}$	65
default	$\frac{ax\operatorname{arcsinh}(ax)^{4}-4\operatorname{arcsinh}(ax)^{3}\sqrt{a^{2}x^{2}+1}+12ax\operatorname{arcsinh}(ax)^{2}-24\operatorname{arcsinh}(ax)\sqrt{a^{2}x^{2}+1}+24ax}{a}$	65

```
[In] int(arcsinh(a*x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(a*x*arcsinh(a*x)^4-4*arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)+12*a*x*arcsinh(a
*x)^2-24*arcsinh(a*x)*(a^2*x^2+1)^(1/2)+24*a*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.67

$$\int \operatorname{arcsinh}(ax)^{4}\,dx$$
$$=\frac{ax\log\left(ax+\sqrt{a^{2}x^{2}+1}\right)^{4}+12\,ax\log\left(ax+\sqrt{a^{2}x^{2}+1}\right)^{2}-4\sqrt{a^{2}x^{2}+1}\log\left(ax+\sqrt{a^{2}x^{2}+1}\right)^{3}+24\,ax-}{a}$$

```
[In] integrate(arcsinh(a*x)^4,x, algorithm="fricas")
```

```
[Out] (a*x*log(a*x + sqrt(a^2*x^2 + 1))^4 + 12*a*x*log(a*x + sqrt(a^2*x^2 + 1))^2
 - 4*sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))^3 + 24*a*x - 24*sqrt(a^
2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1)))/a
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.97

$$\int \operatorname{arcsinh}(ax)^{4}\,dx$$
$$=\begin{cases} x\operatorname{asinh}^{4}(ax)+12x\operatorname{asinh}^{2}(ax)+24x-\frac{4\sqrt{a^{2}x^{2}+1}\operatorname{asinh}^{3}(ax)}{a}-\frac{24\sqrt{a^{2}x^{2}+1}\operatorname{asinh}(ax)}{a} & \text{for } a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

```
[In] integrate(asinh(a*x)**4,x)
```

```
[Out] Piecewise((x*asinh(a*x)**4 + 12*x*asinh(a*x)**2 + 24*x - 4*sqrt(a**2*x**2 +
 1)*asinh(a*x)**3/a - 24*sqrt(a**2*x**2 + 1)*asinh(a*x)/a, Ne(a, 0)), (0, T
rue))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.09

$$\int \operatorname{arcsinh}(ax)^4\,dx = x\operatorname{arsinh}\left(ax\right)^4 - \frac{4\,\sqrt{a^2x^2+1}\operatorname{arsinh}\left(ax\right)^3}{a} + 12\left(\frac{x\operatorname{arsinh}\left(ax\right)^2}{a} + \frac{2\left(x - \frac{\sqrt{a^2x^2+1}\operatorname{arsinh}(ax)}{a}\right)}{a}\right)a$$

```
[In] integrate(arcsinh(a*x)^4,x, algorithm="maxima")
```

```
[Out] x*arcsinh(a*x)^4 - 4*sqrt(a^2*x^2 + 1)*arcsinh(a*x)^3/a + 12*(x*arcsinh(a*x
)^2/a + 2*(x - sqrt(a^2*x^2 + 1)*arcsinh(a*x)/a)/a)*a
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.87

$$\int \operatorname{arcsinh}(ax)^4\,dx = x\log\left(ax+\sqrt{a^2x^2+1}\right)^4 - 4\left(\frac{\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)^3}{a^2} - \frac{3\left(x\log\left(ax+\sqrt{a^2x^2+1}\right)^2 + 2\,a\left(\frac{x}{a} - \frac{\sqrt{a^2x^2+1}\log\left(ax+\sqrt{a^2x^2+1}\right)}{a^2}\right.\right.}{a}\right.$$

```
[In] integrate(arcsinh(a*x)^4,x, algorithm="giac")
```

```
[Out] x*log(a*x + sqrt(a^2*x^2 + 1))^4 - 4*(sqrt(a^2*x^2 + 1)*log(a*x + sqrt(a^2*
x^2 + 1))^3/a^2 - 3*(x*log(a*x + sqrt(a^2*x^2 + 1))^2 + 2*a*(x/a - sqrt(a^2
*x^2 + 1)*log(a*x + sqrt(a^2*x^2 + 1))/a^2))/a)*a
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{arcsinh}(ax)^4\,dx = \int \operatorname{asinh}(a\,x)^4\,dx$$

```
[In] int(asinh(a*x)^4,x)
```

```
[Out] int(asinh(a*x)^4, x)
```

3.38 $\int \frac{\operatorname{arcsinh}(ax)^4}{x}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 97

$$\begin{aligned}\int \frac{\operatorname{arcsinh}(ax)^4}{x}\,dx = &-\frac{1}{5}\operatorname{arcsinh}(ax)^5+\operatorname{arcsinh}(ax)^4\log\left(1-e^{2\operatorname{arcsinh}(ax)}\right)\\&+2\operatorname{arcsinh}(ax)^3\operatorname{PolyLog}\left(2,e^{2\operatorname{arcsinh}(ax)}\right)\\&-3\operatorname{arcsinh}(ax)^2\operatorname{PolyLog}\left(3,e^{2\operatorname{arcsinh}(ax)}\right)\\&+3\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(4,e^{2\operatorname{arcsinh}(ax)}\right)-\frac{3}{2}\operatorname{PolyLog}\left(5,e^{2\operatorname{arcsinh}(ax)}\right)\end{aligned}$$

```
[Out] -1/5*arcsinh(a*x)^5+arcsinh(a*x)^4*ln(1-(a*x+(a^2*x^2+1)^(1/2))^2)+2*arcsin
h(a*x)^3*polylog(2,(a*x+(a^2*x^2+1)^(1/2))^2)-3*arcsinh(a*x)^2*polylog(3,(a
*x+(a^2*x^2+1)^(1/2))^2)+3*arcsinh(a*x)*polylog(4,(a*x+(a^2*x^2+1)^(1/2))^2
)-3/2*polylog(5,(a*x+(a^2*x^2+1)^(1/2))^2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {5775, 3797, 2221, 2611, 6744, 2320, 6724}

$$\begin{aligned}\int \frac{\operatorname{arcsinh}(ax)^4}{x}\,dx = &\ 2\operatorname{arcsinh}(ax)^3\operatorname{PolyLog}\left(2,e^{2\operatorname{arcsinh}(ax)}\right)\\&-3\operatorname{arcsinh}(ax)^2\operatorname{PolyLog}\left(3,e^{2\operatorname{arcsinh}(ax)}\right)\\&+3\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(4,e^{2\operatorname{arcsinh}(ax)}\right)-\frac{3}{2}\operatorname{PolyLog}\left(5,e^{2\operatorname{arcsinh}(ax)}\right)\\&-\frac{1}{5}\operatorname{arcsinh}(ax)^5+\operatorname{arcsinh}(ax)^4\log\left(1-e^{2\operatorname{arcsinh}(ax)}\right)\end{aligned}$$

```
[In] Int[ArcSinh[a*x]^4/x,x]
```

```
[Out] -1/5*ArcSinh[a*x]^5 + ArcSinh[a*x]^4*Log[1 - E^(2*ArcSinh[a*x])] + 2*ArcSin
h[a*x]^3*PolyLog[2, E^(2*ArcSinh[a*x])] - 3*ArcSinh[a*x]^2*PolyLog[3, E^(2*
ArcSinh[a*x])] + 3*ArcSinh[a*x]*PolyLog[4, E^(2*ArcSinh[a*x])] - (3*PolyLog
[5, E^(2*ArcSinh[a*x])])/2
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int x^4\coth(x)\,dx, x, \text{arcsinh}(ax)\right)\\
&= -\frac{1}{5}\text{arcsinh}(ax)^5 - 2\text{Subst}\left(\int \frac{e^{2x}x^4}{1-e^{2x}}\,dx, x, \text{arcsinh}(ax)\right)\\
&= -\frac{1}{5}\text{arcsinh}(ax)^5 + \text{arcsinh}(ax)^4\log\left(1-e^{2\text{arcsinh}(ax)}\right)\\
&\quad - 4\text{Subst}\left(\int x^3\log\left(1-e^{2x}\right)\,dx, x, \text{arcsinh}(ax)\right)\\
&= -\frac{1}{5}\text{arcsinh}(ax)^5 + \text{arcsinh}(ax)^4\log\left(1-e^{2\text{arcsinh}(ax)}\right)\\
&\quad + 2\text{arcsinh}(ax)^3\,\text{PolyLog}\left(2, e^{2\text{arcsinh}(ax)}\right)\\
&\quad - 6\text{Subst}\left(\int x^2\,\text{PolyLog}\left(2, e^{2x}\right)\,dx, x, \text{arcsinh}(ax)\right)\\
&= -\frac{1}{5}\text{arcsinh}(ax)^5 + \text{arcsinh}(ax)^4\log\left(1-e^{2\text{arcsinh}(ax)}\right)\\
&\quad + 2\text{arcsinh}(ax)^3\,\text{PolyLog}\left(2, e^{2\text{arcsinh}(ax)}\right) - 3\text{arcsinh}(ax)^2\,\text{PolyLog}\left(3, e^{2\text{arcsinh}(ax)}\right)\\
&\quad + 6\text{Subst}\left(\int x\,\text{PolyLog}\left(3, e^{2x}\right)\,dx, x, \text{arcsinh}(ax)\right)\\
&= -\frac{1}{5}\text{arcsinh}(ax)^5 + \text{arcsinh}(ax)^4\log\left(1-e^{2\text{arcsinh}(ax)}\right)\\
&\quad + 2\text{arcsinh}(ax)^3\,\text{PolyLog}\left(2, e^{2\text{arcsinh}(ax)}\right)\\
&\quad - 3\text{arcsinh}(ax)^2\,\text{PolyLog}\left(3, e^{2\text{arcsinh}(ax)}\right) + 3\text{arcsinh}(ax)\,\text{PolyLog}\left(4, e^{2\text{arcsinh}(ax)}\right)\\
&\quad - 3\text{Subst}\left(\int \text{PolyLog}\left(4, e^{2x}\right)\,dx, x, \text{arcsinh}(ax)\right)\\
&= -\frac{1}{5}\text{arcsinh}(ax)^5 + \text{arcsinh}(ax)^4\log\left(1-e^{2\text{arcsinh}(ax)}\right)\\
&\quad + 2\text{arcsinh}(ax)^3\,\text{PolyLog}\left(2, e^{2\text{arcsinh}(ax)}\right)\\
&\quad - 3\text{arcsinh}(ax)^2\,\text{PolyLog}\left(3, e^{2\text{arcsinh}(ax)}\right) + 3\text{arcsinh}(ax)\,\text{PolyLog}\left(4, e^{2\text{arcsinh}(ax)}\right)\\
&\quad - \frac{3}{2}\text{Subst}\left(\int \frac{\text{PolyLog}(4,x)}{x}\,dx, x, e^{2\text{arcsinh}(ax)}\right)
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{1}{5}\text{arcsinh}(ax)^5 + \text{arcsinh}(ax)^4 \log\left(1 - e^{2\text{arcsinh}(ax)}\right) \\
&\quad + 2\text{arcsinh}(ax)^3 \operatorname{PolyLog}\left(2, e^{2\text{arcsinh}(ax)}\right) - 3\text{arcsinh}(ax)^2 \operatorname{PolyLog}\left(3, e^{2\text{arcsinh}(ax)}\right) \\
&\quad + 3\text{arcsinh}(ax) \operatorname{PolyLog}\left(4, e^{2\text{arcsinh}(ax)}\right) - \frac{3}{2} \operatorname{PolyLog}\left(5, e^{2\text{arcsinh}(ax)}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00

$$
\begin{aligned}
\int \frac{\text{arcsinh}(ax)^4}{x}\, dx = &-\frac{1}{5}\text{arcsinh}(ax)^5 + \text{arcsinh}(ax)^4 \log\left(1 - e^{2\text{arcsinh}(ax)}\right) \\
&+ 2\text{arcsinh}(ax)^3 \operatorname{PolyLog}\left(2, e^{2\text{arcsinh}(ax)}\right) \\
&- 3\text{arcsinh}(ax)^2 \operatorname{PolyLog}\left(3, e^{2\text{arcsinh}(ax)}\right) \\
&+ 3\text{arcsinh}(ax) \operatorname{PolyLog}\left(4, e^{2\text{arcsinh}(ax)}\right) - \frac{3}{2} \operatorname{PolyLog}\left(5, e^{2\text{arcsinh}(ax)}\right)
\end{aligned}
$$

```
[In] Integrate[ArcSinh[a*x]^4/x,x]
```

```
[Out] -1/5*ArcSinh[a*x]^5 + ArcSinh[a*x]^4*Log[1 - E^(2*ArcSinh[a*x])] + 2*ArcSin
h[a*x]^3*PolyLog[2, E^(2*ArcSinh[a*x])] - 3*ArcSinh[a*x]^2*PolyLog[3, E^(2*
ArcSinh[a*x])] + 3*ArcSinh[a*x]*PolyLog[4, E^(2*ArcSinh[a*x])] - (3*PolyLog
[5, E^(2*ArcSinh[a*x])])/2
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.65

method	result
derivativedivides	$-\frac{\text{arcsinh}(ax)^5}{5} + \text{arcsinh}\left(ax\right)^4 \ln\left(1 + ax + \sqrt{a^2x^2+1}\right) + 4\,\text{arcsinh}\left(ax\right)^3 \operatorname{polylog}\left(2, -ax\right.$
default	$-\frac{\text{arcsinh}(ax)^5}{5} + \text{arcsinh}\left(ax\right)^4 \ln\left(1 + ax + \sqrt{a^2x^2+1}\right) + 4\,\text{arcsinh}\left(ax\right)^3 \operatorname{polylog}\left(2, -ax\right.$

```
[In] int(arcsinh(a*x)^4/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*arcsinh(a*x)^5+arcsinh(a*x)^4*ln(1+a*x+(a^2*x^2+1)^(1/2))+4*arcsinh(a*
x)^3*polylog(2,-a*x-(a^2*x^2+1)^(1/2))-12*arcsinh(a*x)^2*polylog(3,-a*x-(a^
2*x^2+1)^(1/2))+24*arcsinh(a*x)*polylog(4,-a*x-(a^2*x^2+1)^(1/2))-24*polylo
g(5,-a*x-(a^2*x^2+1)^(1/2))+arcsinh(a*x)^4*ln(1-a*x-(a^2*x^2+1)^(1/2))+4*ar
csinh(a*x)^3*polylog(2,a*x+(a^2*x^2+1)^(1/2))-12*arcsinh(a*x)^2*polylog(3,a
*x+(a^2*x^2+1)^(1/2))+24*arcsinh(a*x)*polylog(4,a*x+(a^2*x^2+1)^(1/2))-24*p
olylog(5,a*x+(a^2*x^2+1)^(1/2))
```

Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax)^4}{x}\,dx=\int \frac{\operatorname{arsinh}\left(ax\right)^4}{x}\,dx$$

```
[In] integrate(arcsinh(a*x)^4/x,x, algorithm="fricas")
[Out] integral(arcsinh(a*x)^4/x, x)
```

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)^4}{x}\,dx=\int \frac{\operatorname{asinh}^4\left(ax\right)}{x}\,dx$$

```
[In] integrate(asinh(a*x)**4/x,x)
[Out] Integral(asinh(a*x)**4/x, x)
```

Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax)^4}{x}\,dx=\int \frac{\operatorname{arsinh}\left(ax\right)^4}{x}\,dx$$

```
[In] integrate(arcsinh(a*x)^4/x,x, algorithm="maxima")
[Out] integrate(arcsinh(a*x)^4/x, x)
```

Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax)^4}{x}\,dx=\int \frac{\operatorname{arsinh}\left(ax\right)^4}{x}\,dx$$

```
[In] integrate(arcsinh(a*x)^4/x,x, algorithm="giac")
[Out] integrate(arcsinh(a*x)^4/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^4}{x}\,dx=\int \frac{\operatorname{asinh}(a\,x)^4}{x}\,dx$$

```
[In] int(asinh(a*x)^4/x,x)
[Out] int(asinh(a*x)^4/x, x)
```

3.39 $\int \frac{\operatorname{arcsinh}(ax)^4}{x^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 120

$$\begin{aligned}\int \frac{\operatorname{arcsinh}(ax)^4}{x^2}\,dx = {}& -\frac{\operatorname{arcsinh}(ax)^4}{x} - 8a\operatorname{arcsinh}(ax)^3\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(ax)}\right) \\ & - 12a\operatorname{arcsinh}(ax)^2\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(ax)}\right) \\ & + 12a\operatorname{arcsinh}(ax)^2\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(ax)}\right) \\ & + 24a\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(3,-e^{\operatorname{arcsinh}(ax)}\right) \\ & - 24a\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(3,e^{\operatorname{arcsinh}(ax)}\right) \\ & - 24a\operatorname{PolyLog}\left(4,-e^{\operatorname{arcsinh}(ax)}\right) + 24a\operatorname{PolyLog}\left(4,e^{\operatorname{arcsinh}(ax)}\right)\end{aligned}$$

```
[Out] -arcsinh(a*x)^4/x-8*a*arcsinh(a*x)^3*arctanh(a*x+(a^2*x^2+1)^(1/2))-12*a*ar
csinh(a*x)^2*polylog(2,-a*x-(a^2*x^2+1)^(1/2))+12*a*arcsinh(a*x)^2*polylog(
2,a*x+(a^2*x^2+1)^(1/2))+24*a*arcsinh(a*x)*polylog(3,-a*x-(a^2*x^2+1)^(1/2)
)-24*a*arcsinh(a*x)*polylog(3,a*x+(a^2*x^2+1)^(1/2))-24*a*polylog(4,-a*x-(a
^2*x^2+1)^(1/2))+24*a*polylog(4,a*x+(a^2*x^2+1)^(1/2))
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used

$= \{5776, 5816, 4267, 2611, 6744, 2320, 6724\}$

$$\begin{aligned}\int \frac{\operatorname{arcsinh}(ax)^4}{x^2}\,dx = &-8a\operatorname{arcsinh}(ax)^3\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(ax)}\right)\\ &- 12a\operatorname{arcsinh}(ax)^2\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(ax)}\right)\\ &+ 12a\operatorname{arcsinh}(ax)^2\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(ax)}\right)\\ &+ 24a\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(3,-e^{\operatorname{arcsinh}(ax)}\right)\\ &- 24a\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(3,e^{\operatorname{arcsinh}(ax)}\right)\\ &- 24a\operatorname{PolyLog}\left(4,-e^{\operatorname{arcsinh}(ax)}\right)\\ &+ 24a\operatorname{PolyLog}\left(4,e^{\operatorname{arcsinh}(ax)}\right) - \frac{\operatorname{arcsinh}(ax)^4}{x}\end{aligned}$$

```
[In] Int[ArcSinh[a*x]^4/x^2,x]
```

```
[Out] -(ArcSinh[a*x]^4/x) - 8*a*ArcSinh[a*x]^3*ArcTanh[E^ArcSinh[a*x]] - 12*a*Arc
Sinh[a*x]^2*PolyLog[2, -E^ArcSinh[a*x]] + 12*a*ArcSinh[a*x]^2*PolyLog[2, E^
ArcSinh[a*x]] + 24*a*ArcSinh[a*x]*PolyLog[3, -E^ArcSinh[a*x]] - 24*a*ArcSin
h[a*x]*PolyLog[3, E^ArcSinh[a*x]] - 24*a*PolyLog[4, -E^ArcSinh[a*x]] + 24*a
*PolyLog[4, E^ArcSinh[a*x]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\operatorname{arcsinh}(ax)^4}{x} + (4a)\int \frac{\operatorname{arcsinh}(ax)^3}{x\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{\operatorname{arcsinh}(ax)^4}{x} + (4a)\text{Subst}\left(\int x^3\operatorname{csch}(x)\,dx, x, \operatorname{arcsinh}(ax)\right) \\
&= -\frac{\operatorname{arcsinh}(ax)^4}{x} - 8a\operatorname{arcsinh}(ax)^3\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(ax)}\right) \\
&\quad - (12a)\text{Subst}\left(\int x^2\log\left(1-e^x\right)\,dx, x, \operatorname{arcsinh}(ax)\right) \\
&\quad + (12a)\text{Subst}\left(\int x^2\log\left(1+e^x\right)\,dx, x, \operatorname{arcsinh}(ax)\right)
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{\operatorname{arcsinh}(ax)^4}{x} - 8a\operatorname{arcsinh}(ax)^3\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(ax)}\right) \\
&\quad - 12a\operatorname{arcsinh}(ax)^2 \operatorname{PolyLog}\left(2, -e^{\operatorname{arcsinh}(ax)}\right) \\
&\quad + 12a\operatorname{arcsinh}(ax)^2 \operatorname{PolyLog}\left(2, e^{\operatorname{arcsinh}(ax)}\right) \\
&\quad + (24a)\operatorname{Subst}\left(\int x \operatorname{PolyLog}\left(2, -e^x\right) \, dx, x, \operatorname{arcsinh}(ax)\right) \\
&\quad - (24a)\operatorname{Subst}\left(\int x \operatorname{PolyLog}\left(2, e^x\right) \, dx, x, \operatorname{arcsinh}(ax)\right)
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{\operatorname{arcsinh}(ax)^4}{x} - 8a\operatorname{arcsinh}(ax)^3\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(ax)}\right) \\
&\quad - 12a\operatorname{arcsinh}(ax)^2 \operatorname{PolyLog}\left(2, -e^{\operatorname{arcsinh}(ax)}\right) \\
&\quad + 12a\operatorname{arcsinh}(ax)^2 \operatorname{PolyLog}\left(2, e^{\operatorname{arcsinh}(ax)}\right) \\
&\quad + 24a\operatorname{arcsinh}(ax) \operatorname{PolyLog}\left(3, -e^{\operatorname{arcsinh}(ax)}\right) \\
&\quad - 24a\operatorname{arcsinh}(ax) \operatorname{PolyLog}\left(3, e^{\operatorname{arcsinh}(ax)}\right) \\
&\quad - (24a)\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(3, -e^x\right) \, dx, x, \operatorname{arcsinh}(ax)\right) \\
&\quad + (24a)\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(3, e^x\right) \, dx, x, \operatorname{arcsinh}(ax)\right)
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{\operatorname{arcsinh}(ax)^4}{x} - 8a\operatorname{arcsinh}(ax)^3\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(ax)}\right) \\
&\quad - 12a\operatorname{arcsinh}(ax)^2 \operatorname{PolyLog}\left(2, -e^{\operatorname{arcsinh}(ax)}\right) \\
&\quad + 12a\operatorname{arcsinh}(ax)^2 \operatorname{PolyLog}\left(2, e^{\operatorname{arcsinh}(ax)}\right) \\
&\quad + 24a\operatorname{arcsinh}(ax) \operatorname{PolyLog}\left(3, -e^{\operatorname{arcsinh}(ax)}\right) \\
&\quad - 24a\operatorname{arcsinh}(ax) \operatorname{PolyLog}\left(3, e^{\operatorname{arcsinh}(ax)}\right) \\
&\quad - (24a)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(3, -x)}{x} \, dx, x, e^{\operatorname{arcsinh}(ax)}\right) \\
&\quad + (24a)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(3, x)}{x} \, dx, x, e^{\operatorname{arcsinh}(ax)}\right)
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{\operatorname{arcsinh}(ax)^4}{x} - 8a\operatorname{arcsinh}(ax)^3\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(ax)}\right) \\
&\quad - 12a\operatorname{arcsinh}(ax)^2 \operatorname{PolyLog}\left(2, -e^{\operatorname{arcsinh}(ax)}\right) \\
&\quad + 12a\operatorname{arcsinh}(ax)^2 \operatorname{PolyLog}\left(2, e^{\operatorname{arcsinh}(ax)}\right) \\
&\quad + 24a\operatorname{arcsinh}(ax) \operatorname{PolyLog}\left(3, -e^{\operatorname{arcsinh}(ax)}\right) \\
&\quad - 24a\operatorname{arcsinh}(ax) \operatorname{PolyLog}\left(3, e^{\operatorname{arcsinh}(ax)}\right) \\
&\quad - 24a \operatorname{PolyLog}\left(4, -e^{\operatorname{arcsinh}(ax)}\right) + 24a \operatorname{PolyLog}\left(4, e^{\operatorname{arcsinh}(ax)}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.34

$$
\begin{aligned}
\int \frac{\operatorname{arcsinh}(ax)^4}{x^2}\,dx = \frac{1}{2}a\bigg(\pi^4 - 2\operatorname{arcsinh}(ax)^4 - \frac{2\operatorname{arcsinh}(ax)^4}{ax}& \\
- 8\operatorname{arcsinh}(ax)^3\log\left(1+e^{-\operatorname{arcsinh}(ax)}\right)& \\
+ 8\operatorname{arcsinh}(ax)^3\log\left(1-e^{\operatorname{arcsinh}(ax)}\right)& \\
+ 24\operatorname{arcsinh}(ax)^2\operatorname{PolyLog}\left(2,-e^{-\operatorname{arcsinh}(ax)}\right)& \\
+ 24\operatorname{arcsinh}(ax)^2\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(ax)}\right)& \\
+ 48\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(3,-e^{-\operatorname{arcsinh}(ax)}\right)& \\
- 48\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(3,e^{\operatorname{arcsinh}(ax)}\right)& \\
+ 48\operatorname{PolyLog}\left(4,-e^{-\operatorname{arcsinh}(ax)}\right) + 48\operatorname{PolyLog}\left(4,e^{\operatorname{arcsinh}(ax)}\right)\bigg)&
\end{aligned}
$$

```
[In] Integrate[ArcSinh[a*x]^4/x^2,x]
```

```
[Out] (a*(Pi^4 - 2*ArcSinh[a*x]^4 - (2*ArcSinh[a*x]^4)/(a*x) - 8*ArcSinh[a*x]^3*L
og[1 + E^(-ArcSinh[a*x])] + 8*ArcSinh[a*x]^3*Log[1 - E^ArcSinh[a*x]] + 24*A
rcSinh[a*x]^2*PolyLog[2, -E^(-ArcSinh[a*x])] + 24*ArcSinh[a*x]^2*PolyLog[2,
 E^ArcSinh[a*x]] + 48*ArcSinh[a*x]*PolyLog[3, -E^(-ArcSinh[a*x])] - 48*ArcS
inh[a*x]*PolyLog[3, E^ArcSinh[a*x]] + 48*PolyLog[4, -E^(-ArcSinh[a*x])] + 4
8*PolyLog[4, E^ArcSinh[a*x]]))/2
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.78

method	result
derivativedivides	$a\Big(-\frac{\operatorname{arcsinh}(ax)^4}{ax} - 4\operatorname{arcsinh}(ax)^3\ln\left(1+ax+\sqrt{a^2x^2+1}\right) - 12\operatorname{arcsinh}(ax)^2\operatorname{polylog}(2,$
default	$a\Big(-\frac{\operatorname{arcsinh}(ax)^4}{ax} - 4\operatorname{arcsinh}(ax)^3\ln\left(1+ax+\sqrt{a^2x^2+1}\right) - 12\operatorname{arcsinh}(ax)^2\operatorname{polylog}(2,$

```
[In] int(arcsinh(a*x)^4/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] a*(-arcsinh(a*x)^4/a/x-4*arcsinh(a*x)^3*ln(1+a*x+(a^2*x^2+1)^(1/2))-12*arcs
inh(a*x)^2*polylog(2,-a*x-(a^2*x^2+1)^(1/2))+24*arcsinh(a*x)*polylog(3,-a*x
-(a^2*x^2+1)^(1/2))-24*polylog(4,-a*x-(a^2*x^2+1)^(1/2))+4*arcsinh(a*x)^3*l
n(1-a*x-(a^2*x^2+1)^(1/2))+12*arcsinh(a*x)^2*polylog(2,a*x+(a^2*x^2+1)^(1/2
))-24*arcsinh(a*x)*polylog(3,a*x+(a^2*x^2+1)^(1/2))+24*polylog(4,a*x+(a^2*x
^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax)^4}{x^2}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^4}{x^2}\,dx$$

```
[In] integrate(arcsinh(a*x)^4/x^2,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x)^4/x^2, x)
```

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)^4}{x^2}\,dx = \int \frac{\operatorname{asinh}^4\left(ax\right)}{x^2}\,dx$$

```
[In] integrate(asinh(a*x)**4/x**2,x)
```

```
[Out] Integral(asinh(a*x)**4/x**2, x)
```

Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax)^4}{x^2}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^4}{x^2}\,dx$$

```
[In] integrate(arcsinh(a*x)^4/x^2,x, algorithm="maxima")
```

```
[Out] -log(a*x + sqrt(a^2*x^2 + 1))^4/x + integrate(4*(a^3*x^2 + sqrt(a^2*x^2 + 1
)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))^3/(a^3*x^4 + a*x^2 + (a^2*x^3 + x
)*sqrt(a^2*x^2 + 1)), x)
```

Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax)^4}{x^2}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^4}{x^2}\,dx$$

```
[In] integrate(arcsinh(a*x)^4/x^2,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^4/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^4}{x^2}\,dx = \int \frac{\mathrm{asinh}(a\,x)^4}{x^2}\,dx$$

```
[In] int(asinh(a*x)^4/x^2,x)
```

```
[Out] int(asinh(a*x)^4/x^2, x)
```

3.40 $\int \frac{\operatorname{arcsinh}(ax)^4}{x^3}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 108

$$\begin{aligned}\int \frac{\operatorname{arcsinh}(ax)^4}{x^3}\,dx = {} & -2a^2\operatorname{arcsinh}(ax)^3 - \frac{2a\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}{x} \\ & - \frac{\operatorname{arcsinh}(ax)^4}{2x^2} + 6a^2\operatorname{arcsinh}(ax)^2\log\left(1-e^{2\operatorname{arcsinh}(ax)}\right) \\ & + 6a^2\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(2,e^{2\operatorname{arcsinh}(ax)}\right) \\ & - 3a^2\operatorname{PolyLog}\left(3,e^{2\operatorname{arcsinh}(ax)}\right)\end{aligned}$$

```
[Out] -2*a^2*arcsinh(a*x)^3-1/2*arcsinh(a*x)^4/x^2+6*a^2*arcsinh(a*x)^2*ln(1-(a*x
+(a^2*x^2+1)^(1/2))^2)+6*a^2*arcsinh(a*x)*polylog(2,(a*x+(a^2*x^2+1)^(1/2))
^2)-3*a^2*polylog(3,(a*x+(a^2*x^2+1)^(1/2))^2)-2*a*arcsinh(a*x)^3*(a^2*x^2+
1)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {5776, 5800, 5775, 3797, 2221, 2611, 2320, 6724}

$$\begin{aligned}\int \frac{\operatorname{arcsinh}(ax)^4}{x^3}\,dx = {} & 6a^2\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(2,e^{2\operatorname{arcsinh}(ax)}\right) - 3a^2\operatorname{PolyLog}\left(3,e^{2\operatorname{arcsinh}(ax)}\right) \\ & - \frac{2a\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^3}{x} - 2a^2\operatorname{arcsinh}(ax)^3 \\ & + 6a^2\operatorname{arcsinh}(ax)^2\log\left(1-e^{2\operatorname{arcsinh}(ax)}\right) - \frac{\operatorname{arcsinh}(ax)^4}{2x^2}\end{aligned}$$

```
[In] Int[ArcSinh[a*x]^4/x^3,x]
```

```
[Out] -2*a^2*ArcSinh[a*x]^3 - (2*a*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/x - ArcSinh[
a*x]^4/(2*x^2) + 6*a^2*ArcSinh[a*x]^2*Log[1 - E^(2*ArcSinh[a*x])] + 6*a^2*A
rcSinh[a*x]*PolyLog[2, E^(2*ArcSinh[a*x])] - 3*a^2*PolyLog[3, E^(2*ArcSinh[
a*x])]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
```

```
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5800

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] - Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e
*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*
ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[
e, c^2*d] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\operatorname{arcsinh}(ax)^4}{2x^2} + (2a)\int \frac{\operatorname{arcsinh}(ax)^3}{x^2\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{2a\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}{x} - \frac{\operatorname{arcsinh}(ax)^4}{2x^2} + \left(6a^2\right)\int \frac{\operatorname{arcsinh}(ax)^2}{x}\,dx \\
&= -\frac{2a\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}{x} - \frac{\operatorname{arcsinh}(ax)^4}{2x^2} + \left(6a^2\right)\operatorname{Subst}\left(\int x^2\coth(x)\,dx, x, \operatorname{arcsinh}(ax)\right) \\
&= -2a^2\operatorname{arcsinh}(ax)^3 - \frac{2a\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}{x} - \frac{\operatorname{arcsinh}(ax)^4}{2x^2} \\
&\quad - \left(12a^2\right)\operatorname{Subst}\left(\int \frac{e^{2x}x^2}{1-e^{2x}}\,dx, x, \operatorname{arcsinh}(ax)\right) \\
&= -2a^2\operatorname{arcsinh}(ax)^3 - \frac{2a\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}{x} \\
&\quad - \frac{\operatorname{arcsinh}(ax)^4}{2x^2} + 6a^2\operatorname{arcsinh}(ax)^2\log\left(1-e^{2\operatorname{arcsinh}(ax)}\right) \\
&\quad - \left(12a^2\right)\operatorname{Subst}\left(\int x\log\left(1-e^{2x}\right)\,dx, x, \operatorname{arcsinh}(ax)\right) \\
&= -2a^2\operatorname{arcsinh}(ax)^3 - \frac{2a\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}{x} - \frac{\operatorname{arcsinh}(ax)^4}{2x^2} \\
&\quad + 6a^2\operatorname{arcsinh}(ax)^2\log\left(1-e^{2\operatorname{arcsinh}(ax)}\right) + 6a^2\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(2, e^{2\operatorname{arcsinh}(ax)}\right) \\
&\quad - \left(6a^2\right)\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(2, e^{2x}\right)\,dx, x, \operatorname{arcsinh}(ax)\right)
\end{aligned}
$$

$$
\begin{aligned}
&= -2a^2\text{arcsinh}(ax)^3 - \frac{2a\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{x} - \frac{\text{arcsinh}(ax)^4}{2x^2} \\
&\quad + 6a^2\text{arcsinh}(ax)^2 \log\left(1-e^{2\text{arcsinh}(ax)}\right) + 6a^2\text{arcsinh}(ax)\,\text{PolyLog}\left(2, e^{2\text{arcsinh}(ax)}\right) \\
&\quad - \left(3a^2\right) \text{Subst}\left(\int \frac{\text{PolyLog}(2,x)}{x}\,dx, x, e^{2\text{arcsinh}(ax)}\right)
\end{aligned}
$$

$$
\begin{aligned}
&= -2a^2\text{arcsinh}(ax)^3 - \frac{2a\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{x} \\
&\quad - \frac{\text{arcsinh}(ax)^4}{2x^2} + 6a^2\text{arcsinh}(ax)^2 \log\left(1-e^{2\text{arcsinh}(ax)}\right) \\
&\quad + 6a^2\text{arcsinh}(ax)\,\text{PolyLog}\left(2, e^{2\text{arcsinh}(ax)}\right) - 3a^2\,\text{PolyLog}\left(3, e^{2\text{arcsinh}(ax)}\right)
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.05

$$
\begin{aligned}
\int \frac{\text{arcsinh}(ax)^4}{x^3}\,dx = -\frac{\text{arcsinh}(ax)^4}{2x^2} + \frac{1}{4}a^2\Bigg(i\pi^3 - 8\text{arcsinh}(ax)^3 - \frac{8\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{ax} \\
+ 24\text{arcsinh}(ax)^2 \log\left(1-e^{2\text{arcsinh}(ax)}\right) \\
+ 24\text{arcsinh}(ax)\,\text{PolyLog}\left(2, e^{2\text{arcsinh}(ax)}\right) \\
- 12\,\text{PolyLog}\left(3, e^{2\text{arcsinh}(ax)}\right)\Bigg)
\end{aligned}
$$

```
[In] Integrate[ArcSinh[a*x]^4/x^3,x]
```

```
[Out] -1/2*ArcSinh[a*x]^4/x^2 + (a^2*(I*Pi^3 - 8*ArcSinh[a*x]^3 - (8*Sqrt[1 + a^2
*x^2]*ArcSinh[a*x]^3)/(a*x) + 24*ArcSinh[a*x]^2*Log[1 - E^(2*ArcSinh[a*x])]
 + 24*ArcSinh[a*x]*PolyLog[2, E^(2*ArcSinh[a*x])] - 12*PolyLog[3, E^(2*ArcS
inh[a*x])]))/4
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.84

method	result
derivativedivides	$a^2 \Big(-\frac{\text{arcsinh}(ax)^3 \left(-4a^2x^2+4ax\sqrt{a^2x^2+1}+\text{arcsinh}(ax)\right)}{2a^2x^2} - 4\,\text{arcsinh}\,(ax)^3 + 6\,\text{arcsinh}\,(ax)^2 \ln\,(1+$
default	$a^2 \Big(-\frac{\text{arcsinh}(ax)^3 \left(-4a^2x^2+4ax\sqrt{a^2x^2+1}+\text{arcsinh}(ax)\right)}{2a^2x^2} - 4\,\text{arcsinh}\,(ax)^3 + 6\,\text{arcsinh}\,(ax)^2 \ln\,(1+$

```
[In] int(arcsinh(a*x)^4/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(-1/2*arcsinh(a*x)^3*(-4*a^2*x^2+4*a*x*(a^2*x^2+1)^(1/2)+arcsinh(a*x))/
a^2/x^2-4*arcsinh(a*x)^3+6*arcsinh(a*x)^2*ln(1+a*x+(a^2*x^2+1)^(1/2))+12*ar
csinh(a*x)*polylog(2,-a*x-(a^2*x^2+1)^(1/2))-12*polylog(3,-a*x-(a^2*x^2+1)^
(1/2))+6*arcsinh(a*x)^2*ln(1-a*x-(a^2*x^2+1)^(1/2))+12*arcsinh(a*x)*polylog
(2,a*x+(a^2*x^2+1)^(1/2))-12*polylog(3,a*x+(a^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax)^4}{x^3}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^4}{x^3}\,dx$$

```
[In] integrate(arcsinh(a*x)^4/x^3,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x)^4/x^3, x)
```

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)^4}{x^3}\,dx = \int \frac{\operatorname{asinh}^4\left(ax\right)}{x^3}\,dx$$

```
[In] integrate(asinh(a*x)**4/x**3,x)
```

```
[Out] Integral(asinh(a*x)**4/x**3, x)
```

Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax)^4}{x^3}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^4}{x^3}\,dx$$

```
[In] integrate(arcsinh(a*x)^4/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*log(a*x + sqrt(a^2*x^2 + 1))^4/x^2 + integrate(2*(a^3*x^2 + sqrt(a^2*x
^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))^3/(a^3*x^5 + a*x^3 + (a^2*x
^4 + x^2)*sqrt(a^2*x^2 + 1)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\operatorname{arcsinh}(ax)^4}{x^3}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(arcsinh(a*x)^4/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^4}{x^3}\,dx = \int \frac{\operatorname{asinh}(a\,x)^4}{x^3}\,dx$$

```
[In] int(asinh(a*x)^4/x^3,x)
```

```
[Out] int(asinh(a*x)^4/x^3, x)
```

3.41 $\int \frac{\operatorname{arcsinh}(ax)^4}{x^4}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 223

$$\begin{aligned}\int \frac{\operatorname{arcsinh}(ax)^4}{x^4}\,dx = & -\frac{2a^2\operatorname{arcsinh}(ax)^2}{x}-\frac{2a\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}{3x^2}\\ & -\frac{\operatorname{arcsinh}(ax)^4}{3x^3}-8a^3\operatorname{arcsinh}(ax)\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(ax)}\right)\\ & +\frac{4}{3}a^3\operatorname{arcsinh}(ax)^3\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(ax)}\right)-4a^3\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(ax)}\right)\\ & +2a^3\operatorname{arcsinh}(ax)^2\operatorname{PolyLog}\left(2,-e^{\operatorname{arcsinh}(ax)}\right)\\ & +4a^3\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(ax)}\right)-2a^3\operatorname{arcsinh}(ax)^2\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(ax)}\right)\\ & -4a^3\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(3,-e^{\operatorname{arcsinh}(ax)}\right)\\ & +4a^3\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(3,e^{\operatorname{arcsinh}(ax)}\right)\\ & +4a^3\operatorname{PolyLog}\left(4,-e^{\operatorname{arcsinh}(ax)}\right)-4a^3\operatorname{PolyLog}\left(4,e^{\operatorname{arcsinh}(ax)}\right)\end{aligned}$$

```
[Out] -2*a^2*arcsinh(a*x)^2/x-1/3*arcsinh(a*x)^4/x^3-8*a^3*arcsinh(a*x)*arctanh(a
*x+(a^2*x^2+1)^(1/2))+4/3*a^3*arcsinh(a*x)^3*arctanh(a*x+(a^2*x^2+1)^(1/2))
-4*a^3*polylog(2,-a*x-(a^2*x^2+1)^(1/2))+2*a^3*arcsinh(a*x)^2*polylog(2,-a*
x-(a^2*x^2+1)^(1/2))+4*a^3*polylog(2,a*x+(a^2*x^2+1)^(1/2))-2*a^3*arcsinh(a
*x)^2*polylog(2,a*x+(a^2*x^2+1)^(1/2))-4*a^3*arcsinh(a*x)*polylog(3,-a*x-(a
^2*x^2+1)^(1/2))+4*a^3*arcsinh(a*x)*polylog(3,a*x+(a^2*x^2+1)^(1/2))+4*a^3*
polylog(4,-a*x-(a^2*x^2+1)^(1/2))-4*a^3*polylog(4,a*x+(a^2*x^2+1)^(1/2))-2/
3*a*arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5776, 5809, 5816, 4267, 2611, 6744, 2320, 6724, 2317, 2438}

$$\begin{aligned}\int \frac{\text{arcsinh}(ax)^4}{x^4}\,dx = {} & \frac{4}{3}a^3\text{arcsinh}(ax)^3\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right) \\ & - 8a^3\text{arcsinh}(ax)\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right) \\ & + 2a^3\text{arcsinh}(ax)^2\operatorname{PolyLog}\left(2,-e^{\text{arcsinh}(ax)}\right) \\ & - 2a^3\text{arcsinh}(ax)^2\operatorname{PolyLog}\left(2,e^{\text{arcsinh}(ax)}\right) \\ & - 4a^3\text{arcsinh}(ax)\operatorname{PolyLog}\left(3,-e^{\text{arcsinh}(ax)}\right) \\ & + 4a^3\text{arcsinh}(ax)\operatorname{PolyLog}\left(3,e^{\text{arcsinh}(ax)}\right) \\ & - 4a^3\operatorname{PolyLog}\left(2,-e^{\text{arcsinh}(ax)}\right) + 4a^3\operatorname{PolyLog}\left(2,e^{\text{arcsinh}(ax)}\right) \\ & + 4a^3\operatorname{PolyLog}\left(4,-e^{\text{arcsinh}(ax)}\right) - 4a^3\operatorname{PolyLog}\left(4,e^{\text{arcsinh}(ax)}\right) \\ & - \frac{2a\sqrt{a^2x^2+1}\text{arcsinh}(ax)^3}{3x^2} - \frac{2a^2\text{arcsinh}(ax)^2}{x} - \frac{\text{arcsinh}(ax)^4}{3x^3}\end{aligned}$$

```
[In] Int[ArcSinh[a*x]^4/x^4,x]
```

```
[Out] (-2*a^2*ArcSinh[a*x]^2)/x - (2*a*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^3)/(3*x^2)
- ArcSinh[a*x]^4/(3*x^3) - 8*a^3*ArcSinh[a*x]*ArcTanh[E^ArcSinh[a*x]] + (4*
a^3*ArcSinh[a*x]^3*ArcTanh[E^ArcSinh[a*x]])/3 - 4*a^3*PolyLog[2, -E^ArcSinh
[a*x]] + 2*a^3*ArcSinh[a*x]^2*PolyLog[2, -E^ArcSinh[a*x]] + 4*a^3*PolyLog[2
, E^ArcSinh[a*x]] - 2*a^3*ArcSinh[a*x]^2*PolyLog[2, E^ArcSinh[a*x]] - 4*a^3
*ArcSinh[a*x]*PolyLog[3, -E^ArcSinh[a*x]] + 4*a^3*ArcSinh[a*x]*PolyLog[3, E
^ArcSinh[a*x]] + 4*a^3*PolyLog[4, -E^ArcSinh[a*x]] - 4*a^3*PolyLog[4, E^Arc
Sinh[a*x]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5809

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a +
b*ArcSinh[c*x])^n/(d*f*(m + 1))), x] + (-Dist[c^2*((m + 2*p + 3)/(f^2*(m +
1))), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x] - Dist
[b*c*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(f*x)^(m + 1)
*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 5816

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)
*(x_)^2], x_Symbol] :> Dist[(1/c^(m + 1))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e
*x^2]], Subst[Int[(a + b*x)^n*Sinh[x]^m, x], x, ArcSinh[c*x]], x] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{arcsinh}(ax)^4}{3x^3}+\frac{1}{3}(4a)\int\frac{\text{arcsinh}(ax)^3}{x^3\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{2a\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{3x^2}-\frac{\text{arcsinh}(ax)^4}{3x^3}\\
&\quad+\left(2a^2\right)\int\frac{\text{arcsinh}(ax)^2}{x^2}\,dx-\frac{1}{3}\left(2a^3\right)\int\frac{\text{arcsinh}(ax)^3}{x\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{2a^2\text{arcsinh}(ax)^2}{x}-\frac{2a\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{3x^2}-\frac{\text{arcsinh}(ax)^4}{3x^3}\\
&\quad-\frac{1}{3}\left(2a^3\right)\text{Subst}\left(\int x^3\text{csch}(x)\,dx,x,\text{arcsinh}(ax)\right)+\left(4a^3\right)\int\frac{\text{arcsinh}(ax)}{x\sqrt{1+a^2x^2}}\,dx\\
&= -\frac{2a^2\text{arcsinh}(ax)^2}{x}-\frac{2a\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}{3x^2}\\
&\quad-\frac{\text{arcsinh}(ax)^4}{3x^3}+\frac{4}{3}a^3\text{arcsinh}(ax)^3\text{arctanh}\left(e^{\text{arcsinh}(ax)}\right)\\
&\quad+\left(2a^3\right)\text{Subst}\left(\int x^2\log\left(1-e^x\right)\,dx,x,\text{arcsinh}(ax)\right)\\
&\quad-\left(2a^3\right)\text{Subst}\left(\int x^2\log\left(1+e^x\right)\,dx,x,\text{arcsinh}(ax)\right)\\
&\quad+\left(4a^3\right)\text{Subst}(\int x\text{csch}(x)\,dx,x,\text{arcsinh}(ax))
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2a^2\operatorname{arcsinh}(ax)^2}{x} - \frac{2a\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}{3x^2} - \frac{\operatorname{arcsinh}(ax)^4}{3x^3} \\
&\quad - 8a^3\operatorname{arcsinh}(ax)\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(ax)}\right) + \frac{4}{3}a^3\operatorname{arcsinh}(ax)^3\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(ax)}\right) \\
&\quad + 2a^3\operatorname{arcsinh}(ax)^2\operatorname{PolyLog}\left(2, -e^{\operatorname{arcsinh}(ax)}\right) \\
&\quad - 2a^3\operatorname{arcsinh}(ax)^2\operatorname{PolyLog}\left(2, e^{\operatorname{arcsinh}(ax)}\right) \\
&\quad - \left(4a^3\right)\operatorname{Subst}\left(\int \log\left(1-e^x\right)\,dx, x, \operatorname{arcsinh}(ax)\right) \\
&\quad + \left(4a^3\right)\operatorname{Subst}\left(\int \log\left(1+e^x\right)\,dx, x, \operatorname{arcsinh}(ax)\right) \\
&\quad - \left(4a^3\right)\operatorname{Subst}\left(\int x\operatorname{PolyLog}\left(2, -e^x\right)\,dx, x, \operatorname{arcsinh}(ax)\right) \\
&\quad + \left(4a^3\right)\operatorname{Subst}\left(\int x\operatorname{PolyLog}\left(2, e^x\right)\,dx, x, \operatorname{arcsinh}(ax)\right)
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2a^2\operatorname{arcsinh}(ax)^2}{x} - \frac{2a\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}{3x^2} - \frac{\operatorname{arcsinh}(ax)^4}{3x^3} \\
&\quad - 8a^3\operatorname{arcsinh}(ax)\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(ax)}\right) + \frac{4}{3}a^3\operatorname{arcsinh}(ax)^3\operatorname{arctanh}\left(e^{\operatorname{arcsinh}(ax)}\right) \\
&\quad + 2a^3\operatorname{arcsinh}(ax)^2\operatorname{PolyLog}\left(2, -e^{\operatorname{arcsinh}(ax)}\right) \\
&\quad - 2a^3\operatorname{arcsinh}(ax)^2\operatorname{PolyLog}\left(2, e^{\operatorname{arcsinh}(ax)}\right) \\
&\quad - 4a^3\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(3, -e^{\operatorname{arcsinh}(ax)}\right) \\
&\quad + 4a^3\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(3, e^{\operatorname{arcsinh}(ax)}\right) \\
&\quad - \left(4a^3\right)\operatorname{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{\operatorname{arcsinh}(ax)}\right) \\
&\quad + \left(4a^3\right)\operatorname{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{\operatorname{arcsinh}(ax)}\right) \\
&\quad + \left(4a^3\right)\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(3, -e^x\right)\,dx, x, \operatorname{arcsinh}(ax)\right) \\
&\quad - \left(4a^3\right)\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(3, e^x\right)\,dx, x, \operatorname{arcsinh}(ax)\right)
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2a^2\mathrm{arcsinh}(ax)^2}{x} - \frac{2a\sqrt{1+a^2x^2}\mathrm{arcsinh}(ax)^3}{3x^2} - \frac{\mathrm{arcsinh}(ax)^4}{3x^3} \\
&\quad - 8a^3\mathrm{arcsinh}(ax)\mathrm{arctanh}\big(e^{\mathrm{arcsinh}(ax)}\big) + \frac{4}{3}a^3\mathrm{arcsinh}(ax)^3\mathrm{arctanh}\big(e^{\mathrm{arcsinh}(ax)}\big) \\
&\quad - 4a^3\,\mathrm{PolyLog}\left(2,-e^{\mathrm{arcsinh}(ax)}\right) + 2a^3\mathrm{arcsinh}(ax)^2\,\mathrm{PolyLog}\left(2,-e^{\mathrm{arcsinh}(ax)}\right) \\
&\quad + 4a^3\,\mathrm{PolyLog}\left(2,e^{\mathrm{arcsinh}(ax)}\right) - 2a^3\mathrm{arcsinh}(ax)^2\,\mathrm{PolyLog}\left(2,e^{\mathrm{arcsinh}(ax)}\right) \\
&\quad - 4a^3\mathrm{arcsinh}(ax)\,\mathrm{PolyLog}\left(3,-e^{\mathrm{arcsinh}(ax)}\right) \\
&\quad + 4a^3\mathrm{arcsinh}(ax)\,\mathrm{PolyLog}\left(3,e^{\mathrm{arcsinh}(ax)}\right) \\
&\quad + \left(4a^3\right)\mathrm{Subst}\left(\int\frac{\mathrm{PolyLog}(3,-x)}{x}\,dx, x, e^{\mathrm{arcsinh}(ax)}\right) \\
&\quad - \left(4a^3\right)\mathrm{Subst}\left(\int\frac{\mathrm{PolyLog}(3,x)}{x}\,dx, x, e^{\mathrm{arcsinh}(ax)}\right)
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2a^2\mathrm{arcsinh}(ax)^2}{x} - \frac{2a\sqrt{1+a^2x^2}\mathrm{arcsinh}(ax)^3}{3x^2} - \frac{\mathrm{arcsinh}(ax)^4}{3x^3} \\
&\quad - 8a^3\mathrm{arcsinh}(ax)\mathrm{arctanh}\big(e^{\mathrm{arcsinh}(ax)}\big) + \frac{4}{3}a^3\mathrm{arcsinh}(ax)^3\mathrm{arctanh}\big(e^{\mathrm{arcsinh}(ax)}\big) \\
&\quad - 4a^3\,\mathrm{PolyLog}\left(2,-e^{\mathrm{arcsinh}(ax)}\right) + 2a^3\mathrm{arcsinh}(ax)^2\,\mathrm{PolyLog}\left(2,-e^{\mathrm{arcsinh}(ax)}\right) \\
&\quad + 4a^3\,\mathrm{PolyLog}\left(2,e^{\mathrm{arcsinh}(ax)}\right) - 2a^3\mathrm{arcsinh}(ax)^2\,\mathrm{PolyLog}\left(2,e^{\mathrm{arcsinh}(ax)}\right) \\
&\quad - 4a^3\mathrm{arcsinh}(ax)\,\mathrm{PolyLog}\left(3,-e^{\mathrm{arcsinh}(ax)}\right) \\
&\quad + 4a^3\mathrm{arcsinh}(ax)\,\mathrm{PolyLog}\left(3,e^{\mathrm{arcsinh}(ax)}\right) \\
&\quad + 4a^3\,\mathrm{PolyLog}\left(4,-e^{\mathrm{arcsinh}(ax)}\right) - 4a^3\,\mathrm{PolyLog}\left(4,e^{\mathrm{arcsinh}(ax)}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.86 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.59

$$\int \frac{\operatorname{arcsinh}(ax)^4}{x^4}\,dx = \frac{1}{24}a^3\left(-2\pi^4+4\operatorname{arcsinh}(ax)^4-24\operatorname{arcsinh}(ax)^2\coth\left(\frac{1}{2}\operatorname{arcsinh}(ax)\right)\right.$$
$$+2\operatorname{arcsinh}(ax)^4\coth\left(\frac{1}{2}\operatorname{arcsinh}(ax)\right)$$
$$-4\operatorname{arcsinh}(ax)^3\operatorname{csch}^2\left(\frac{1}{2}\operatorname{arcsinh}(ax)\right)$$
$$-\frac{1}{2}ax\operatorname{arcsinh}(ax)^4\operatorname{csch}^4\left(\frac{1}{2}\operatorname{arcsinh}(ax)\right)$$
$$+96\operatorname{arcsinh}(ax)\log\left(1-e^{-\operatorname{arcsinh}(ax)}\right)$$
$$-96\operatorname{arcsinh}(ax)\log\left(1+e^{-\operatorname{arcsinh}(ax)}\right)$$
$$+16\operatorname{arcsinh}(ax)^3\log\left(1+e^{-\operatorname{arcsinh}(ax)}\right)$$
$$-16\operatorname{arcsinh}(ax)^3\log\left(1-e^{\operatorname{arcsinh}(ax)}\right)$$
$$-48\left(-2+\operatorname{arcsinh}(ax)^2\right)\operatorname{PolyLog}\left(2,-e^{-\operatorname{arcsinh}(ax)}\right)$$
$$-96\operatorname{PolyLog}\left(2,e^{-\operatorname{arcsinh}(ax)}\right)-48\operatorname{arcsinh}(ax)^2\operatorname{PolyLog}\left(2,e^{\operatorname{arcsinh}(ax)}\right)$$
$$-96\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(3,-e^{-\operatorname{arcsinh}(ax)}\right)$$
$$+96\operatorname{arcsinh}(ax)\operatorname{PolyLog}\left(3,e^{\operatorname{arcsinh}(ax)}\right)$$
$$-96\operatorname{PolyLog}\left(4,-e^{-\operatorname{arcsinh}(ax)}\right)-96\operatorname{PolyLog}\left(4,e^{\operatorname{arcsinh}(ax)}\right)$$
$$-4\operatorname{arcsinh}(ax)^3\operatorname{sech}^2\left(\frac{1}{2}\operatorname{arcsinh}(ax)\right)$$
$$-\frac{8\operatorname{arcsinh}(ax)^4\sinh^4\left(\frac{1}{2}\operatorname{arcsinh}(ax)\right)}{a^3x^3}$$
$$+24\operatorname{arcsinh}(ax)^2\tanh\left(\frac{1}{2}\operatorname{arcsinh}(ax)\right)$$
$$\left.-2\operatorname{arcsinh}(ax)^4\tanh\left(\frac{1}{2}\operatorname{arcsinh}(ax)\right)\right)$$

```
[In] Integrate[ArcSinh[a*x]^4/x^4,x]
```

```
[Out] (a^3*(-2*Pi^4 + 4*ArcSinh[a*x]^4 - 24*ArcSinh[a*x]^2*Coth[ArcSinh[a*x]/2] +
 2*ArcSinh[a*x]^4*Coth[ArcSinh[a*x]/2] - 4*ArcSinh[a*x]^3*Csch[ArcSinh[a*x]
/2]^2 - (a*x*ArcSinh[a*x]^4*Csch[ArcSinh[a*x]/2]^4)/2 + 96*ArcSinh[a*x]*Log
[1 - E^(-ArcSinh[a*x])] - 96*ArcSinh[a*x]*Log[1 + E^(-ArcSinh[a*x])] + 16*A
rcSinh[a*x]^3*Log[1 + E^(-ArcSinh[a*x])] - 16*ArcSinh[a*x]^3*Log[1 - E^ArcS
inh[a*x]] - 48*(-2 + ArcSinh[a*x]^2)*PolyLog[2, -E^(-ArcSinh[a*x])] - 96*Po
lyLog[2, E^(-ArcSinh[a*x])] - 48*ArcSinh[a*x]^2*PolyLog[2, E^ArcSinh[a*x]]
- 96*ArcSinh[a*x]*PolyLog[3, -E^(-ArcSinh[a*x])] + 96*ArcSinh[a*x]*PolyLog[
3, E^ArcSinh[a*x]] - 96*PolyLog[4, -E^(-ArcSinh[a*x])] - 96*PolyLog[4, E^Ar
```

```
cSinh[a*x]] - 4*ArcSinh[a*x]^3*Sech[ArcSinh[a*x]/2]^2 - (8*ArcSinh[a*x]^4*S
inh[ArcSinh[a*x]/2]^4)/(a^3*x^3) + 24*ArcSinh[a*x]^2*Tanh[ArcSinh[a*x]/2] -
 2*ArcSinh[a*x]^4*Tanh[ArcSinh[a*x]/2]))/24
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.52

method	result
derivativedivides	$a^3\Bigg(-\frac{\operatorname{arcsinh}(ax)^2\left(2\operatorname{arcsinh}(ax)\sqrt{a^2x^2+1}\,ax+\operatorname{arcsinh}(ax)^2+6a^2x^2\right)}{3a^3x^3}+\frac{2\operatorname{arcsinh}(ax)^3\ln\left(1+ax+\sqrt{a^2x^2+1}\right)}{3}+2$
default	$a^3\Bigg(-\frac{\operatorname{arcsinh}(ax)^2\left(2\operatorname{arcsinh}(ax)\sqrt{a^2x^2+1}\,ax+\operatorname{arcsinh}(ax)^2+6a^2x^2\right)}{3a^3x^3}+\frac{2\operatorname{arcsinh}(ax)^3\ln\left(1+ax+\sqrt{a^2x^2+1}\right)}{3}+2$

```
[In] int(arcsinh(a*x)^4/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*(-1/3/a^3/x^3*arcsinh(a*x)^2*(2*arcsinh(a*x)*(a^2*x^2+1)^(1/2)*a*x+arcs
inh(a*x)^2+6*a^2*x^2)+2/3*arcsinh(a*x)^3*ln(1+a*x+(a^2*x^2+1)^(1/2))+2*arcs
inh(a*x)^2*polylog(2,-a*x-(a^2*x^2+1)^(1/2))-4*arcsinh(a*x)*polylog(3,-a*x-
(a^2*x^2+1)^(1/2))+4*polylog(4,-a*x-(a^2*x^2+1)^(1/2))-2/3*arcsinh(a*x)^3*l
n(1-a*x-(a^2*x^2+1)^(1/2))-2*arcsinh(a*x)^2*polylog(2,a*x+(a^2*x^2+1)^(1/2)
)+4*arcsinh(a*x)*polylog(3,a*x+(a^2*x^2+1)^(1/2))-4*polylog(4,a*x+(a^2*x^2+
1)^(1/2))-4*arcsinh(a*x)*ln(1+a*x+(a^2*x^2+1)^(1/2))-4*polylog(2,-a*x-(a^2*
x^2+1)^(1/2))+4*arcsinh(a*x)*ln(1-a*x-(a^2*x^2+1)^(1/2))+4*polylog(2,a*x+(a
^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{\operatorname{arcsinh}(ax)^4}{x^4}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^4}{x^4}\,dx$$

```
[In] integrate(arcsinh(a*x)^4/x^4,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x)^4/x^4, x)
```

Sympy [F]

$$\int \frac{\operatorname{arcsinh}(ax)^4}{x^4}\,dx = \int \frac{\operatorname{asinh}^4{(ax)}}{x^4}\,dx$$

```
[In] integrate(asinh(a*x)**4/x**4,x)
```

```
[Out] Integral(asinh(a*x)**4/x**4, x)
```

Maxima [F]

$$\int \frac{\operatorname{arcsinh}(ax)^4}{x^4}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^4}{x^4}\,dx$$

```
[In] integrate(arcsinh(a*x)^4/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*log(a*x + sqrt(a^2*x^2 + 1))^4/x^3 + integrate(4/3*(a^3*x^2 + sqrt(a^2
*x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))^3/(a^3*x^6 + a*x^4 + (a^2
*x^5 + x^3)*sqrt(a^2*x^2 + 1)), x)
```

Giac [F]

$$\int \frac{\operatorname{arcsinh}(ax)^4}{x^4}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^4}{x^4}\,dx$$

```
[In] integrate(arcsinh(a*x)^4/x^4,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^4/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arcsinh}(ax)^4}{x^4}\,dx = \int \frac{{\mathrm{asinh}\left(a\,x\right)}^4}{x^4}\,dx$$

```
[In] int(asinh(a*x)^4/x^4,x)
```

```
[Out] int(asinh(a*x)^4/x^4, x)
```

3.42 $\int \frac{x^6}{\operatorname{arcsinh}(ax)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 55

$$\int \frac{x^6}{\operatorname{arcsinh}(ax)}\,dx = -\frac{5\operatorname{Chi}(\operatorname{arcsinh}(ax))}{64a^7} + \frac{9\operatorname{Chi}(3\operatorname{arcsinh}(ax))}{64a^7} - \frac{5\operatorname{Chi}(5\operatorname{arcsinh}(ax))}{64a^7} + \frac{\operatorname{Chi}(7\operatorname{arcsinh}(ax))}{64a^7}$$

```
[Out] -5/64*Chi(arcsinh(a*x))/a^7+9/64*Chi(3*arcsinh(a*x))/a^7-5/64*Chi(5*arcsinh
(a*x))/a^7+1/64*Chi(7*arcsinh(a*x))/a^7
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5780, 5556, 3382}

$$\int \frac{x^6}{\operatorname{arcsinh}(ax)}\,dx = -\frac{5\operatorname{Chi}(\operatorname{arcsinh}(ax))}{64a^7} + \frac{9\operatorname{Chi}(3\operatorname{arcsinh}(ax))}{64a^7} - \frac{5\operatorname{Chi}(5\operatorname{arcsinh}(ax))}{64a^7} + \frac{\operatorname{Chi}(7\operatorname{arcsinh}(ax))}{64a^7}$$

```
[In] Int[x^6/ArcSinh[a*x],x]
```

```
[Out] (-5*CoshIntegral[ArcSinh[a*x]])/(64*a^7) + (9*CoshIntegral[3*ArcSinh[a*x]])
/(64*a^7) - (5*CoshIntegral[5*ArcSinh[a*x]])/(64*a^7) + CoshIntegral[7*ArcS
inh[a*x]]/(64*a^7)
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\cosh(x)\sinh^6(x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{a^7} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{5\cosh(x)}{64x}+\frac{9\cosh(3x)}{64x}-\frac{5\cosh(5x)}{64x}+\frac{\cosh(7x)}{64x}\right)dx, x, \text{arcsinh}(ax)\right)}{a^7} \\
&= \frac{\text{Subst}\left(\int \frac{\cosh(7x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{64a^7} - \frac{5\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{64a^7} \\
&\quad - \frac{5\text{Subst}\left(\int \frac{\cosh(5x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{64a^7} + \frac{9\text{Subst}\left(\int \frac{\cosh(3x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{64a^7} \\
&= -\frac{5\text{Chi}(\text{arcsinh}(ax))}{64a^7}+\frac{9\text{Chi}(3\text{arcsinh}(ax))}{64a^7}-\frac{5\text{Chi}(5\text{arcsinh}(ax))}{64a^7}+\frac{\text{Chi}(7\text{arcsinh}(ax))}{64a^7}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.73

$$
\int \frac{x^6}{\text{arcsinh}(ax)}\,dx = \frac{-5\text{Chi}(\text{arcsinh}(ax))+9\text{Chi}(3\text{arcsinh}(ax))-5\text{Chi}(5\text{arcsinh}(ax))+\text{Chi}(7\text{arcsinh}(ax))}{64a^7}
$$

```
[In] Integrate[x^6/ArcSinh[a*x],x]
```

```
[Out] (-5*CoshIntegral[ArcSinh[a*x]] + 9*CoshIntegral[3*ArcSinh[a*x]] - 5*CoshInt
egral[5*ArcSinh[a*x]] + CoshIntegral[7*ArcSinh[a*x]])/(64*a^7)
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$\frac{-\frac{5\,\mathrm{Chi}(\operatorname{arcsinh}(ax))}{64}+\frac{9\,\mathrm{Chi}(3\operatorname{arcsinh}(ax))}{64}-\frac{5\,\mathrm{Chi}(5\operatorname{arcsinh}(ax))}{64}+\frac{\mathrm{Chi}(7\operatorname{arcsinh}(ax))}{64}}{a^7}$	40
default	$\frac{-\frac{5\,\mathrm{Chi}(\operatorname{arcsinh}(ax))}{64}+\frac{9\,\mathrm{Chi}(3\operatorname{arcsinh}(ax))}{64}-\frac{5\,\mathrm{Chi}(5\operatorname{arcsinh}(ax))}{64}+\frac{\mathrm{Chi}(7\operatorname{arcsinh}(ax))}{64}}{a^7}$	40

```
[In] int(x^6/arcsinh(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^7*(-5/64*Chi(arcsinh(a*x))+9/64*Chi(3*arcsinh(a*x))-5/64*Chi(5*arcsinh(
a*x))+1/64*Chi(7*arcsinh(a*x)))
```

Fricas [F]

$$\int \frac{x^6}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{x^6}{\operatorname{arsinh}\left(ax\right)}\,dx$$

```
[In] integrate(x^6/arcsinh(a*x),x, algorithm="fricas")
```

```
[Out] integral(x^6/arcsinh(a*x), x)
```

Sympy [F]

$$\int \frac{x^6}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{x^6}{\operatorname{asinh}\left(ax\right)}\,dx$$

```
[In] integrate(x**6/asinh(a*x),x)
```

```
[Out] Integral(x**6/asinh(a*x), x)
```

Maxima [F]

$$\int \frac{x^6}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{x^6}{\operatorname{arsinh}\left(ax\right)}\,dx$$

```
[In] integrate(x^6/arcsinh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^6/arcsinh(a*x), x)
```

Giac [F]

$$\int \frac{x^6}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{x^6}{\operatorname{arsinh}\left(ax\right)}\,dx$$

```
[In] integrate(x^6/arcsinh(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^6/arcsinh(a*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{x^6}{\operatorname{asinh}\left(a\,x\right)}\,dx$$

```
[In] int(x^6/asinh(a*x),x)
```

```
[Out] int(x^6/asinh(a*x), x)
```

3.43 $\int \frac{x^5}{\operatorname{arcsinh}(ax)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 43

$$\int \frac{x^5}{\operatorname{arcsinh}(ax)}\,dx=\frac{5\operatorname{Shi}(2\operatorname{arcsinh}(ax))}{32a^6}-\frac{\operatorname{Shi}(4\operatorname{arcsinh}(ax))}{8a^6}+\frac{\operatorname{Shi}(6\operatorname{arcsinh}(ax))}{32a^6}$$

```
[Out] 5/32*Shi(2*arcsinh(a*x))/a^6-1/8*Shi(4*arcsinh(a*x))/a^6+1/32*Shi(6*arcsinh
(a*x))/a^6
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5780, 5556, 3379}

$$\int \frac{x^5}{\operatorname{arcsinh}(ax)}\,dx=\frac{5\operatorname{Shi}(2\operatorname{arcsinh}(ax))}{32a^6}-\frac{\operatorname{Shi}(4\operatorname{arcsinh}(ax))}{8a^6}+\frac{\operatorname{Shi}(6\operatorname{arcsinh}(ax))}{32a^6}$$

```
[In] Int[x^5/ArcSinh[a*x],x]
```

```
[Out] (5*SinhIntegral[2*ArcSinh[a*x]])/(32*a^6) - SinhIntegral[4*ArcSinh[a*x]]/(8
*a^6) + SinhIntegral[6*ArcSinh[a*x]]/(32*a^6)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\Big(\int \frac{\cosh(x)\sinh^5(x)}{x}\,dx, x, \operatorname{arcsinh}(ax)\Big)}{a^6} \\
&= \frac{\text{Subst}\Big(\int \Big(\frac{5\sinh(2x)}{32x} - \frac{\sinh(4x)}{8x} + \frac{\sinh(6x)}{32x}\Big)\,dx, x, \operatorname{arcsinh}(ax)\Big)}{a^6} \\
&= \frac{\text{Subst}\Big(\int \frac{\sinh(6x)}{x}\,dx, x, \operatorname{arcsinh}(ax)\Big)}{32a^6} - \frac{\text{Subst}\Big(\int \frac{\sinh(4x)}{x}\,dx, x, \operatorname{arcsinh}(ax)\Big)}{8a^6} \\
&\quad + \frac{5\text{Subst}\Big(\int \frac{\sinh(2x)}{x}\,dx, x, \operatorname{arcsinh}(ax)\Big)}{32a^6} \\
&= \frac{5\text{Shi}(2\operatorname{arcsinh}(ax))}{32a^6} - \frac{\text{Shi}(4\operatorname{arcsinh}(ax))}{8a^6} + \frac{\text{Shi}(6\operatorname{arcsinh}(ax))}{32a^6}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

$$\int \frac{x^5}{\operatorname{arcsinh}(ax)}\,dx = \frac{5\text{Shi}(2\operatorname{arcsinh}(ax)) - 4\text{Shi}(4\operatorname{arcsinh}(ax)) + \text{Shi}(6\operatorname{arcsinh}(ax))}{32a^6}$$

```
[In] Integrate[x^5/ArcSinh[a*x],x]
```

```
[Out] (5*SinhIntegral[2*ArcSinh[a*x]] - 4*SinhIntegral[4*ArcSinh[a*x]] + SinhInte
gral[6*ArcSinh[a*x]])/(32*a^6)
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{\frac{5\,\mathrm{Shi}(2\,\mathrm{arcsinh}(ax))}{32}-\frac{\mathrm{Shi}(4\,\mathrm{arcsinh}(ax))}{8}+\frac{\mathrm{Shi}(6\,\mathrm{arcsinh}(ax))}{32}}{a^6}$	33
default	$\frac{\frac{5\,\mathrm{Shi}(2\,\mathrm{arcsinh}(ax))}{32}-\frac{\mathrm{Shi}(4\,\mathrm{arcsinh}(ax))}{8}+\frac{\mathrm{Shi}(6\,\mathrm{arcsinh}(ax))}{32}}{a^6}$	33

```
[In] int(x^5/arcsinh(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^6*(5/32*Shi(2*arcsinh(a*x))-1/8*Shi(4*arcsinh(a*x))+1/32*Shi(6*arcsinh(
a*x)))
```

Fricas [F]

$$\int \frac{x^5}{\operatorname{arcsinh}(ax)}\,dx=\int \frac{x^5}{\operatorname{arsinh}\left(ax\right)}\,dx$$

```
[In] integrate(x^5/arcsinh(a*x),x, algorithm="fricas")
```

```
[Out] integral(x^5/arcsinh(a*x), x)
```

Sympy [F]

$$\int \frac{x^5}{\operatorname{arcsinh}(ax)}\,dx=\int \frac{x^5}{\operatorname{asinh}\left(ax\right)}\,dx$$

```
[In] integrate(x**5/asinh(a*x),x)
```

```
[Out] Integral(x**5/asinh(a*x), x)
```

Maxima [F]

$$\int \frac{x^5}{\operatorname{arcsinh}(ax)}\,dx=\int \frac{x^5}{\operatorname{arsinh}\left(ax\right)}\,dx$$

```
[In] integrate(x^5/arcsinh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^5/arcsinh(a*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^5}{\operatorname{arcsinh}(ax)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^5/arcsinh(a*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{x^5}{\operatorname{asinh}(a\,x)}\,dx$$

```
[In] int(x^5/asinh(a*x),x)
```

```
[Out] int(x^5/asinh(a*x), x)
```

3.44 $\int \frac{x^4}{\operatorname{arcsinh}(ax)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 41

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)}\,dx=\frac{\operatorname{Chi}(\operatorname{arcsinh}(ax))}{8a^5}-\frac{3\operatorname{Chi}(3\operatorname{arcsinh}(ax))}{16a^5}+\frac{\operatorname{Chi}(5\operatorname{arcsinh}(ax))}{16a^5}$$

```
[Out] 1/8*Chi(arcsinh(a*x))/a^5-3/16*Chi(3*arcsinh(a*x))/a^5+1/16*Chi(5*arcsinh(a
*x))/a^5
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5780, 5556, 3382}

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)}\,dx=\frac{\operatorname{Chi}(\operatorname{arcsinh}(ax))}{8a^5}-\frac{3\operatorname{Chi}(3\operatorname{arcsinh}(ax))}{16a^5}+\frac{\operatorname{Chi}(5\operatorname{arcsinh}(ax))}{16a^5}$$

```
[In] Int[x^4/ArcSinh[a*x],x]
```

```
[Out] CoshIntegral[ArcSinh[a*x]]/(8*a^5) - (3*CoshIntegral[3*ArcSinh[a*x]])/(16*a
^5) + CoshIntegral[5*ArcSinh[a*x]]/(16*a^5)
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{\cosh(x)\sinh^4(x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{a^5}\\ &= \frac{\text{Subst}\left(\int \left(\frac{\cosh(x)}{8x} - \frac{3\cosh(3x)}{16x} + \frac{\cosh(5x)}{16x}\right)\,dx, x, \text{arcsinh}(ax)\right)}{a^5}\\ &= \frac{\text{Subst}\left(\int \frac{\cosh(5x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{16a^5} + \frac{\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{8a^5}\\ &\quad - \frac{3\text{Subst}\left(\int \frac{\cosh(3x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{16a^5}\\ &= \frac{\text{Chi}(\text{arcsinh}(ax))}{8a^5} - \frac{3\text{Chi}(3\text{arcsinh}(ax))}{16a^5} + \frac{\text{Chi}(5\text{arcsinh}(ax))}{16a^5}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \frac{x^4}{\text{arcsinh}(ax)}\,dx = \frac{2\text{Chi}(\text{arcsinh}(ax)) - 3\text{Chi}(3\text{arcsinh}(ax)) + \text{Chi}(5\text{arcsinh}(ax))}{16a^5}$$

```
[In] Integrate[x^4/ArcSinh[a*x],x]
```

```
[Out] (2*CoshIntegral[ArcSinh[a*x]] - 3*CoshIntegral[3*ArcSinh[a*x]] + CoshIntegr
al[5*ArcSinh[a*x]])/(16*a^5)
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$\frac{\frac{\mathrm{Chi}(\mathrm{arcsinh}(ax))}{8}-\frac{3\,\mathrm{Chi}(3\,\mathrm{arcsinh}(ax))}{16}+\frac{\mathrm{Chi}(5\,\mathrm{arcsinh}(ax))}{16}}{a^5}$	31
default	$\frac{\frac{\mathrm{Chi}(\mathrm{arcsinh}(ax))}{8}-\frac{3\,\mathrm{Chi}(3\,\mathrm{arcsinh}(ax))}{16}+\frac{\mathrm{Chi}(5\,\mathrm{arcsinh}(ax))}{16}}{a^5}$	31

```
[In] int(x^4/arcsinh(a*x),x,method=_RETURNVERBOSE)
[Out] 1/a^5*(1/8*Chi(arcsinh(a*x))-3/16*Chi(3*arcsinh(a*x))+1/16*Chi(5*arcsinh(a*
x)))
```

Fricas [F]

$$\int \frac{x^4}{\mathrm{arcsinh}(ax)}\,dx=\int \frac{x^4}{\mathrm{arsinh}\,(ax)}\,dx$$

```
[In] integrate(x^4/arcsinh(a*x),x, algorithm="fricas")
[Out] integral(x^4/arcsinh(a*x), x)
```

Sympy [F]

$$\int \frac{x^4}{\mathrm{arcsinh}(ax)}\,dx=\int \frac{x^4}{\mathrm{asinh}\,(ax)}\,dx$$

```
[In] integrate(x**4/asinh(a*x),x)
[Out] Integral(x**4/asinh(a*x), x)
```

Maxima [F]

$$\int \frac{x^4}{\mathrm{arcsinh}(ax)}\,dx=\int \frac{x^4}{\mathrm{arsinh}\,(ax)}\,dx$$

```
[In] integrate(x^4/arcsinh(a*x),x, algorithm="maxima")
[Out] integrate(x^4/arcsinh(a*x), x)
```

Giac [F]

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{x^4}{\operatorname{arsinh}\left(ax\right)}\,dx$$

```
[In] integrate(x^4/arcsinh(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^4/arcsinh(a*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{x^4}{\operatorname{asinh}\left(a\,x\right)}\,dx$$

```
[In] int(x^4/asinh(a*x),x)
```

```
[Out] int(x^4/asinh(a*x), x)
```

3.45 $\int \frac{x^3}{\operatorname{arcsinh}(ax)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 29

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)}\,dx=-\frac{\operatorname{Shi}(2\operatorname{arcsinh}(ax))}{4a^4}+\frac{\operatorname{Shi}(4\operatorname{arcsinh}(ax))}{8a^4}$$

```
[Out] -1/4*Shi(2*arcsinh(a*x))/a^4+1/8*Shi(4*arcsinh(a*x))/a^4
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5780, 5556, 3379}

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)}\,dx=\frac{\operatorname{Shi}(4\operatorname{arcsinh}(ax))}{8a^4}-\frac{\operatorname{Shi}(2\operatorname{arcsinh}(ax))}{4a^4}$$

```
[In] Int[x^3/ArcSinh[a*x],x]
```

```
[Out] -1/4*SinhIntegral[2*ArcSinh[a*x]]/a^4 + SinhIntegral[4*ArcSinh[a*x]]/(8*a^4
)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
```

```
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{\cosh(x)\sinh^3(x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{a^4}\\ &= \frac{\text{Subst}\left(\int \left(-\frac{\sinh(2x)}{4x}+\frac{\sinh(4x)}{8x}\right)\,dx, x, \text{arcsinh}(ax)\right)}{a^4}\\ &= \frac{\text{Subst}\left(\int \frac{\sinh(4x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{8a^4}-\frac{\text{Subst}\left(\int \frac{\sinh(2x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{4a^4}\\ &= -\frac{\text{Shi}(2\text{arcsinh}(ax))}{4a^4}+\frac{\text{Shi}(4\text{arcsinh}(ax))}{8a^4}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{x^3}{\text{arcsinh}(ax)}\,dx = \frac{-2\text{Shi}(2\text{arcsinh}(ax))+\text{Shi}(4\text{arcsinh}(ax))}{8a^4}$$

```
[In] Integrate[x^3/ArcSinh[a*x],x]
[Out] (-2*SinhIntegral[2*ArcSinh[a*x]] + SinhIntegral[4*ArcSinh[a*x]])/(8*a^4)
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{-\frac{\text{Shi}(2\ \text{arcsinh}(ax))}{4}+\frac{\text{Shi}(4\ \text{arcsinh}(ax))}{8}}{a^4}$	24
default	$\frac{-\frac{\text{Shi}(2\ \text{arcsinh}(ax))}{4}+\frac{\text{Shi}(4\ \text{arcsinh}(ax))}{8}}{a^4}$	24

```
[In] int(x^3/arcsinh(a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*(-1/4*Shi(2*arcsinh(a*x))+1/8*Shi(4*arcsinh(a*x)))
```

Fricas [F]

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{x^3}{\operatorname{arsinh}\left(ax\right)}\,dx$$

```
[In] integrate(x^3/arcsinh(a*x),x, algorithm="fricas")
```

```
[Out] integral(x^3/arcsinh(a*x), x)
```

Sympy [F]

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{x^3}{\operatorname{asinh}\left(ax\right)}\,dx$$

```
[In] integrate(x**3/asinh(a*x),x)
```

```
[Out] Integral(x**3/asinh(a*x), x)
```

Maxima [F]

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{x^3}{\operatorname{arsinh}\left(ax\right)}\,dx$$

```
[In] integrate(x^3/arcsinh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^3/arcsinh(a*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3/arcsinh(a*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{x^3}{\operatorname{asinh}\left(a\,x\right)}\,dx$$

```
[In] int(x^3/asinh(a*x),x)
```

```
[Out] int(x^3/asinh(a*x), x)
```

3.46 $\int \frac{x^2}{\operatorname{arcsinh}(ax)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 27

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)}\,dx = -\frac{\operatorname{Chi}(\operatorname{arcsinh}(ax))}{4a^3} + \frac{\operatorname{Chi}(3\operatorname{arcsinh}(ax))}{4a^3}$$

```
[Out] -1/4*Chi(arcsinh(a*x))/a^3+1/4*Chi(3*arcsinh(a*x))/a^3
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5780, 5556, 3382}

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)}\,dx = \frac{\operatorname{Chi}(3\operatorname{arcsinh}(ax))}{4a^3} - \frac{\operatorname{Chi}(\operatorname{arcsinh}(ax))}{4a^3}$$

```
[In] Int[x^2/ArcSinh[a*x],x]
```

```
[Out] -1/4*CoshIntegral[ArcSinh[a*x]]/a^3 + CoshIntegral[3*ArcSinh[a*x]]/(4*a^3)
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
```

```
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{a^3} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{\cosh(x)}{4x} + \frac{\cosh(3x)}{4x}\right)\,dx, x, \text{arcsinh}(ax)\right)}{a^3} \\
&= -\frac{\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{4a^3} + \frac{\text{Subst}\left(\int \frac{\cosh(3x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{4a^3} \\
&= -\frac{\text{Chi}(\text{arcsinh}(ax))}{4a^3} + \frac{\text{Chi}(3\text{arcsinh}(ax))}{4a^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{x^2}{\text{arcsinh}(ax)}\,dx = \frac{-\text{Chi}(\text{arcsinh}(ax)) + \text{Chi}(3\text{arcsinh}(ax))}{4a^3}$$

```
[In] Integrate[x^2/ArcSinh[a*x],x]
[Out] (-CoshIntegral[ArcSinh[a*x]] + CoshIntegral[3*ArcSinh[a*x]])/(4*a^3)
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{-\frac{\text{Chi}(\text{arcsinh}(ax))}{4}+\frac{\text{Chi}(3\ \text{arcsinh}(ax))}{4}}{a^3}$	22
default	$\frac{-\frac{\text{Chi}(\text{arcsinh}(ax))}{4}+\frac{\text{Chi}(3\ \text{arcsinh}(ax))}{4}}{a^3}$	22

```
[In] int(x^2/arcsinh(a*x),x,method=_RETURNVERBOSE)
[Out] 1/a^3*(-1/4*Chi(arcsinh(a*x))+1/4*Chi(3*arcsinh(a*x)))
```

Fricas [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{x^2}{\operatorname{arsinh}\left(ax\right)}\,dx$$

```
[In] integrate(x^2/arcsinh(a*x),x, algorithm="fricas")
[Out] integral(x^2/arcsinh(a*x), x)
```

Sympy [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{x^2}{\operatorname{asinh}\left(ax\right)}\,dx$$

```
[In] integrate(x**2/asinh(a*x),x)
[Out] Integral(x**2/asinh(a*x), x)
```

Maxima [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{x^2}{\operatorname{arsinh}\left(ax\right)}\,dx$$

```
[In] integrate(x^2/arcsinh(a*x),x, algorithm="maxima")
[Out] integrate(x^2/arcsinh(a*x), x)
```

Giac [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{x^2}{\operatorname{arsinh}\left(ax\right)}\,dx$$

```
[In] integrate(x^2/arcsinh(a*x),x, algorithm="giac")
[Out] integrate(x^2/arcsinh(a*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{x^2}{\operatorname{asinh}\left(a\,x\right)}\,dx$$

```
[In] int(x^2/asinh(a*x),x)
```

```
[Out] int(x^2/asinh(a*x), x)
```

3.47 $\int \frac{x}{\operatorname{arcsinh}(ax)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 14

$$\int \frac{x}{\operatorname{arcsinh}(ax)}\,dx=\frac{\operatorname{Shi}(2\operatorname{arcsinh}(ax))}{2a^2}$$

```
[Out] 1/2*Shi(2*arcsinh(a*x))/a^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5780, 5556, 12, 3379}

$$\int \frac{x}{\operatorname{arcsinh}(ax)}\,dx=\frac{\operatorname{Shi}(2\operatorname{arcsinh}(ax))}{2a^2}$$

```
[In] Int[x/ArcSinh[a*x],x]
```

```
[Out] SinhIntegral[2*ArcSinh[a*x]]/(2*a^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{a^2}\\ &= \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{2x}\,dx, x, \text{arcsinh}(ax)\right)}{a^2}\\ &= \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{2a^2}\\ &= \frac{\text{Shi}(2\text{arcsinh}(ax))}{2a^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{x}{\text{arcsinh}(ax)}\,dx = \frac{\text{Shi}(2\text{arcsinh}(ax))}{2a^2}$$

```
[In] Integrate[x/ArcSinh[a*x],x]
[Out] SinhIntegral[2*ArcSinh[a*x]]/(2*a^2)
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\mathrm{Shi}(2\,\mathrm{arcsinh}(ax))}{2a^2}$	13
default	$\frac{\mathrm{Shi}(2\,\mathrm{arcsinh}(ax))}{2a^2}$	13

```
[In] int(x/arcsinh(a*x),x,method=_RETURNVERBOSE)
[Out] 1/2*Shi(2*arcsinh(a*x))/a^2
```

Fricas [F]

$$\int \frac{x}{\mathrm{arcsinh}(ax)}\,dx = \int \frac{x}{\mathrm{arsinh}\,(ax)}\,dx$$

```
[In] integrate(x/arcsinh(a*x),x, algorithm="fricas")
[Out] integral(x/arcsinh(a*x), x)
```

Sympy [F]

$$\int \frac{x}{\mathrm{arcsinh}(ax)}\,dx = \int \frac{x}{\mathrm{asinh}\,(ax)}\,dx$$

```
[In] integrate(x/asinh(a*x),x)
[Out] Integral(x/asinh(a*x), x)
```

Maxima [F]

$$\int \frac{x}{\mathrm{arcsinh}(ax)}\,dx = \int \frac{x}{\mathrm{arsinh}\,(ax)}\,dx$$

```
[In] integrate(x/arcsinh(a*x),x, algorithm="maxima")
[Out] integrate(x/arcsinh(a*x), x)
```

Giac [F]

$$\int \frac{x}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{x}{\operatorname{arsinh}\left(ax\right)}\,dx$$

```
[In] integrate(x/arcsinh(a*x),x, algorithm="giac")
[Out] integrate(x/arcsinh(a*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{x}{\operatorname{asinh}\left(a\,x\right)}\,dx$$

```
[In] int(x/asinh(a*x),x)
[Out] int(x/asinh(a*x), x)
```

3.48 $\int \frac{1}{\operatorname{arcsinh}(ax)}\,dx$

Optimal result

Integrand size = 6, antiderivative size = 9

$$\int \frac{1}{\operatorname{arcsinh}(ax)}\,dx = \frac{\operatorname{Chi}(\operatorname{arcsinh}(ax))}{a}$$

```
[Out] Chi(arcsinh(a*x))/a
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5774, 3382}

$$\int \frac{1}{\operatorname{arcsinh}(ax)}\,dx = \frac{\operatorname{Chi}(\operatorname{arcsinh}(ax))}{a}$$

```
[In] Int[ArcSinh[a*x]^(-1),x]
```

```
[Out] CoshIntegral[ArcSinh[a*x]]/a
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \operatorname{arcsinh}(ax)\right)}{a}\\ &= \frac{\text{Chi}(\operatorname{arcsinh}(ax))}{a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{\operatorname{arcsinh}(ax)}\,dx = \frac{\text{Chi}(\operatorname{arcsinh}(ax))}{a}$$

```
[In] Integrate[ArcSinh[a*x]^(-1),x]
[Out] CoshIntegral[ArcSinh[a*x]]/a
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

method	result	size
derivativedivides	$\frac{\text{Chi}(\operatorname{arcsinh}(ax))}{a}$	10
default	$\frac{\text{Chi}(\operatorname{arcsinh}(ax))}{a}$	10

```
[In] int(1/arcsinh(a*x),x,method=_RETURNVERBOSE)
[Out] Chi(arcsinh(a*x))/a
```

Fricas [F]

$$\int \frac{1}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{1}{\operatorname{arsinh}(ax)}\,dx$$

```
[In] integrate(1/arcsinh(a*x),x, algorithm="fricas")
[Out] integral(1/arcsinh(a*x), x)
```

Sympy [F]

$$\int \frac{1}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{1}{\operatorname{asinh}{\left(ax \right)}}\,dx$$

```
[In] integrate(1/asinh(a*x),x)
[Out] Integral(1/asinh(a*x), x)
```

Maxima [F]

$$\int \frac{1}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{1}{\operatorname{arsinh}\left(ax\right)}\,dx$$

```
[In] integrate(1/arcsinh(a*x),x, algorithm="maxima")
[Out] integrate(1/arcsinh(a*x), x)
```

Giac [F]

$$\int \frac{1}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{1}{\operatorname{arsinh}\left(ax\right)}\,dx$$

```
[In] integrate(1/arcsinh(a*x),x, algorithm="giac")
[Out] integrate(1/arcsinh(a*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{1}{\operatorname{asinh}\left(a\,x\right)}\,dx$$

```
[In] int(1/asinh(a*x),x)
[Out] int(1/asinh(a*x), x)
```

3.49 $\int \frac{1}{x\mathbf{arcsinh}(ax)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{1}{x\operatorname{arcsinh}(ax)}\,dx = \operatorname{Int}\left(\frac{1}{x\operatorname{arcsinh}(ax)},x\right)$$

```
[Out] Unintegrable(1/x/arcsinh(a*x),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\operatorname{arcsinh}(ax)}\,dx = \int \frac{1}{x\operatorname{arcsinh}(ax)}\,dx$$

```
[In] Int[1/(x*ArcSinh[a*x]),x]
```

```
[Out] Defer[Int][1/(x*ArcSinh[a*x]), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x\operatorname{arcsinh}(ax)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\text{arcsinh}(ax)}\,dx = \int \frac{1}{x\text{arcsinh}(ax)}\,dx$$

```
[In] Integrate[1/(x*ArcSinh[a*x]),x]
[Out] Integrate[1/(x*ArcSinh[a*x]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\ \text{arcsinh}\left(ax\right)}dx$$

```
[In] int(1/x/arcsinh(a*x),x)
[Out] int(1/x/arcsinh(a*x),x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\text{arcsinh}(ax)}\,dx = \int \frac{1}{x\ \text{arsinh}\left(ax\right)}\,dx$$

```
[In] integrate(1/x/arcsinh(a*x),x, algorithm="fricas")
[Out] integral(1/(x*arcsinh(a*x)), x)
```

Sympy [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{x\text{arcsinh}(ax)}\,dx = \int \frac{1}{x\ \text{asinh}\left(ax\right)}\,dx$$

```
[In] integrate(1/x/asinh(a*x),x)
[Out] Integral(1/(x*asinh(a*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\mathrm{arcsinh}(ax)}\,dx = \int \frac{1}{x\,\mathrm{arsinh}\left(ax\right)}\,dx$$

```
[In] integrate(1/x/arcsinh(a*x),x, algorithm="maxima")
[Out] integrate(1/(x*arcsinh(a*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\mathrm{arcsinh}(ax)}\,dx = \int \frac{1}{x\,\mathrm{arsinh}\left(ax\right)}\,dx$$

```
[In] integrate(1/x/arcsinh(a*x),x, algorithm="giac")
[Out] integrate(1/(x*arcsinh(a*x)), x)
```

Mupad [N/A]

Not integrable

Time = 2.57 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\mathrm{arcsinh}(ax)}\,dx = \int \frac{1}{x\,\mathrm{asinh}\left(a\,x\right)}\,dx$$

```
[In] int(1/(x*asinh(a*x)),x)
[Out] int(1/(x*asinh(a*x)), x)
```

3.50 $\int \frac{1}{x^2\textbf{arcsinh}(ax)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{1}{x^2\text{arcsinh}(ax)}\,dx = \text{Int}\left(\frac{1}{x^2\text{arcsinh}(ax)},x\right)$$

```
[Out] Unintegrable(1/x^2/arcsinh(a*x),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2\text{arcsinh}(ax)}\,dx = \int \frac{1}{x^2\text{arcsinh}(ax)}\,dx$$

```
[In] Int[1/(x^2*ArcSinh[a*x]),x]
```

```
[Out] Defer[Int][1/(x^2*ArcSinh[a*x]), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x^2\text{arcsinh}(ax)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2\text{arcsinh}(ax)}\,dx = \int \frac{1}{x^2\text{arcsinh}(ax)}\,dx$$

```
[In] Integrate[1/(x^2*ArcSinh[a*x]),x]
[Out] Integrate[1/(x^2*ArcSinh[a*x]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\,\text{arcsinh}\left(ax\right)}dx$$

```
[In] int(1/x^2/arcsinh(a*x),x)
[Out] int(1/x^2/arcsinh(a*x),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2\text{arcsinh}(ax)}\,dx = \int \frac{1}{x^2\,\text{arsinh}\left(ax\right)}\,dx$$

```
[In] integrate(1/x^2/arcsinh(a*x),x, algorithm="fricas")
[Out] integral(1/(x^2*arcsinh(a*x)), x)
```

Sympy [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\text{arcsinh}(ax)}\,dx = \int \frac{1}{x^2\,\text{asinh}\left(ax\right)}\,dx$$

```
[In] integrate(1/x**2/asinh(a*x),x)
[Out] Integral(1/(x**2*asinh(a*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \operatorname{arcsinh}(ax)}\,dx = \int \frac{1}{x^2 \operatorname{arsinh}\left(ax\right)}\,dx$$

```
[In] integrate(1/x^2/arcsinh(a*x),x, algorithm="maxima")
[Out] integrate(1/(x^2*arcsinh(a*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \operatorname{arcsinh}(ax)}\,dx = \int \frac{1}{x^2 \operatorname{arsinh}\left(ax\right)}\,dx$$

```
[In] integrate(1/x^2/arcsinh(a*x),x, algorithm="giac")
[Out] integrate(1/(x^2*arcsinh(a*x)), x)
```

Mupad [N/A]

Not integrable

Time = 2.66 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \operatorname{arcsinh}(ax)}\,dx = \int \frac{1}{x^2 \operatorname{asinh}\left(a\,x\right)}\,dx$$

```
[In] int(1/(x^2*asinh(a*x)),x)
[Out] int(1/(x^2*asinh(a*x)), x)
```

3.51 $\int \frac{x^6}{\text{arcsinh}(ax)^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 82

$$\int \frac{x^6}{\text{arcsinh}(ax)^2}\,dx = -\frac{x^6\sqrt{1+a^2x^2}}{a\text{arcsinh}(ax)} - \frac{5\text{Shi}(\text{arcsinh}(ax))}{64a^7} + \frac{27\text{Shi}(3\text{arcsinh}(ax))}{64a^7} - \frac{25\text{Shi}(5\text{arcsinh}(ax))}{64a^7} + \frac{7\text{Shi}(7\text{arcsinh}(ax))}{64a^7}$$

```
[Out] -5/64*Shi(arcsinh(a*x))/a^7+27/64*Shi(3*arcsinh(a*x))/a^7-25/64*Shi(5*arcsi
nh(a*x))/a^7+7/64*Shi(7*arcsinh(a*x))/a^7-x^6*(a^2*x^2+1)^(1/2)/a/arcsinh(a
*x)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5778, 3379}

$$\int \frac{x^6}{\text{arcsinh}(ax)^2}\,dx = -\frac{5\text{Shi}(\text{arcsinh}(ax))}{64a^7} + \frac{27\text{Shi}(3\text{arcsinh}(ax))}{64a^7} - \frac{25\text{Shi}(5\text{arcsinh}(ax))}{64a^7} + \frac{7\text{Shi}(7\text{arcsinh}(ax))}{64a^7} - \frac{x^6\sqrt{a^2x^2+1}}{a\text{arcsinh}(ax)}$$

```
[In] Int[x^6/ArcSinh[a*x]^2,x]
```

```
[Out] -((x^6*Sqrt[1 + a^2*x^2])/(a*ArcSinh[a*x])) - (5*SinhIntegral[ArcSinh[a*x]]
)/(64*a^7) + (27*SinhIntegral[3*ArcSinh[a*x]])/(64*a^7) - (25*SinhIntegral[
5*ArcSinh[a*x]])/(64*a^7) + (7*SinhIntegral[7*ArcSinh[a*x]])/(64*a^7)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{x^6\sqrt{1+a^2x^2}}{a\text{arcsinh}(ax)} \\
&\quad + \frac{\text{Subst}\left(\int \left(-\frac{5\sinh(x)}{64x}+\frac{27\sinh(3x)}{64x}-\frac{25\sinh(5x)}{64x}+\frac{7\sinh(7x)}{64x}\right)\,dx, x, \text{arcsinh}(ax)\right)}{a^7} \\
&= -\frac{x^6\sqrt{1+a^2x^2}}{a\text{arcsinh}(ax)} - \frac{5\text{Subst}\left(\int \frac{\sinh(x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{64a^7} \\
&\quad + \frac{7\text{Subst}\left(\int \frac{\sinh(7x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{64a^7} - \frac{25\text{Subst}\left(\int \frac{\sinh(5x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{64a^7} \\
&\quad + \frac{27\text{Subst}\left(\int \frac{\sinh(3x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{64a^7} \\
&= -\frac{x^6\sqrt{1+a^2x^2}}{a\text{arcsinh}(ax)} - \frac{5\text{Shi}(\text{arcsinh}(ax))}{64a^7} + \frac{27\text{Shi}(3\text{arcsinh}(ax))}{64a^7} \\
&\quad - \frac{25\text{Shi}(5\text{arcsinh}(ax))}{64a^7} + \frac{7\text{Shi}(7\text{arcsinh}(ax))}{64a^7}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.04

$$
\int \frac{x^6}{\text{arcsinh}(ax)^2}\,dx = \\
-\frac{64a^6x^6\sqrt{1+a^2x^2} + 5\text{arcsinh}(ax)\text{Shi}(\text{arcsinh}(ax)) - 27\text{arcsinh}(ax)\text{Shi}(3\text{arcsinh}(ax)) + 25\text{arcsinh}(ax)\text{S}}{64a^7\text{arcsinh}(ax)}
$$

```
[In] Integrate[x^6/ArcSinh[a*x]^2,x]
```

```
[Out] -1/64*(64*a^6*x^6*Sqrt[1 + a^2*x^2] + 5*ArcSinh[a*x]*SinhIntegral[ArcSinh[a
*x]] - 27*ArcSinh[a*x]*SinhIntegral[3*ArcSinh[a*x]] + 25*ArcSinh[a*x]*SinhI
ntegral[5*ArcSinh[a*x]] - 7*ArcSinh[a*x]*SinhIntegral[7*ArcSinh[a*x]])/(a^7
*ArcSinh[a*x])
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.27

method	result
derivativedivides	$\frac{\frac{5\sqrt{a^2x^2+1}}{64\ \mathrm{arcsinh}(ax)}-\frac{5\ \mathrm{Shi}(\mathrm{arcsinh}(ax))}{64}-\frac{9\cosh(3\ \mathrm{arcsinh}(ax))}{64\ \mathrm{arcsinh}(ax)}+\frac{27\ \mathrm{Shi}(3\ \mathrm{arcsinh}(ax))}{64}+\frac{5\cosh(5\ \mathrm{arcsinh}(ax))}{64\ \mathrm{arcsinh}(ax)}-\frac{25\ \mathrm{Shi}(5\ \mathrm{arcsinh}(ax))}{64}}{a^7}$
default	$\frac{\frac{5\sqrt{a^2x^2+1}}{64\ \mathrm{arcsinh}(ax)}-\frac{5\ \mathrm{Shi}(\mathrm{arcsinh}(ax))}{64}-\frac{9\cosh(3\ \mathrm{arcsinh}(ax))}{64\ \mathrm{arcsinh}(ax)}+\frac{27\ \mathrm{Shi}(3\ \mathrm{arcsinh}(ax))}{64}+\frac{5\cosh(5\ \mathrm{arcsinh}(ax))}{64\ \mathrm{arcsinh}(ax)}-\frac{25\ \mathrm{Shi}(5\ \mathrm{arcsinh}(ax))}{64}}{a^7}$

```
[In] int(x^6/arcsinh(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^7*(5/64/arcsinh(a*x)*(a^2*x^2+1)^(1/2)-5/64*Shi(arcsinh(a*x))-9/64/arcs
inh(a*x)*cosh(3*arcsinh(a*x))+27/64*Shi(3*arcsinh(a*x))+5/64/arcsinh(a*x)*c
osh(5*arcsinh(a*x))-25/64*Shi(5*arcsinh(a*x))-1/64/arcsinh(a*x)*cosh(7*arcs
inh(a*x))+7/64*Shi(7*arcsinh(a*x)))
```

Fricas [F]

$$\int \frac{x^6}{\mathrm{arcsinh}(ax)^2}\,dx = \int \frac{x^6}{\mathrm{arsinh}\left(ax\right)^2}\,dx$$

```
[In] integrate(x^6/arcsinh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^6/arcsinh(a*x)^2, x)
```

Sympy [F]

$$\int \frac{x^6}{\mathrm{arcsinh}(ax)^2}\,dx = \int \frac{x^6}{\mathrm{asinh}^2\left(ax\right)}\,dx$$

```
[In] integrate(x**6/asinh(a*x)**2,x)
```

```
[Out] Integral(x**6/asinh(a*x)**2, x)
```

Maxima [F]

$$\int \frac{x^6}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{x^6}{\operatorname{arsinh}\left(ax\right)^2}\,dx$$

```
[In] integrate(x^6/arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^3*x^9 + a*x^7 + (a^2*x^8 + x^6)*sqrt(a^2*x^2 + 1))/((a^3*x^2 + sqrt(a^2
*x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))) + integrate((7*a^5*x^10
+ 14*a^3*x^8 + 7*a*x^6 + (7*a^3*x^8 + 5*a*x^6)*(a^2*x^2 + 1) + (14*a^4*x^9
+ 19*a^2*x^7 + 6*x^5)*sqrt(a^2*x^2 + 1))/((a^5*x^4 + (a^2*x^2 + 1)*a^3*x^2
+ 2*a^3*x^2 + 2*(a^4*x^3 + a^2*x)*sqrt(a^2*x^2 + 1) + a)*log(a*x + sqrt(a^2
*x^2 + 1))), x)
```

Giac [F]

$$\int \frac{x^6}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{x^6}{\operatorname{arsinh}\left(ax\right)^2}\,dx$$

```
[In] integrate(x^6/arcsinh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^6/arcsinh(a*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{x^6}{\operatorname{asinh}\left(a\,x\right)^2}\,dx$$

```
[In] int(x^6/asinh(a*x)^2,x)
```

```
[Out] int(x^6/asinh(a*x)^2, x)
```

3.52 $\int \frac{x^5}{\mathbf{arcsinh}(ax)^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 70

$$\int \frac{x^5}{\operatorname{arcsinh}(ax)^2}\,dx = -\frac{x^5\sqrt{1+a^2x^2}}{a\operatorname{arcsinh}(ax)}+\frac{5\operatorname{Chi}(2\operatorname{arcsinh}(ax))}{16a^6}-\frac{\operatorname{Chi}(4\operatorname{arcsinh}(ax))}{2a^6}+\frac{3\operatorname{Chi}(6\operatorname{arcsinh}(ax))}{16a^6}$$

```
[Out] 5/16*Chi(2*arcsinh(a*x))/a^6-1/2*Chi(4*arcsinh(a*x))/a^6+3/16*Chi(6*arcsinh
(a*x))/a^6-x^5*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5778, 3382}

$$\int \frac{x^5}{\operatorname{arcsinh}(ax)^2}\,dx = \frac{5\operatorname{Chi}(2\operatorname{arcsinh}(ax))}{16a^6}-\frac{\operatorname{Chi}(4\operatorname{arcsinh}(ax))}{2a^6}+\frac{3\operatorname{Chi}(6\operatorname{arcsinh}(ax))}{16a^6}-\frac{x^5\sqrt{a^2x^2+1}}{a\operatorname{arcsinh}(ax)}$$

```
[In] Int[x^5/ArcSinh[a*x]^2,x]
```

```
[Out] -((x^5*Sqrt[1 + a^2*x^2])/(a*ArcSinh[a*x])) + (5*CoshIntegral[2*ArcSinh[a*x
]])/(16*a^6) - CoshIntegral[4*ArcSinh[a*x]]/(2*a^6) + (3*CoshIntegral[6*Arc
Sinh[a*x]])/(16*a^6)
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{x^5\sqrt{1+a^2x^2}}{a\text{arcsinh}(ax)}+\frac{\text{Subst}\left(\int\left(\frac{5\cosh(2x)}{16x}-\frac{\cosh(4x)}{2x}+\frac{3\cosh(6x)}{16x}\right)dx,x,\text{arcsinh}(ax)\right)}{a^6}\\
&= -\frac{x^5\sqrt{1+a^2x^2}}{a\text{arcsinh}(ax)}+\frac{3\text{Subst}\left(\int\frac{\cosh(6x)}{x}dx,x,\text{arcsinh}(ax)\right)}{16a^6}\\
&\quad+\frac{5\text{Subst}\left(\int\frac{\cosh(2x)}{x}dx,x,\text{arcsinh}(ax)\right)}{16a^6}-\frac{\text{Subst}\left(\int\frac{\cosh(4x)}{x}dx,x,\text{arcsinh}(ax)\right)}{2a^6}\\
&= -\frac{x^5\sqrt{1+a^2x^2}}{a\text{arcsinh}(ax)}+\frac{5\text{Chi}(2\text{arcsinh}(ax))}{16a^6}-\frac{\text{Chi}(4\text{arcsinh}(ax))}{2a^6}+\frac{3\text{Chi}(6\text{arcsinh}(ax))}{16a^6}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.11

$$
\int\frac{x^5}{\text{arcsinh}(ax)^2}dx = -\frac{-10\text{arcsinh}(ax)\text{Chi}(2\text{arcsinh}(ax))+16\text{arcsinh}(ax)\text{Chi}(4\text{arcsinh}(ax))-6\text{arcsinh}(ax)\text{Chi}(6\text{arcsinh}(ax))}{32a^6\text{arcsinh}(ax)}
$$

```
[In] Integrate[x^5/ArcSinh[a*x]^2,x]
```

```
[Out] -1/32*(-10*ArcSinh[a*x]*CoshIntegral[2*ArcSinh[a*x]] + 16*ArcSinh[a*x]*Cosh
Integral[4*ArcSinh[a*x]] - 6*ArcSinh[a*x]*CoshIntegral[6*ArcSinh[a*x]] + 5*
Sinh[2*ArcSinh[a*x]] - 4*Sinh[4*ArcSinh[a*x]] + Sinh[6*ArcSinh[a*x]])/(a^6*
ArcSinh[a*x])
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.11

method	result
derivativedivides	$\frac{-\frac{5 \sinh(2 \operatorname{arcsinh}(ax))}{32 \operatorname{arcsinh}(ax)}+\frac{5 \operatorname{Chi}(2 \operatorname{arcsinh}(ax))}{16}+\frac{\sinh(4 \operatorname{arcsinh}(ax))}{8 \operatorname{arcsinh}(ax)}-\frac{\operatorname{Chi}(4 \operatorname{arcsinh}(ax))}{2}-\frac{\sinh(6 \operatorname{arcsinh}(ax))}{32 \operatorname{arcsinh}(ax)}+\frac{3 \operatorname{Chi}(6 \operatorname{arcsinh}(a}{16}}{a^6}$
default	$\frac{-\frac{5 \sinh(2 \operatorname{arcsinh}(ax))}{32 \operatorname{arcsinh}(ax)}+\frac{5 \operatorname{Chi}(2 \operatorname{arcsinh}(ax))}{16}+\frac{\sinh(4 \operatorname{arcsinh}(ax))}{8 \operatorname{arcsinh}(ax)}-\frac{\operatorname{Chi}(4 \operatorname{arcsinh}(ax))}{2}-\frac{\sinh(6 \operatorname{arcsinh}(ax))}{32 \operatorname{arcsinh}(ax)}+\frac{3 \operatorname{Chi}(6 \operatorname{arcsinh}(a}{16}}{a^6}$

```
[In] int(x^5/arcsinh(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^6*(-5/32/arcsinh(a*x)*sinh(2*arcsinh(a*x))+5/16*Chi(2*arcsinh(a*x))+1/8
/arcsinh(a*x)*sinh(4*arcsinh(a*x))-1/2*Chi(4*arcsinh(a*x))-1/32/arcsinh(a*x
)*sinh(6*arcsinh(a*x))+3/16*Chi(6*arcsinh(a*x)))
```

Fricas [F]

$$\int \frac{x^5}{\operatorname{arcsinh}(ax)^2} \, dx = \int \frac{x^5}{\operatorname{arsinh}\left(ax\right)^2} \, dx$$

```
[In] integrate(x^5/arcsinh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^5/arcsinh(a*x)^2, x)
```

Sympy [F]

$$\int \frac{x^5}{\operatorname{arcsinh}(ax)^2} \, dx = \int \frac{x^5}{\operatorname{asinh}^2\left(ax\right)} \, dx$$

```
[In] integrate(x**5/asinh(a*x)**2,x)
```

```
[Out] Integral(x**5/asinh(a*x)**2, x)
```

Maxima [F]

$$\int \frac{x^5}{\operatorname{arcsinh}(ax)^2} \, dx = \int \frac{x^5}{\operatorname{arsinh}\left(ax\right)^2} \, dx$$

```
[In] integrate(x^5/arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^3*x^8 + a*x^6 + (a^2*x^7 + x^5)*sqrt(a^2*x^2 + 1))/((a^3*x^2 + sqrt(a^2
*x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))) + integrate((6*a^5*x^9 +
```

```
 12*a^3*x^7 + 6*a*x^5 + 2*(3*a^3*x^7 + 2*a*x^5)*(a^2*x^2 + 1) + (12*a^4*x^8
 + 16*a^2*x^6 + 5*x^4)*sqrt(a^2*x^2 + 1))/((a^5*x^4 + (a^2*x^2 + 1)*a^3*x^2
 + 2*a^3*x^2 + 2*(a^4*x^3 + a^2*x)*sqrt(a^2*x^2 + 1) + a)*log(a*x + sqrt(a^
2*x^2 + 1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^5}{\operatorname{arcsinh}(ax)^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^5/arcsinh(a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{x^5}{{\mathrm{asinh}\left(a\,x\right)}^2} \,d x$$

```
[In] int(x^5/asinh(a*x)^2,x)
```

```
[Out] int(x^5/asinh(a*x)^2, x)
```

3.53 $\int \frac{x^4}{\operatorname{arcsinh}(ax)^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 68

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^2}\,dx = -\frac{x^4\sqrt{1+a^2x^2}}{a\operatorname{arcsinh}(ax)} + \frac{\operatorname{Shi}(\operatorname{arcsinh}(ax))}{8a^5} - \frac{9\operatorname{Shi}(3\operatorname{arcsinh}(ax))}{16a^5} + \frac{5\operatorname{Shi}(5\operatorname{arcsinh}(ax))}{16a^5}$$

```
[Out] 1/8*Shi(arcsinh(a*x))/a^5-9/16*Shi(3*arcsinh(a*x))/a^5+5/16*Shi(5*arcsinh(a
*x))/a^5-x^4*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5778, 3379}

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^2}\,dx = \frac{\operatorname{Shi}(\operatorname{arcsinh}(ax))}{8a^5} - \frac{9\operatorname{Shi}(3\operatorname{arcsinh}(ax))}{16a^5} + \frac{5\operatorname{Shi}(5\operatorname{arcsinh}(ax))}{16a^5} - \frac{x^4\sqrt{a^2x^2+1}}{a\operatorname{arcsinh}(ax)}$$

```
[In] Int[x^4/ArcSinh[a*x]^2,x]
```

```
[Out] -((x^4*Sqrt[1 + a^2*x^2])/(a*ArcSinh[a*x])) + SinhIntegral[ArcSinh[a*x]]/(8
*a^5) - (9*SinhIntegral[3*ArcSinh[a*x]])/(16*a^5) + (5*SinhIntegral[5*ArcSi
nh[a*x]])/(16*a^5)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{x^4\sqrt{1+a^2x^2}}{a\text{arcsinh}(ax)}+\frac{\text{Subst}\left(\int\left(\frac{\sinh(x)}{8x}-\frac{9\sinh(3x)}{16x}+\frac{5\sinh(5x)}{16x}\right)dx,x,\text{arcsinh}(ax)\right)}{a^5}\\
&= -\frac{x^4\sqrt{1+a^2x^2}}{a\text{arcsinh}(ax)}+\frac{\text{Subst}\left(\int\frac{\sinh(x)}{x}\,dx,x,\text{arcsinh}(ax)\right)}{8a^5}\\
&\quad+\frac{5\text{Subst}\left(\int\frac{\sinh(5x)}{x}\,dx,x,\text{arcsinh}(ax)\right)}{16a^5}-\frac{9\text{Subst}\left(\int\frac{\sinh(3x)}{x}\,dx,x,\text{arcsinh}(ax)\right)}{16a^5}\\
&= -\frac{x^4\sqrt{1+a^2x^2}}{a\text{arcsinh}(ax)}+\frac{\text{Shi}(\text{arcsinh}(ax))}{8a^5}-\frac{9\text{Shi}(3\text{arcsinh}(ax))}{16a^5}+\frac{5\text{Shi}(5\text{arcsinh}(ax))}{16a^5}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.88

$$\int\frac{x^4}{\text{arcsinh}(ax)^2}\,dx = \frac{-\frac{16a^4x^4\sqrt{1+a^2x^2}}{\text{arcsinh}(ax)}+2\text{Shi}(\text{arcsinh}(ax))-9\text{Shi}(3\text{arcsinh}(ax))+5\text{Shi}(5\text{arcsinh}(ax))}{16a^5}$$

```
[In] Integrate[x^4/ArcSinh[a*x]^2,x]
```

```
[Out] ((-16*a^4*x^4*Sqrt[1 + a^2*x^2])/ArcSinh[a*x] + 2*SinhIntegral[ArcSinh[a*x]
] - 9*SinhIntegral[3*ArcSinh[a*x]] + 5*SinhIntegral[5*ArcSinh[a*x]])/(16*a^
5)
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.18

method	result	s
derivativedivides	$\frac{-\frac{\sqrt{a^{2}x^{2}+1}}{8\,\operatorname{arcsinh}(ax)}+\frac{\operatorname{Shi}(\operatorname{arcsinh}(ax))}{8}+\frac{3\cosh(3\,\operatorname{arcsinh}(ax))}{16\,\operatorname{arcsinh}(ax)}-\frac{9\,\operatorname{Shi}(3\,\operatorname{arcsinh}(ax))}{16}-\frac{\cosh(5\,\operatorname{arcsinh}(ax))}{16\,\operatorname{arcsinh}(ax)}+\frac{5\,\operatorname{Shi}(5\,\operatorname{arcsinh}(ax))}{16}}{a^{5}}$	8
default	$\frac{-\frac{\sqrt{a^{2}x^{2}+1}}{8\,\operatorname{arcsinh}(ax)}+\frac{\operatorname{Shi}(\operatorname{arcsinh}(ax))}{8}+\frac{3\cosh(3\,\operatorname{arcsinh}(ax))}{16\,\operatorname{arcsinh}(ax)}-\frac{9\,\operatorname{Shi}(3\,\operatorname{arcsinh}(ax))}{16}-\frac{\cosh(5\,\operatorname{arcsinh}(ax))}{16\,\operatorname{arcsinh}(ax)}+\frac{5\,\operatorname{Shi}(5\,\operatorname{arcsinh}(ax))}{16}}{a^{5}}$	8

```
[In] int(x^4/arcsinh(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^5*(-1/8/arcsinh(a*x)*(a^2*x^2+1)^(1/2)+1/8*Shi(arcsinh(a*x))+3/16/arcsi
nh(a*x)*cosh(3*arcsinh(a*x))-9/16*Shi(3*arcsinh(a*x))-1/16/arcsinh(a*x)*cos
h(5*arcsinh(a*x))+5/16*Shi(5*arcsinh(a*x)))
```

Fricas [F]

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{x^{4}}{\operatorname{arsinh}\left(ax\right)^{2}}\,dx$$

```
[In] integrate(x^4/arcsinh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^4/arcsinh(a*x)^2, x)
```

Sympy [F]

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{x^{4}}{\operatorname{asinh}^{2}\left(ax\right)}\,dx$$

```
[In] integrate(x**4/asinh(a*x)**2,x)
```

```
[Out] Integral(x**4/asinh(a*x)**2, x)
```

Maxima [F]

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{x^{4}}{\operatorname{arsinh}\left(ax\right)^{2}}\,dx$$

```
[In] integrate(x^4/arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^3*x^7 + a*x^5 + (a^2*x^6 + x^4)*sqrt(a^2*x^2 + 1))/((a^3*x^2 + sqrt(a^2
*x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))) + integrate((5*a^5*x^8 +
```

```
 10*a^3*x^6 + 5*a*x^4 + (5*a^3*x^6 + 3*a*x^4)*(a^2*x^2 + 1) + (10*a^4*x^7 +
 13*a^2*x^5 + 4*x^3)*sqrt(a^2*x^2 + 1))/((a^5*x^4 + (a^2*x^2 + 1)*a^3*x^2 +
 2*a^3*x^2 + 2*(a^4*x^3 + a^2*x)*sqrt(a^2*x^2 + 1) + a)*log(a*x + sqrt(a^2*
x^2 + 1))), x)
```

Giac [F]

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^2}\,dx=\int \frac{x^4}{\operatorname{arsinh}\left(ax\right)^2}\,dx$$

```
[In] integrate(x^4/arcsinh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^4/arcsinh(a*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^2}\,dx=\int \frac{x^4}{\operatorname{asinh}\left(a\,x\right)^2}\,dx$$

```
[In] int(x^4/asinh(a*x)^2,x)
```

```
[Out] int(x^4/asinh(a*x)^2, x)
```

3.54 $\int \frac{x^3}{\operatorname{arcsinh}(ax)^2} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 56

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^2} \, dx = -\frac{x^3\sqrt{1+a^2x^2}}{a\operatorname{arcsinh}(ax)} - \frac{\operatorname{Chi}(2\operatorname{arcsinh}(ax))}{2a^4} + \frac{\operatorname{Chi}(4\operatorname{arcsinh}(ax))}{2a^4}$$

```
[Out] -1/2*Chi(2*arcsinh(a*x))/a^4+1/2*Chi(4*arcsinh(a*x))/a^4-x^3*(a^2*x^2+1)^(1
/2)/a/arcsinh(a*x)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5778, 3382}

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^2} \, dx = -\frac{\operatorname{Chi}(2\operatorname{arcsinh}(ax))}{2a^4} + \frac{\operatorname{Chi}(4\operatorname{arcsinh}(ax))}{2a^4} - \frac{x^3\sqrt{a^2x^2+1}}{a\operatorname{arcsinh}(ax)}$$

```
[In] Int[x^3/ArcSinh[a*x]^2,x]
```

```
[Out] -((x^3*Sqrt[1 + a^2*x^2])/(a*ArcSinh[a*x])) - CoshIntegral[2*ArcSinh[a*x]]/
(2*a^4) + CoshIntegral[4*ArcSinh[a*x]]/(2*a^4)
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^3\sqrt{1+a^2x^2}}{a\text{arcsinh}(ax)}+\frac{\text{Subst}\left(\int\left(-\frac{\cosh(2x)}{2x}+\frac{\cosh(4x)}{2x}\right)dx,x,\text{arcsinh}(ax)\right)}{a^4}\\
&= -\frac{x^3\sqrt{1+a^2x^2}}{a\text{arcsinh}(ax)}-\frac{\text{Subst}\left(\int\frac{\cosh(2x)}{x}\,dx,x,\text{arcsinh}(ax)\right)}{2a^4}+\frac{\text{Subst}\left(\int\frac{\cosh(4x)}{x}\,dx,x,\text{arcsinh}(ax)\right)}{2a^4}\\
&= -\frac{x^3\sqrt{1+a^2x^2}}{a\text{arcsinh}(ax)}-\frac{\text{Chi}(2\text{arcsinh}(ax))}{2a^4}+\frac{\text{Chi}(4\text{arcsinh}(ax))}{2a^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int\frac{x^3}{\text{arcsinh}(ax)^2}\,dx = -\frac{4\text{arcsinh}(ax)\text{Chi}(2\text{arcsinh}(ax))-4\text{arcsinh}(ax)\text{Chi}(4\text{arcsinh}(ax))-2\sinh(2\text{arcsinh}(ax))+\sinh(4\text{arcsi}}{8a^4\text{arcsinh}(ax)}$$

```
[In] Integrate[x^3/ArcSinh[a*x]^2,x]
[Out] -1/8*(4*ArcSinh[a*x]*CoshIntegral[2*ArcSinh[a*x]] - 4*ArcSinh[a*x]*CoshInte
gral[4*ArcSinh[a*x]] - 2*Sinh[2*ArcSinh[a*x]] + Sinh[4*ArcSinh[a*x]])/(a^4*
ArcSinh[a*x])
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{\frac{\sinh(2\,\text{arcsinh}(ax))}{4\,\text{arcsinh}(ax)}-\frac{\text{Chi}(2\,\text{arcsinh}(ax))}{2}-\frac{\sinh(4\,\text{arcsinh}(ax))}{8\,\text{arcsinh}(ax)}+\frac{\text{Chi}(4\,\text{arcsinh}(ax))}{2}}{a^4}$	54
default	$\frac{\frac{\sinh(2\,\text{arcsinh}(ax))}{4\,\text{arcsinh}(ax)}-\frac{\text{Chi}(2\,\text{arcsinh}(ax))}{2}-\frac{\sinh(4\,\text{arcsinh}(ax))}{8\,\text{arcsinh}(ax)}+\frac{\text{Chi}(4\,\text{arcsinh}(ax))}{2}}{a^4}$	54

```
[In] int(x^3/arcsinh(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*(1/4/arcsinh(a*x)*sinh(2*arcsinh(a*x))-1/2*Chi(2*arcsinh(a*x))-1/8/ar
csinh(a*x)*sinh(4*arcsinh(a*x))+1/2*Chi(4*arcsinh(a*x)))
```

Fricas [F]

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{x^3}{\operatorname{arsinh}\left(ax\right)^2}\,dx$$

```
[In] integrate(x^3/arcsinh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^3/arcsinh(a*x)^2, x)
```

Sympy [F]

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{x^3}{\operatorname{asinh}^2\left(ax\right)}\,dx$$

```
[In] integrate(x**3/asinh(a*x)**2,x)
```

```
[Out] Integral(x**3/asinh(a*x)**2, x)
```

Maxima [F]

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{x^3}{\operatorname{arsinh}\left(ax\right)^2}\,dx$$

```
[In] integrate(x^3/arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^3*x^6 + a*x^4 + (a^2*x^5 + x^3)*sqrt(a^2*x^2 + 1))/((a^3*x^2 + sqrt(a^2
*x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))) + integrate((4*a^5*x^7 +
 8*a^3*x^5 + 4*a*x^3 + 2*(2*a^3*x^5 + a*x^3)*(a^2*x^2 + 1) + (8*a^4*x^6 + 1
0*a^2*x^4 + 3*x^2)*sqrt(a^2*x^2 + 1))/((a^5*x^4 + (a^2*x^2 + 1)*a^3*x^2 + 2
*a^3*x^2 + 2*(a^4*x^3 + a^2*x)*sqrt(a^2*x^2 + 1) + a)*log(a*x + sqrt(a^2*x^
2 + 1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3/arcsinh(a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{x^3}{{\mathrm{asinh}\left(a\,x\right)}^2}\,dx$$

```
[In] int(x^3/asinh(a*x)^2,x)
```

```
[Out] int(x^3/asinh(a*x)^2, x)
```

3.55 $\int \frac{x^2}{\text{arcsinh}(ax)^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 54

$$\int \frac{x^2}{\text{arcsinh}(ax)^2}\,dx=-\frac{x^2\sqrt{1+a^2x^2}}{a\text{arcsinh}(ax)}-\frac{\text{Shi}(\text{arcsinh}(ax))}{4a^3}+\frac{3\text{Shi}(3\text{arcsinh}(ax))}{4a^3}$$

```
[Out] -1/4*Shi(arcsinh(a*x))/a^3+3/4*Shi(3*arcsinh(a*x))/a^3-x^2*(a^2*x^2+1)^(1/2
)/a/arcsinh(a*x)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5778, 3379}

$$\int \frac{x^2}{\text{arcsinh}(ax)^2}\,dx=-\frac{\text{Shi}(\text{arcsinh}(ax))}{4a^3}+\frac{3\text{Shi}(3\text{arcsinh}(ax))}{4a^3}-\frac{x^2\sqrt{a^2x^2+1}}{a\text{arcsinh}(ax)}$$

```
[In] Int[x^2/ArcSinh[a*x]^2,x]
```

```
[Out] -((x^2*Sqrt[1 + a^2*x^2])/(a*ArcSinh[a*x])) - SinhIntegral[ArcSinh[a*x]]/(4
*a^3) + (3*SinhIntegral[3*ArcSinh[a*x]])/(4*a^3)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{x^2\sqrt{1+a^2x^2}}{a\text{arcsinh}(ax)}+\frac{\text{Subst}\left(\int\left(-\frac{\sinh(x)}{4x}+\frac{3\sinh(3x)}{4x}\right)dx,x,\text{arcsinh}(ax)\right)}{a^3}\\
&= -\frac{x^2\sqrt{1+a^2x^2}}{a\text{arcsinh}(ax)}-\frac{\text{Subst}\left(\int\frac{\sinh(x)}{x}\,dx,x,\text{arcsinh}(ax)\right)}{4a^3}+\frac{3\text{Subst}\left(\int\frac{\sinh(3x)}{x}\,dx,x,\text{arcsinh}(ax)\right)}{4a^3}\\
&= -\frac{x^2\sqrt{1+a^2x^2}}{a\text{arcsinh}(ax)}-\frac{\text{Shi}(\text{arcsinh}(ax))}{4a^3}+\frac{3\text{Shi}(3\text{arcsinh}(ax))}{4a^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int\frac{x^2}{\text{arcsinh}(ax)^2}\,dx=-\frac{\frac{4a^2x^2\sqrt{1+a^2x^2}}{\text{arcsinh}(ax)}+\text{Shi}(\text{arcsinh}(ax))-3\text{Shi}(3\text{arcsinh}(ax))}{4a^3}$$

```
[In] Integrate[x^2/ArcSinh[a*x]^2,x]
```

```
[Out] -1/4*((4*a^2*x^2*Sqrt[1 + a^2*x^2])/ArcSinh[a*x] + SinhIntegral[ArcSinh[a*x
]] - 3*SinhIntegral[3*ArcSinh[a*x]])/a^3
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$\frac{\frac{\sqrt{a^2x^2+1}}{4\,\text{arcsinh}(ax)}-\frac{\text{Shi}(\text{arcsinh}(ax))}{4}-\frac{\cosh(3\,\text{arcsinh}(ax))}{4\,\text{arcsinh}(ax)}+\frac{3\,\text{Shi}(3\,\text{arcsinh}(ax))}{4}}{a^3}$	56
default	$\frac{\frac{\sqrt{a^2x^2+1}}{4\,\text{arcsinh}(ax)}-\frac{\text{Shi}(\text{arcsinh}(ax))}{4}-\frac{\cosh(3\,\text{arcsinh}(ax))}{4\,\text{arcsinh}(ax)}+\frac{3\,\text{Shi}(3\,\text{arcsinh}(ax))}{4}}{a^3}$	56

```
[In] int(x^2/arcsinh(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(1/4/arcsinh(a*x)*(a^2*x^2+1)^(1/2)-1/4*Shi(arcsinh(a*x))-1/4/arcsinh
(a*x)*cosh(3*arcsinh(a*x))+3/4*Shi(3*arcsinh(a*x)))
```

Fricas [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{x^2}{\operatorname{arsinh}\left(ax\right)^2}\,dx$$

```
[In] integrate(x^2/arcsinh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^2/arcsinh(a*x)^2, x)
```

Sympy [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{x^2}{\operatorname{asinh}^2\left(ax\right)}\,dx$$

```
[In] integrate(x**2/asinh(a*x)**2,x)
```

```
[Out] Integral(x**2/asinh(a*x)**2, x)
```

Maxima [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{x^2}{\operatorname{arsinh}\left(ax\right)^2}\,dx$$

```
[In] integrate(x^2/arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^3*x^5 + a*x^3 + (a^2*x^4 + x^2)*sqrt(a^2*x^2 + 1))/((a^3*x^2 + sqrt(a^2
*x^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))) + integrate((3*a^5*x^6 +
 6*a^3*x^4 + 3*a*x^2 + (3*a^3*x^4 + a*x^2)*(a^2*x^2 + 1) + (6*a^4*x^5 + 7*a
^2*x^3 + 2*x)*sqrt(a^2*x^2 + 1))/((a^5*x^4 + (a^2*x^2 + 1)*a^3*x^2 + 2*a^3*
x^2 + 2*(a^4*x^3 + a^2*x)*sqrt(a^2*x^2 + 1) + a)*log(a*x + sqrt(a^2*x^2 + 1
))), x)
```

Giac [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{x^2}{\operatorname{arsinh}\left(ax\right)^2}\,dx$$

```
[In] integrate(x^2/arcsinh(a*x)^2,x, algorithm="giac")
[Out] integrate(x^2/arcsinh(a*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{x^2}{\operatorname{asinh}\left(a\,x\right)^2}\,dx$$

```
[In] int(x^2/asinh(a*x)^2,x)
[Out] int(x^2/asinh(a*x)^2, x)
```

3.56 $\int \frac{x}{\textbf{arcsinh}(ax)^2} \, dx$

Optimal result

Integrand size = 8, antiderivative size = 37

$$\int \frac{x}{\operatorname{arcsinh}(ax)^2} \, dx = -\frac{x\sqrt{1+a^2x^2}}{a\operatorname{arcsinh}(ax)} + \frac{\operatorname{Chi}(2\operatorname{arcsinh}(ax))}{a^2}$$

```
[Out] Chi(2*arcsinh(a*x))/a^2-x*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5778, 3382}

$$\int \frac{x}{\operatorname{arcsinh}(ax)^2} \, dx = \frac{\operatorname{Chi}(2\operatorname{arcsinh}(ax))}{a^2} - \frac{x\sqrt{a^2x^2+1}}{a\operatorname{arcsinh}(ax)}$$

```
[In] Int[x/ArcSinh[a*x]^2,x]
```

```
[Out] -((x*Sqrt[1 + a^2*x^2])/(a*ArcSinh[a*x])) + CoshIntegral[2*ArcSinh[a*x]]/a^
2
```

<u>Rule 3382</u>

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

<u>Rule 5778</u>

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
```

```
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{x\sqrt{1+a^2x^2}}{a\text{arcsinh}(ax)}+\frac{\text{Subst}\left(\int\frac{\cosh(2x)}{x}\,dx,x,\text{arcsinh}(ax)\right)}{a^2}\\ &= -\frac{x\sqrt{1+a^2x^2}}{a\text{arcsinh}(ax)}+\frac{\text{Chi}(2\text{arcsinh}(ax))}{a^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int\frac{x}{\text{arcsinh}(ax)^2}\,dx=\frac{\text{Chi}(2\text{arcsinh}(ax))-\frac{\sinh(2\text{arcsinh}(ax))}{2\text{arcsinh}(ax)}}{a^2}$$

```
[In] Integrate[x/ArcSinh[a*x]^2,x]
[Out] (CoshIntegral[2*ArcSinh[a*x]] - Sinh[2*ArcSinh[a*x]]/(2*ArcSinh[a*x]))/a^2
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$\frac{-\frac{\sinh(2\,\text{arcsinh}(ax))}{2\,\text{arcsinh}(ax)}+\text{Chi}(2\,\text{arcsinh}(ax))}{a^2}$	28
default	$\frac{-\frac{\sinh(2\,\text{arcsinh}(ax))}{2\,\text{arcsinh}(ax)}+\text{Chi}(2\,\text{arcsinh}(ax))}{a^2}$	28

```
[In] int(x/arcsinh(a*x)^2,x,method=_RETURNVERBOSE)
[Out] 1/a^2*(-1/2/arcsinh(a*x)*sinh(2*arcsinh(a*x))+Chi(2*arcsinh(a*x)))
```

Fricas [F]

$$\int \frac{x}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{x}{\operatorname{arsinh}\left(ax\right)^2}\,dx$$

```
[In] integrate(x/arcsinh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x/arcsinh(a*x)^2, x)
```

Sympy [F]

$$\int \frac{x}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{x}{\operatorname{asinh}^2\left(ax\right)}\,dx$$

```
[In] integrate(x/asinh(a*x)**2,x)
```

```
[Out] Integral(x/asinh(a*x)**2, x)
```

Maxima [F]

$$\int \frac{x}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{x}{\operatorname{arsinh}\left(ax\right)^2}\,dx$$

```
[In] integrate(x/arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^3*x^4 + a*x^2 + (a^2*x^3 + x)*sqrt(a^2*x^2 + 1))/((a^3*x^2 + sqrt(a^2*x
^2 + 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))) + integrate((2*a^5*x^5 + 2
*(a^2*x^2 + 1)*a^3*x^3 + 4*a^3*x^3 + 2*a*x + (4*a^4*x^4 + 4*a^2*x^2 + 1)*sq
rt(a^2*x^2 + 1))/((a^5*x^4 + (a^2*x^2 + 1)*a^3*x^2 + 2*a^3*x^2 + 2*(a^4*x^3
 + a^2*x)*sqrt(a^2*x^2 + 1) + a)*log(a*x + sqrt(a^2*x^2 + 1))), x)
```

Giac [F]

$$\int \frac{x}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{x}{\operatorname{arsinh}\left(ax\right)^2}\,dx$$

```
[In] integrate(x/arcsinh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x/arcsinh(a*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{x}{\operatorname{asinh}\left(a\,x\right)^2}\,dx$$

```
[In] int(x/asinh(a*x)^2,x)
[Out] int(x/asinh(a*x)^2, x)
```

3.57 $\int \frac{1}{\operatorname{arcsinh}(ax)^2}\,dx$

Optimal result

Integrand size = 6, antiderivative size = 34

$$\int \frac{1}{\operatorname{arcsinh}(ax)^2}\,dx = -\frac{\sqrt{1+a^2x^2}}{a\operatorname{arcsinh}(ax)}+\frac{\operatorname{Shi}(\operatorname{arcsinh}(ax))}{a}$$

```
[Out] Shi(arcsinh(a*x))/a-(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5773, 5819, 3379}

$$\int \frac{1}{\operatorname{arcsinh}(ax)^2}\,dx = \frac{\operatorname{Shi}(\operatorname{arcsinh}(ax))}{a}-\frac{\sqrt{a^2x^2+1}}{a\operatorname{arcsinh}(ax)}$$

```
[In] Int[ArcSinh[a*x]^(-2),x]
```

```
[Out] -(Sqrt[1 + a^2*x^2]/(a*ArcSinh[a*x])) + SinhIntegral[ArcSinh[a*x]]/a
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
```

```
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\sqrt{1+a^2x^2}}{a\text{arcsinh}(ax)}+a\int\frac{x}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx\\ &= -\frac{\sqrt{1+a^2x^2}}{a\text{arcsinh}(ax)}+\frac{\text{Subst}\left(\int\frac{\sinh(x)}{x}\,dx,x,\text{arcsinh}(ax)\right)}{a}\\ &= -\frac{\sqrt{1+a^2x^2}}{a\text{arcsinh}(ax)}+\frac{\text{Shi}(\text{arcsinh}(ax))}{a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int\frac{1}{\text{arcsinh}(ax)^2}\,dx=\frac{-\frac{\sqrt{1+a^2x^2}}{\text{arcsinh}(ax)}+\text{Shi}(\text{arcsinh}(ax))}{a}$$

```
[In] Integrate[ArcSinh[a*x]^(-2),x]
[Out] (-(Sqrt[1 + a^2*x^2]/ArcSinh[a*x]) + SinhIntegral[ArcSinh[a*x]])/a
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{-\frac{\sqrt{a^2x^2+1}}{\text{arcsinh}(ax)}+\text{Shi}(\text{arcsinh}(ax))}{a}$	30
default	$\frac{-\frac{\sqrt{a^2x^2+1}}{\text{arcsinh}(ax)}+\text{Shi}(\text{arcsinh}(ax))}{a}$	30

```
[In] int(1/arcsinh(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(-1/arcsinh(a*x)*(a^2*x^2+1)^(1/2)+Shi(arcsinh(a*x)))
```

Fricas [F]

$$\int \frac{1}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{1}{\operatorname{arsinh}\left(ax\right)^2}\,dx$$

```
[In] integrate(1/arcsinh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x)^(-2), x)
```

Sympy [F]

$$\int \frac{1}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{1}{\operatorname{asinh}^2\left(ax\right)}\,dx$$

```
[In] integrate(1/asinh(a*x)**2,x)
```

```
[Out] Integral(asinh(a*x)**(-2), x)
```

Maxima [F]

$$\int \frac{1}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{1}{\operatorname{arsinh}\left(ax\right)^2}\,dx$$

```
[In] integrate(1/arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^3*x^3 + a*x + (a^2*x^2 + 1)^(3/2))/((a^3*x^2 + sqrt(a^2*x^2 + 1)*a^2*x
+ a)*log(a*x + sqrt(a^2*x^2 + 1))) + integrate((a^4*x^4 + 2*a^2*x^2 + (a^2*
x^2 + 1)*(a^2*x^2 - 1) + (2*a^3*x^3 + a*x)*sqrt(a^2*x^2 + 1) + 1)/((a^4*x^4
 + (a^2*x^2 + 1)*a^2*x^2 + 2*a^2*x^2 + 2*(a^3*x^3 + a*x)*sqrt(a^2*x^2 + 1)
+ 1)*log(a*x + sqrt(a^2*x^2 + 1))), x)
```

Giac [F]

$$\int \frac{1}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{1}{\operatorname{arsinh}\left(ax\right)^2}\,dx$$

```
[In] integrate(1/arcsinh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^(-2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{1}{\operatorname{asinh}\left(a\,x\right)^2}\,dx$$

```
[In] int(1/asinh(a*x)^2,x)
```

```
[Out] int(1/asinh(a*x)^2, x)
```

3.58 $\int \frac{1}{x\textbf{arcsinh}(ax)^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{1}{x\text{arcsinh}(ax)^2}\,dx = \text{Int}\left(\frac{1}{x\text{arcsinh}(ax)^2},x\right)$$

```
[Out] Unintegrable(1/x/arcsinh(a*x)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\text{arcsinh}(ax)^2}\,dx = \int \frac{1}{x\text{arcsinh}(ax)^2}\,dx$$

```
[In] Int[1/(x*ArcSinh[a*x]^2),x]
```

```
[Out] Defer[Int][1/(x*ArcSinh[a*x]^2), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x\text{arcsinh}(ax)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\text{arcsinh}(ax)^2}\,dx = \int \frac{1}{x\text{arcsinh}(ax)^2}\,dx$$

```
[In] Integrate[1/(x*ArcSinh[a*x]^2),x]
[Out] Integrate[1/(x*ArcSinh[a*x]^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\,\text{arcsinh}\left(ax\right)^2}dx$$

```
[In] int(1/x/arcsinh(a*x)^2,x)
[Out] int(1/x/arcsinh(a*x)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\text{arcsinh}(ax)^2}\,dx = \int \frac{1}{x\,\text{arsinh}\left(ax\right)^2}\,dx$$

```
[In] integrate(1/x/arcsinh(a*x)^2,x, algorithm="fricas")
[Out] integral(1/(x*arcsinh(a*x)^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\text{arcsinh}(ax)^2}\,dx = \int \frac{1}{x\,\text{asinh}^2\left(ax\right)}\,dx$$

```
[In] integrate(1/x/asinh(a*x)**2,x)
[Out] Integral(1/(x*asinh(a*x)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 200, normalized size of antiderivative = 20.00

$$\int \frac{1}{x\text{arcsinh}(ax)^2}\,dx = \int \frac{1}{x\,\text{arsinh}\left(ax\right)^2}\,dx$$

```
[In] integrate(1/x/arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^3*x^3 + a*x + (a^2*x^2 + 1)^(3/2))/((a^3*x^3 + sqrt(a^2*x^2 + 1)*a^2*x^
2 + a*x)*log(a*x + sqrt(a^2*x^2 + 1))) - integrate((2*(a^2*x^2 + 1)*a*x + (
2*a^2*x^2 + 1)*sqrt(a^2*x^2 + 1))/((a^5*x^6 + (a^2*x^2 + 1)*a^3*x^4 + 2*a^3
*x^4 + a*x^2 + 2*(a^4*x^5 + a^2*x^3)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*
x^2 + 1))), x)
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\text{arcsinh}(ax)^2}\,dx = \int \frac{1}{x\,\text{arsinh}\left(ax\right)^2}\,dx$$

```
[In] integrate(1/x/arcsinh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/(x*arcsinh(a*x)^2), x)
```

Mupad [N/A]

Not integrable

Time = 2.54 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\text{arcsinh}(ax)^2}\,dx = \int \frac{1}{x\,\text{asinh}\left(a\,x\right)^2}\,dx$$

```
[In] int(1/(x*asinh(a*x)^2),x)
```

```
[Out] int(1/(x*asinh(a*x)^2), x)
```

3.59 $\int \frac{1}{x^2\textbf{arcsinh}(ax)^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{1}{x^2\operatorname{arcsinh}(ax)^2}\,dx = \operatorname{Int}\left(\frac{1}{x^2\operatorname{arcsinh}(ax)^2},x\right)$$

```
[Out] Unintegrable(1/x^2/arcsinh(a*x)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{1}{x^2\operatorname{arcsinh}(ax)^2}\,dx$$

```
[In] Int[1/(x^2*ArcSinh[a*x]^2),x]
```

```
[Out] Defer[Int][1/(x^2*ArcSinh[a*x]^2), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x^2\operatorname{arcsinh}(ax)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 5.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2\text{arcsinh}(ax)^2}\,dx = \int \frac{1}{x^2\text{arcsinh}(ax)^2}\,dx$$

```
[In] Integrate[1/(x^2*ArcSinh[a*x]^2),x]
[Out] Integrate[1/(x^2*ArcSinh[a*x]^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\operatorname{arcsinh}\left(ax\right)^2}dx$$

```
[In] int(1/x^2/arcsinh(a*x)^2,x)
[Out] int(1/x^2/arcsinh(a*x)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2\text{arcsinh}(ax)^2}\,dx = \int \frac{1}{x^2\operatorname{arsinh}\left(ax\right)^2}\,dx$$

```
[In] integrate(1/x^2/arcsinh(a*x)^2,x, algorithm="fricas")
[Out] integral(1/(x^2*arcsinh(a*x)^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2\text{arcsinh}(ax)^2}\,dx = \int \frac{1}{x^2\operatorname{asinh}^2\left(ax\right)}\,dx$$

```
[In] integrate(1/x**2/asinh(a*x)**2,x)
[Out] Integral(1/(x**2*asinh(a*x)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 237, normalized size of antiderivative = 23.70

$$\int \frac{1}{x^2 \text{arcsinh}(ax)^2} \, dx = \int \frac{1}{x^2 \, \text{arsinh}\,(ax)^2} \, dx$$

```
[In] integrate(1/x^2/arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^3*x^3 + a*x + (a^2*x^2 + 1)^(3/2))/((a^3*x^4 + sqrt(a^2*x^2 + 1)*a^2*x^
3 + a*x^2)*log(a*x + sqrt(a^2*x^2 + 1))) - integrate((a^5*x^5 + 2*a^3*x^3 +
 (a^3*x^3 + 3*a*x)*(a^2*x^2 + 1) + a*x + (2*a^4*x^4 + 5*a^2*x^2 + 2)*sqrt(a
^2*x^2 + 1))/((a^5*x^7 + (a^2*x^2 + 1)*a^3*x^5 + 2*a^3*x^5 + a*x^3 + 2*(a^4
*x^6 + a^2*x^4)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))), x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \text{arcsinh}(ax)^2} \, dx = \int \frac{1}{x^2 \, \text{arsinh}\,(ax)^2} \, dx$$

```
[In] integrate(1/x^2/arcsinh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/(x^2*arcsinh(a*x)^2), x)
```

Mupad [N/A]

Not integrable

Time = 2.67 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \text{arcsinh}(ax)^2} \, dx = \int \frac{1}{x^2 \, \text{asinh}\,(a\,x)^2} \, dx$$

```
[In] int(1/(x^2*asinh(a*x)^2),x)
```

```
[Out] int(1/(x^2*asinh(a*x)^2), x)
```

3.60 $\int \frac{x^4}{\mathrm{arcsinh}(ax)^3} dx$

Optimal result

Integrand size = 10, antiderivative size = 97

$$\int \frac{x^4}{\mathrm{arcsinh}(ax)^3} dx = -\frac{x^4\sqrt{1+a^2x^2}}{2a\mathrm{arcsinh}(ax)^2} - \frac{2x^3}{a^2\mathrm{arcsinh}(ax)} - \frac{5x^5}{2\mathrm{arcsinh}(ax)} + \frac{\mathrm{Chi}(\mathrm{arcsinh}(ax))}{16a^5} - \frac{27\mathrm{Chi}(3\mathrm{arcsinh}(ax))}{32a^5} + \frac{25\mathrm{Chi}(5\mathrm{arcsinh}(ax))}{32a^5}$$

```
[Out] -2*x^3/a^2/arcsinh(a*x)-5/2*x^5/arcsinh(a*x)+1/16*Chi(arcsinh(a*x))/a^5-27/
32*Chi(3*arcsinh(a*x))/a^5+25/32*Chi(5*arcsinh(a*x))/a^5-1/2*x^4*(a^2*x^2+1
)^(1/2)/a/arcsinh(a*x)^2
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5779, 5818, 5780, 5556, 3382}

$$\int \frac{x^4}{\mathrm{arcsinh}(ax)^3} dx = \frac{\mathrm{Chi}(\mathrm{arcsinh}(ax))}{16a^5} - \frac{27\mathrm{Chi}(3\mathrm{arcsinh}(ax))}{32a^5} + \frac{25\mathrm{Chi}(5\mathrm{arcsinh}(ax))}{32a^5} - \frac{2x^3}{a^2\mathrm{arcsinh}(ax)} - \frac{x^4\sqrt{a^2x^2+1}}{2a\mathrm{arcsinh}(ax)^2} - \frac{5x^5}{2\mathrm{arcsinh}(ax)}$$

```
[In] Int[x^4/ArcSinh[a*x]^3,x]
```

```
[Out] -1/2*(x^4*Sqrt[1 + a^2*x^2])/(a*ArcSinh[a*x]^2) - (2*x^3)/(a^2*ArcSinh[a*x]
) - (5*x^5)/(2*ArcSinh[a*x]) + CoshIntegral[ArcSinh[a*x]]/(16*a^5) - (27*Co
shIntegral[3*ArcSinh[a*x]])/(32*a^5) + (25*CoshIntegral[5*ArcSinh[a*x]])/(3
2*a^5)
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/S
qrt[1 + c^2*x^2]), x], x] - Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Sinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rubi steps

integral

$$= -\frac{x^4\sqrt{1+a^2x^2}}{2a\text{arcsinh}(ax)^2} + \frac{2\int \frac{x^3}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}\,dx}{a} + \frac{1}{2}(5a)\int \frac{x^5}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}\,dx$$

$$= -\frac{x^4\sqrt{1+a^2x^2}}{2a\text{arcsinh}(ax)^2} - \frac{2x^3}{a^2\text{arcsinh}(ax)} - \frac{5x^5}{2\text{arcsinh}(ax)} + \frac{25}{2}\int \frac{x^4}{\text{arcsinh}(ax)}\,dx + \frac{6\int \frac{x^2}{\text{arcsinh}(ax)}\,dx}{a^2}$$

$$= -\frac{x^4\sqrt{1+a^2x^2}}{2a\text{arcsinh}(ax)^2} - \frac{2x^3}{a^2\text{arcsinh}(ax)} - \frac{5x^5}{2\text{arcsinh}(ax)}$$
$$+ \frac{6\text{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{a^5}$$
$$+ \frac{25\text{Subst}\left(\int \frac{\cosh(x)\sinh^4(x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{2a^5}$$

$$= -\frac{x^4\sqrt{1+a^2x^2}}{2a\text{arcsinh}(ax)^2} - \frac{2x^3}{a^2\text{arcsinh}(ax)} - \frac{5x^5}{2\text{arcsinh}(ax)}$$
$$+ \frac{6\text{Subst}\left(\int \left(-\frac{\cosh(x)}{4x} + \frac{\cosh(3x)}{4x}\right)\,dx, x, \text{arcsinh}(ax)\right)}{a^5}$$
$$+ \frac{25\text{Subst}\left(\int \left(\frac{\cosh(x)}{8x} - \frac{3\cosh(3x)}{16x} + \frac{\cosh(5x)}{16x}\right)\,dx, x, \text{arcsinh}(ax)\right)}{2a^5}$$

$$= -\frac{x^4\sqrt{1+a^2x^2}}{2a\text{arcsinh}(ax)^2} - \frac{2x^3}{a^2\text{arcsinh}(ax)} - \frac{5x^5}{2\text{arcsinh}(ax)} + \frac{25\text{Subst}\left(\int \frac{\cosh(5x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{32a^5}$$
$$- \frac{3\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{2a^5} + \frac{3\text{Subst}\left(\int \frac{\cosh(3x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{2a^5}$$
$$+ \frac{25\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{16a^5} - \frac{75\text{Subst}\left(\int \frac{\cosh(3x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{32a^5}$$

$$= -\frac{x^4\sqrt{1+a^2x^2}}{2a\text{arcsinh}(ax)^2} - \frac{2x^3}{a^2\text{arcsinh}(ax)} - \frac{5x^5}{2\text{arcsinh}(ax)}$$
$$+ \frac{\text{Chi}(\text{arcsinh}(ax))}{16a^5} - \frac{27\text{Chi}(3\text{arcsinh}(ax))}{32a^5} + \frac{25\text{Chi}(5\text{arcsinh}(ax))}{32a^5}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.05

$$\int \frac{x^4}{\text{arcsinh}(ax)^3}\,dx =$$
$$-\frac{16a^4x^4\sqrt{1+a^2x^2} + 64a^3x^3\text{arcsinh}(ax) + 80a^5x^5\text{arcsinh}(ax) - 2\text{arcsinh}(ax)^2\text{Chi}(\text{arcsinh}(ax)) + 27\text{arc}}{32a^5\text{arcsinh}(ax)^2}$$

```
[In] Integrate[x^4/ArcSinh[a*x]^3,x]
```

```
[Out] -1/32*(16*a^4*x^4*Sqrt[1 + a^2*x^2] + 64*a^3*x^3*ArcSinh[a*x] + 80*a^5*x^5*
ArcSinh[a*x] - 2*ArcSinh[a*x]^2*CoshIntegral[ArcSinh[a*x]] + 27*ArcSinh[a*x
]^2*CoshIntegral[3*ArcSinh[a*x]] - 25*ArcSinh[a*x]^2*CoshIntegral[5*ArcSinh
[a*x]])/(a^5*ArcSinh[a*x]^2)
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.24

method	result
derivativedivides	$\frac{-\frac{\sqrt{a^{2} x^{2}+1}}{16 \operatorname{arcsinh}\left(a x\right)^{2}}-\frac{a x}{16 \,\operatorname{arcsinh}\left(a x\right)}+\frac{\operatorname{Chi}\left(\operatorname{arcsinh}\left(a x\right)\right)}{16}+\frac{3 \cosh\left(3 \,\operatorname{arcsinh}\left(a x\right)\right)}{32 \operatorname{arcsinh}\left(a x\right)^{2}}+\frac{9 \sinh\left(3 \,\operatorname{arcsinh}\left(a x\right)\right)}{32 \,\operatorname{arcsinh}\left(a x\right)}-\frac{27 \,\operatorname{Chi}\left(3 \,\operatorname{arcsinh}\left(a x\right)\right)}{32}-\frac{\cosh(}{32\,a}}{a^{5}}$
default	$\frac{-\frac{\sqrt{a^{2} x^{2}+1}}{16 \operatorname{arcsinh}\left(a x\right)^{2}}-\frac{a x}{16 \,\operatorname{arcsinh}\left(a x\right)}+\frac{\operatorname{Chi}\left(\operatorname{arcsinh}\left(a x\right)\right)}{16}+\frac{3 \cosh\left(3 \,\operatorname{arcsinh}\left(a x\right)\right)}{32 \operatorname{arcsinh}\left(a x\right)^{2}}+\frac{9 \sinh\left(3 \,\operatorname{arcsinh}\left(a x\right)\right)}{32 \,\operatorname{arcsinh}\left(a x\right)}-\frac{27 \,\operatorname{Chi}\left(3 \,\operatorname{arcsinh}\left(a x\right)\right)}{32}-\frac{\cosh(}{32\,a}}{a^{5}}$

```
[In] int(x^4/arcsinh(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^5*(-1/16/arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)-1/16*a*x/arcsinh(a*x)+1/16*Ch
i(arcsinh(a*x))+3/32/arcsinh(a*x)^2*cosh(3*arcsinh(a*x))+9/32/arcsinh(a*x)*
sinh(3*arcsinh(a*x))-27/32*Chi(3*arcsinh(a*x))-1/32/arcsinh(a*x)^2*cosh(5*a
rcsinh(a*x))-5/32/arcsinh(a*x)*sinh(5*arcsinh(a*x))+25/32*Chi(5*arcsinh(a*x
)))
```

Fricas [F]

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^3}\,dx = \int \frac{x^{4}}{\operatorname{arsinh}\left(a x\right)^{3}}\,dx$$

```
[In] integrate(x^4/arcsinh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x^4/arcsinh(a*x)^3, x)
```

Sympy [F]

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^3}\,dx = \int \frac{x^{4}}{\operatorname{asinh}^{3}{\left(a x \right)}}\,dx$$

```
[In] integrate(x**4/asinh(a*x)**3,x)
```

```
[Out] Integral(x**4/asinh(a*x)**3, x)
```

Maxima [F]

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^3}\,dx = \int \frac{x^4}{\operatorname{arsinh}\left(ax\right)^3}\,dx$$

```
[In] integrate(x^4/arcsinh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a^8*x^11 + 3*a^6*x^9 + 3*a^4*x^7 + a^2*x^5 + (a^5*x^8 + a^3*x^6)*(a^2
*x^2 + 1)^(3/2) + (3*a^6*x^9 + 5*a^4*x^7 + 2*a^2*x^5)*(a^2*x^2 + 1) + (5*a^
8*x^11 + 15*a^6*x^9 + 15*a^4*x^7 + 5*a^2*x^5 + (5*a^5*x^8 + 8*a^3*x^6 + 3*a
*x^4)*(a^2*x^2 + 1)^(3/2) + (15*a^6*x^9 + 31*a^4*x^7 + 20*a^2*x^5 + 4*x^3)*
(a^2*x^2 + 1) + (15*a^7*x^10 + 38*a^5*x^8 + 32*a^3*x^6 + 9*a*x^4)*sqrt(a^2*
x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1)) + (3*a^7*x^10 + 7*a^5*x^8 + 5*a^3*x^
6 + a*x^4)*sqrt(a^2*x^2 + 1))/((a^8*x^6 + 3*a^6*x^4 + (a^2*x^2 + 1)^(3/2)*a
^5*x^3 + 3*a^4*x^2 + 3*(a^6*x^4 + a^4*x^2)*(a^2*x^2 + 1) + a^2 + 3*(a^7*x^5
 + 2*a^5*x^3 + a^3*x)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))^2) +
integrate(1/2*(25*a^10*x^12 + 100*a^8*x^10 + 150*a^6*x^8 + 100*a^4*x^6 + 25
*a^2*x^4 + (25*a^6*x^8 + 24*a^4*x^6 + 3*a^2*x^4)*(a^2*x^2 + 1)^2 + (100*a^7
*x^9 + 172*a^5*x^7 + 87*a^3*x^5 + 12*a*x^3)*(a^2*x^2 + 1)^(3/2) + 3*(50*a^8
*x^10 + 124*a^6*x^8 + 105*a^4*x^6 + 35*a^2*x^4 + 4*x^2)*(a^2*x^2 + 1) + (10
0*a^9*x^11 + 324*a^7*x^9 + 381*a^5*x^7 + 193*a^3*x^5 + 36*a*x^3)*sqrt(a^2*x
^2 + 1))/((a^10*x^8 + 4*a^8*x^6 + (a^2*x^2 + 1)^2*a^6*x^4 + 6*a^6*x^4 + 4*a
^4*x^2 + 4*(a^7*x^5 + a^5*x^3)*(a^2*x^2 + 1)^(3/2) + 6*(a^8*x^6 + 2*a^6*x^4
 + a^4*x^2)*(a^2*x^2 + 1) + a^2 + 4*(a^9*x^7 + 3*a^7*x^5 + 3*a^5*x^3 + a^3*
x)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))), x)
```

Giac [F]

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^3}\,dx = \int \frac{x^4}{\operatorname{arsinh}\left(ax\right)^3}\,dx$$

```
[In] integrate(x^4/arcsinh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^4/arcsinh(a*x)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^3}\,dx = \int \frac{x^4}{\operatorname{asinh}\left(a\,x\right)^3}\,dx$$

```
[In] int(x^4/asinh(a*x)^3,x)
[Out] int(x^4/asinh(a*x)^3, x)
```

3.61 $\int \frac{x^3}{\operatorname{arcsinh}(ax)^3}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 82

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^3}\,dx = -\frac{x^3\sqrt{1+a^2x^2}}{2a\operatorname{arcsinh}(ax)^2} - \frac{3x^2}{2a^2\operatorname{arcsinh}(ax)} - \frac{2x^4}{\operatorname{arcsinh}(ax)} - \frac{\operatorname{Shi}(2\operatorname{arcsinh}(ax))}{2a^4} + \frac{\operatorname{Shi}(4\operatorname{arcsinh}(ax))}{a^4}$$

```
[Out] -3/2*x^2/a^2/arcsinh(a*x)-2*x^4/arcsinh(a*x)-1/2*Shi(2*arcsinh(a*x))/a^4+Sh
i(4*arcsinh(a*x))/a^4-1/2*x^3*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^2
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5779, 5818, 5780, 5556, 3379, 12}

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^3}\,dx = -\frac{\operatorname{Shi}(2\operatorname{arcsinh}(ax))}{2a^4} + \frac{\operatorname{Shi}(4\operatorname{arcsinh}(ax))}{a^4} - \frac{3x^2}{2a^2\operatorname{arcsinh}(ax)} - \frac{x^3\sqrt{a^2x^2+1}}{2a\operatorname{arcsinh}(ax)^2} - \frac{2x^4}{\operatorname{arcsinh}(ax)}$$

```
[In] Int[x^3/ArcSinh[a*x]^3,x]
```

```
[Out] -1/2*(x^3*Sqrt[1 + a^2*x^2])/(a*ArcSinh[a*x]^2) - (3*x^2)/(2*a^2*ArcSinh[a*
x]) - (2*x^4)/ArcSinh[a*x] - SinhIntegral[2*ArcSinh[a*x]]/(2*a^4) + SinhInt
egral[4*ArcSinh[a*x]]/a^4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/S
qrt[1 + c^2*x^2]), x], x] - Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Sinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rubi steps

$$\text{integral} = -\frac{x^3\sqrt{1+a^2x^2}}{2a\text{arcsinh}(ax)^2} + \frac{3\int \frac{x^2}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}\,dx}{2a} + (2a)\int \frac{x^4}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}\,dx$$

$$= -\frac{x^3\sqrt{1+a^2x^2}}{2a\text{arcsinh}(ax)^2} - \frac{3x^2}{2a^2\text{arcsinh}(ax)} - \frac{2x^4}{\text{arcsinh}(ax)} + 8\int \frac{x^3}{\text{arcsinh}(ax)}\,dx + \frac{3\int \frac{x}{\text{arcsinh}(ax)}\,dx}{a^2}$$

$$= -\frac{x^3\sqrt{1+a^2x^2}}{2a\text{arcsinh}(ax)^2} - \frac{3x^2}{2a^2\text{arcsinh}(ax)} - \frac{2x^4}{\text{arcsinh}(ax)} + \frac{3\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{a^4} + \frac{8\text{Subst}\left(\int \frac{\cosh(x)\sinh^3(x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{a^4}$$

$$= -\frac{x^3\sqrt{1+a^2x^2}}{2a\text{arcsinh}(ax)^2} - \frac{3x^2}{2a^2\text{arcsinh}(ax)} - \frac{2x^4}{\text{arcsinh}(ax)} + \frac{3\text{Subst}\left(\int \frac{\sinh(2x)}{2x}\,dx, x, \text{arcsinh}(ax)\right)}{a^4} + \frac{8\text{Subst}\left(\int \left(-\frac{\sinh(2x)}{4x} + \frac{\sinh(4x)}{8x}\right)\,dx, x, \text{arcsinh}(ax)\right)}{a^4}$$

$$= -\frac{x^3\sqrt{1+a^2x^2}}{2a\text{arcsinh}(ax)^2} - \frac{3x^2}{2a^2\text{arcsinh}(ax)} - \frac{2x^4}{\text{arcsinh}(ax)} + \frac{\text{Subst}\left(\int \frac{\sinh(4x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{a^4} + \frac{3\text{Subst}\left(\int \frac{\sinh(2x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{2a^4} - \frac{2\text{Subst}\left(\int \frac{\sinh(2x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{a^4}$$

$$= -\frac{x^3\sqrt{1+a^2x^2}}{2a\text{arcsinh}(ax)^2} - \frac{3x^2}{2a^2\text{arcsinh}(ax)} - \frac{2x^4}{\text{arcsinh}(ax)} - \frac{\text{Shi}(2\text{arcsinh}(ax))}{2a^4} + \frac{\text{Shi}(4\text{arcsinh}(ax))}{a^4}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.84

$$\int \frac{x^3}{\text{arcsinh}(ax)^3}\,dx = -\frac{\frac{a^2x^2\left(ax\sqrt{1+a^2x^2}+(3+4a^2x^2)\text{arcsinh}(ax)\right)}{\text{arcsinh}(ax)^2} + \text{Shi}(2\text{arcsinh}(ax)) - 2\text{Shi}(4\text{arcsinh}(ax))}{2a^4}$$

```
[In] Integrate[x^3/ArcSinh[a*x]^3,x]
```

```
[Out] -1/2*((a^2*x^2*(a*x*Sqrt[1 + a^2*x^2] + (3 + 4*a^2*x^2)*ArcSinh[a*x]))/ArcS
inh[a*x]^2 + SinhIntegral[2*ArcSinh[a*x]] - 2*SinhIntegral[4*ArcSinh[a*x]])
/a^4
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{\frac{\sinh(2\,\mathrm{arcsinh}(ax))}{8\,\mathrm{arcsinh}(ax)^2}+\frac{\cosh(2\,\mathrm{arcsinh}(ax))}{4\,\mathrm{arcsinh}(ax)}-\frac{\mathrm{Shi}(2\,\mathrm{arcsinh}(ax))}{2}-\frac{\sinh(4\,\mathrm{arcsinh}(ax))}{16\,\mathrm{arcsinh}(ax)^2}-\frac{\cosh(4\,\mathrm{arcsinh}(ax))}{4\,\mathrm{arcsinh}(ax)}+\mathrm{Shi}(4\,\mathrm{arcsinh}(ax))}{a^4}$
default	$\frac{\frac{\sinh(2\,\mathrm{arcsinh}(ax))}{8\,\mathrm{arcsinh}(ax)^2}+\frac{\cosh(2\,\mathrm{arcsinh}(ax))}{4\,\mathrm{arcsinh}(ax)}-\frac{\mathrm{Shi}(2\,\mathrm{arcsinh}(ax))}{2}-\frac{\sinh(4\,\mathrm{arcsinh}(ax))}{16\,\mathrm{arcsinh}(ax)^2}-\frac{\cosh(4\,\mathrm{arcsinh}(ax))}{4\,\mathrm{arcsinh}(ax)}+\mathrm{Shi}(4\,\mathrm{arcsinh}(ax))}{a^4}$

```
[In] int(x^3/arcsinh(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*(1/8/arcsinh(a*x)^2*sinh(2*arcsinh(a*x))+1/4/arcsinh(a*x)*cosh(2*arcs
inh(a*x))-1/2*Shi(2*arcsinh(a*x))-1/16/arcsinh(a*x)^2*sinh(4*arcsinh(a*x))-
1/4/arcsinh(a*x)*cosh(4*arcsinh(a*x))+Shi(4*arcsinh(a*x)))
```

Fricas [F]

$$\int \frac{x^3}{\mathrm{arcsinh}(ax)^3}\,dx = \int \frac{x^3}{\mathrm{arsinh}\,(ax)^3}\,dx$$

```
[In] integrate(x^3/arcsinh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x^3/arcsinh(a*x)^3, x)
```

Sympy [F]

$$\int \frac{x^3}{\mathrm{arcsinh}(ax)^3}\,dx = \int \frac{x^3}{\mathrm{asinh}^3\,(ax)}\,dx$$

```
[In] integrate(x**3/asinh(a*x)**3,x)
```

```
[Out] Integral(x**3/asinh(a*x)**3, x)
```

Maxima [F]

$$\int \frac{x^3}{\mathrm{arcsinh}(ax)^3}\,dx = \int \frac{x^3}{\mathrm{arsinh}\,(ax)^3}\,dx$$

```
[In] integrate(x^3/arcsinh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a^8*x^10 + 3*a^6*x^8 + 3*a^4*x^6 + a^2*x^4 + (a^5*x^7 + a^3*x^5)*(a^2
*x^2 + 1)^(3/2) + (3*a^6*x^8 + 5*a^4*x^6 + 2*a^2*x^4)*(a^2*x^2 + 1) + (4*a^
```

```
8*x^10 + 12*a^6*x^8 + 12*a^4*x^6 + 4*a^2*x^4 + 2*(2*a^5*x^7 + 3*a^3*x^5 + a
*x^3)*(a^2*x^2 + 1)^(3/2) + 3*(4*a^6*x^8 + 8*a^4*x^6 + 5*a^2*x^4 + x^2)*(a^
2*x^2 + 1) + (12*a^7*x^9 + 30*a^5*x^7 + 25*a^3*x^5 + 7*a*x^3)*sqrt(a^2*x^2
+ 1))*log(a*x + sqrt(a^2*x^2 + 1)) + (3*a^7*x^9 + 7*a^5*x^7 + 5*a^3*x^5 + a
*x^3)*sqrt(a^2*x^2 + 1))/((a^8*x^6 + 3*a^6*x^4 + (a^2*x^2 + 1)^(3/2)*a^5*x^
3 + 3*a^4*x^2 + 3*(a^6*x^4 + a^4*x^2)*(a^2*x^2 + 1) + a^2 + 3*(a^7*x^5 + 2*
a^5*x^3 + a^3*x)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))^2) + integ
rate(1/2*(16*a^10*x^11 + 64*a^8*x^9 + 96*a^6*x^7 + 64*a^4*x^5 + 16*a^2*x^3
+ 4*(4*a^6*x^7 + 3*a^4*x^5)*(a^2*x^2 + 1)^2 + (64*a^7*x^8 + 100*a^5*x^6 + 4
2*a^3*x^4 + 3*a*x^2)*(a^2*x^2 + 1)^(3/2) + 6*(16*a^8*x^9 + 38*a^6*x^7 + 30*
a^4*x^5 + 9*a^2*x^3 + x)*(a^2*x^2 + 1) + (64*a^9*x^10 + 204*a^7*x^8 + 234*a
^5*x^6 + 115*a^3*x^4 + 21*a*x^2)*sqrt(a^2*x^2 + 1))/((a^10*x^8 + 4*a^8*x^6
+ (a^2*x^2 + 1)^2*a^6*x^4 + 6*a^6*x^4 + 4*a^4*x^2 + 4*(a^7*x^5 + a^5*x^3)*(
a^2*x^2 + 1)^(3/2) + 6*(a^8*x^6 + 2*a^6*x^4 + a^4*x^2)*(a^2*x^2 + 1) + a^2
+ 4*(a^9*x^7 + 3*a^7*x^5 + 3*a^5*x^3 + a^3*x)*sqrt(a^2*x^2 + 1))*log(a*x +
sqrt(a^2*x^2 + 1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^3} dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3/arcsinh(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^3} dx = \int \frac{x^3}{\operatorname{asinh}(a\,x)^3} \,dx$$

```
[In] int(x^3/asinh(a*x)^3,x)
```

```
[Out] int(x^3/asinh(a*x)^3, x)
```

3.62 $\int \frac{x^2}{\operatorname{arcsinh}(ax)^3}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 81

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^3}\,dx = -\frac{x^2\sqrt{1+a^2x^2}}{2a\operatorname{arcsinh}(ax)^2} - \frac{x}{a^2\operatorname{arcsinh}(ax)} - \frac{3x^3}{2\operatorname{arcsinh}(ax)} - \frac{\operatorname{Chi}(\operatorname{arcsinh}(ax))}{8a^3} + \frac{9\operatorname{Chi}(3\operatorname{arcsinh}(ax))}{8a^3}$$

```
[Out] -x/a^2/arcsinh(a*x)-3/2*x^3/arcsinh(a*x)-1/8*Chi(arcsinh(a*x))/a^3+9/8*Chi(
3*arcsinh(a*x))/a^3-1/2*x^2*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^2
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5779, 5818, 5780, 5556, 3382, 5774}

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^3}\,dx = -\frac{\operatorname{Chi}(\operatorname{arcsinh}(ax))}{8a^3} + \frac{9\operatorname{Chi}(3\operatorname{arcsinh}(ax))}{8a^3} - \frac{x^2\sqrt{a^2x^2+1}}{2a\operatorname{arcsinh}(ax)^2} - \frac{x}{a^2\operatorname{arcsinh}(ax)} - \frac{3x^3}{2\operatorname{arcsinh}(ax)}$$

```
[In] Int[x^2/ArcSinh[a*x]^3,x]
```

```
[Out] -1/2*(x^2*Sqrt[1 + a^2*x^2])/(a*ArcSinh[a*x]^2) - x/(a^2*ArcSinh[a*x]) - (3
*x^3)/(2*ArcSinh[a*x]) - CoshIntegral[ArcSinh[a*x]]/(8*a^3) + (9*CoshIntegr
al[3*ArcSinh[a*x]])/(8*a^3)
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/S
qrt[1 + c^2*x^2]), x], x] - Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Sinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rubi steps

$$\text{integral} = -\frac{x^2\sqrt{1+a^2x^2}}{2a\text{arcsinh}(ax)^2} + \frac{\int \frac{x}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}\,dx}{a} + \frac{1}{2}(3a)\int \frac{x^3}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}\,dx$$

$$= -\frac{x^2\sqrt{1+a^2x^2}}{2a\text{arcsinh}(ax)^2} - \frac{x}{a^2\text{arcsinh}(ax)} - \frac{3x^3}{2\text{arcsinh}(ax)} + \frac{9}{2}\int \frac{x^2}{\text{arcsinh}(ax)}\,dx + \frac{\int \frac{1}{\text{arcsinh}(ax)}\,dx}{a^2}$$

$$= -\frac{x^2\sqrt{1+a^2x^2}}{2a\text{arcsinh}(ax)^2} - \frac{x}{a^2\text{arcsinh}(ax)} - \frac{3x^3}{2\text{arcsinh}(ax)} + \frac{\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{a^3} + \frac{9\text{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{2a^3}$$

$$= -\frac{x^2\sqrt{1+a^2x^2}}{2a\text{arcsinh}(ax)^2} - \frac{x}{a^2\text{arcsinh}(ax)} - \frac{3x^3}{2\text{arcsinh}(ax)} + \frac{\text{Chi}(\text{arcsinh}(ax))}{a^3} + \frac{9\text{Subst}\left(\int \left(-\frac{\cosh(x)}{4x} + \frac{\cosh(3x)}{4x}\right)\,dx, x, \text{arcsinh}(ax)\right)}{2a^3}$$

$$= -\frac{x^2\sqrt{1+a^2x^2}}{2a\text{arcsinh}(ax)^2} - \frac{x}{a^2\text{arcsinh}(ax)} - \frac{3x^3}{2\text{arcsinh}(ax)} + \frac{\text{Chi}(\text{arcsinh}(ax))}{a^3} - \frac{9\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{8a^3} + \frac{9\text{Subst}\left(\int \frac{\cosh(3x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{8a^3}$$

$$= -\frac{x^2\sqrt{1+a^2x^2}}{2a\text{arcsinh}(ax)^2} - \frac{x}{a^2\text{arcsinh}(ax)} - \frac{3x^3}{2\text{arcsinh}(ax)} - \frac{\text{Chi}(\text{arcsinh}(ax))}{8a^3} + \frac{9\text{Chi}(3\text{arcsinh}(ax))}{8a^3}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.79

$$\int \frac{x^2}{\text{arcsinh}(ax)^3}\,dx = -\frac{\frac{4ax\left(ax\sqrt{1+a^2x^2}+(2+3a^2x^2)\text{arcsinh}(ax)\right)}{\text{arcsinh}(ax)^2} + \text{Chi}(\text{arcsinh}(ax)) - 9\text{Chi}(3\text{arcsinh}(ax))}{8a^3}$$

```
[In] Integrate[x^2/ArcSinh[a*x]^3,x]
```

```
[Out] -1/8*((4*a*x*(a*x*Sqrt[1 + a^2*x^2] + (2 + 3*a^2*x^2)*ArcSinh[a*x]))/ArcSin
h[a*x]^2 + CoshIntegral[ArcSinh[a*x]] - 9*CoshIntegral[3*ArcSinh[a*x]])/a^3
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{\frac{\sqrt{a^2x^2+1}}{8\,\text{arcsinh}(ax)^2}+\frac{ax}{8\ \text{arcsinh}(ax)}-\frac{\text{Chi}(\text{arcsinh}(ax))}{8}-\frac{\cosh(3\ \text{arcsinh}(ax))}{8\,\text{arcsinh}(ax)^2}-\frac{3\sinh(3\ \text{arcsinh}(ax))}{8\ \text{arcsinh}(ax)}+\frac{9\ \text{Chi}(3\ \text{arcsinh}(ax))}{8}}{a^3}$	81
default	$\frac{\frac{\sqrt{a^2x^2+1}}{8\,\text{arcsinh}(ax)^2}+\frac{ax}{8\ \text{arcsinh}(ax)}-\frac{\text{Chi}(\text{arcsinh}(ax))}{8}-\frac{\cosh(3\ \text{arcsinh}(ax))}{8\,\text{arcsinh}(ax)^2}-\frac{3\sinh(3\ \text{arcsinh}(ax))}{8\ \text{arcsinh}(ax)}+\frac{9\ \text{Chi}(3\ \text{arcsinh}(ax))}{8}}{a^3}$	81

```
[In] int(x^2/arcsinh(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(1/8/arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)+1/8*a*x/arcsinh(a*x)-1/8*Chi(ar
csinh(a*x))-1/8/arcsinh(a*x)^2*cosh(3*arcsinh(a*x))-3/8/arcsinh(a*x)*sinh(3
*arcsinh(a*x))+9/8*Chi(3*arcsinh(a*x)))
```

Fricas [F]

$$\int \frac{x^2}{\text{arcsinh}(ax)^3}\,dx = \int \frac{x^2}{\text{arsinh}\,(ax)^3}\,dx$$

```
[In] integrate(x^2/arcsinh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x^2/arcsinh(a*x)^3, x)
```

Sympy [F]

$$\int \frac{x^2}{\text{arcsinh}(ax)^3}\,dx = \int \frac{x^2}{\text{asinh}^3\,(ax)}\,dx$$

```
[In] integrate(x**2/asinh(a*x)**3,x)
```

```
[Out] Integral(x**2/asinh(a*x)**3, x)
```

Maxima [F]

$$\int \frac{x^2}{\text{arcsinh}(ax)^3}\,dx = \int \frac{x^2}{\text{arsinh}\,(ax)^3}\,dx$$

```
[In] integrate(x^2/arcsinh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a^8*x^9 + 3*a^6*x^7 + 3*a^4*x^5 + a^2*x^3 + (a^5*x^6 + a^3*x^4)*(a^2*
x^2 + 1)^(3/2) + (3*a^6*x^7 + 5*a^4*x^5 + 2*a^2*x^3)*(a^2*x^2 + 1) + (3*a^8
*x^9 + 9*a^6*x^7 + 9*a^4*x^5 + 3*a^2*x^3 + (3*a^5*x^6 + 4*a^3*x^4 + a*x^2)*
(a^2*x^2 + 1)^(3/2) + (9*a^6*x^7 + 17*a^4*x^5 + 10*a^2*x^3 + 2*x)*(a^2*x^2
+ 1) + (9*a^7*x^8 + 22*a^5*x^6 + 18*a^3*x^4 + 5*a*x^2)*sqrt(a^2*x^2 + 1))*l
og(a*x + sqrt(a^2*x^2 + 1)) + (3*a^7*x^8 + 7*a^5*x^6 + 5*a^3*x^4 + a*x^2)*s
qrt(a^2*x^2 + 1))/((a^8*x^6 + 3*a^6*x^4 + (a^2*x^2 + 1)^(3/2)*a^5*x^3 + 3*a
^4*x^2 + 3*(a^6*x^4 + a^4*x^2)*(a^2*x^2 + 1) + a^2 + 3*(a^7*x^5 + 2*a^5*x^3
 + a^3*x)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))^2) + integrate(1/
2*(9*a^10*x^10 + 36*a^8*x^8 + 54*a^6*x^6 + 36*a^4*x^4 + 9*a^2*x^2 + (9*a^6*
x^6 + 4*a^4*x^4 - a^2*x^2)*(a^2*x^2 + 1)^2 + (36*a^7*x^7 + 48*a^5*x^5 + 13*
a^3*x^3 - 2*a*x)*(a^2*x^2 + 1)^(3/2) + (54*a^8*x^8 + 120*a^6*x^6 + 83*a^4*x
^4 + 19*a^2*x^2 + 2)*(a^2*x^2 + 1) + (36*a^9*x^9 + 112*a^7*x^7 + 123*a^5*x^
5 + 57*a^3*x^3 + 10*a*x)*sqrt(a^2*x^2 + 1))/((a^10*x^8 + 4*a^8*x^6 + (a^2*x
^2 + 1)^2*a^6*x^4 + 6*a^6*x^4 + 4*a^4*x^2 + 4*(a^7*x^5 + a^5*x^3)*(a^2*x^2
+ 1)^(3/2) + 6*(a^8*x^6 + 2*a^6*x^4 + a^4*x^2)*(a^2*x^2 + 1) + a^2 + 4*(a^9
*x^7 + 3*a^7*x^5 + 3*a^5*x^3 + a^3*x)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2
*x^2 + 1))), x)
```

Giac [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^3}\,dx = \int \frac{x^2}{\operatorname{arsinh}\left(ax\right)^3}\,dx$$

```
[In] integrate(x^2/arcsinh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2/arcsinh(a*x)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^3}\,dx = \int \frac{x^2}{\operatorname{asinh}\left(a\,x\right)^3}\,dx$$

```
[In] int(x^2/asinh(a*x)^3,x)
```

```
[Out] int(x^2/asinh(a*x)^3, x)
```

3.63 $\int \frac{x}{\operatorname{arcsinh}(ax)^3}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 63

$$\int \frac{x}{\operatorname{arcsinh}(ax)^3}\,dx=-\frac{x\sqrt{1+a^2x^2}}{2a\operatorname{arcsinh}(ax)^2}-\frac{1}{2a^2\operatorname{arcsinh}(ax)}-\frac{x^2}{\operatorname{arcsinh}(ax)}+\frac{\operatorname{Shi}(2\operatorname{arcsinh}(ax))}{a^2}$$

```
[Out] -1/2/a^2/arcsinh(a*x)-x^2/arcsinh(a*x)+Shi(2*arcsinh(a*x))/a^2-1/2*x*(a^2*x
^2+1)^(1/2)/a/arcsinh(a*x)^2
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {5779, 5818, 5780, 5556, 12, 3379, 5783}

$$\int \frac{x}{\operatorname{arcsinh}(ax)^3}\,dx=\frac{\operatorname{Shi}(2\operatorname{arcsinh}(ax))}{a^2}-\frac{x\sqrt{a^2x^2+1}}{2a\operatorname{arcsinh}(ax)^2}-\frac{1}{2a^2\operatorname{arcsinh}(ax)}-\frac{x^2}{\operatorname{arcsinh}(ax)}$$

```
[In] Int[x/ArcSinh[a*x]^3,x]
```

```
[Out] -1/2*(x*Sqrt[1 + a^2*x^2])/(a*ArcSinh[a*x]^2) - 1/(2*a^2*ArcSinh[a*x]) - x^
2/ArcSinh[a*x] + SinhIntegral[2*ArcSinh[a*x]]/a^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/S
qrt[1 + c^2*x^2]), x], x] - Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Sinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rubi steps

$$\text{integral} = -\frac{x\sqrt{1+a^2x^2}}{2a\operatorname{arcsinh}(ax)^2} + \frac{\int \frac{1}{\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2}\,dx}{2a} + a\int \frac{x^2}{\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2}\,dx$$

$$= -\frac{x\sqrt{1+a^2x^2}}{2a\text{arcsinh}(ax)^2} - \frac{1}{2a^2\text{arcsinh}(ax)} - \frac{x^2}{\text{arcsinh}(ax)} + 2\int \frac{x}{\text{arcsinh}(ax)}\,dx$$

$$= -\frac{x\sqrt{1+a^2x^2}}{2a\text{arcsinh}(ax)^2} - \frac{1}{2a^2\text{arcsinh}(ax)} - \frac{x^2}{\text{arcsinh}(ax)} + \frac{2\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{a^2}$$

$$= -\frac{x\sqrt{1+a^2x^2}}{2a\text{arcsinh}(ax)^2} - \frac{1}{2a^2\text{arcsinh}(ax)} - \frac{x^2}{\text{arcsinh}(ax)} + \frac{2\text{Subst}\left(\int \frac{\sinh(2x)}{2x}\,dx, x, \text{arcsinh}(ax)\right)}{a^2}$$

$$= -\frac{x\sqrt{1+a^2x^2}}{2a\text{arcsinh}(ax)^2} - \frac{1}{2a^2\text{arcsinh}(ax)} - \frac{x^2}{\text{arcsinh}(ax)} + \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{a^2}$$

$$= -\frac{x\sqrt{1+a^2x^2}}{2a\text{arcsinh}(ax)^2} - \frac{1}{2a^2\text{arcsinh}(ax)} - \frac{x^2}{\text{arcsinh}(ax)} + \frac{\text{Shi}(2\text{arcsinh}(ax))}{a^2}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.98

$$\int \frac{x}{\text{arcsinh}(ax)^3}\,dx = -\frac{x\sqrt{1+a^2x^2}}{2a\text{arcsinh}(ax)^2} + \frac{-1-2a^2x^2}{2a^2\text{arcsinh}(ax)} + \frac{\text{Shi}(2\text{arcsinh}(ax))}{a^2}$$

```
[In] Integrate[x/ArcSinh[a*x]^3,x]
```

```
[Out] -1/2*(x*Sqrt[1 + a^2*x^2])/(a*ArcSinh[a*x]^2) + (-1 - 2*a^2*x^2)/(2*a^2*Arc
Sinh[a*x]) + SinhIntegral[2*ArcSinh[a*x]]/a^2
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.68

method	result	size
derivativedivides	$\frac{-\frac{\sinh(2\ \text{arcsinh}(ax))}{4\,\text{arcsinh}(ax)^2}-\frac{\cosh(2\ \text{arcsinh}(ax))}{2\ \text{arcsinh}(ax)}+\text{Shi}(2\ \text{arcsinh}(ax))}{a^2}$	43
default	$\frac{-\frac{\sinh(2\ \text{arcsinh}(ax))}{4\,\text{arcsinh}(ax)^2}-\frac{\cosh(2\ \text{arcsinh}(ax))}{2\ \text{arcsinh}(ax)}+\text{Shi}(2\ \text{arcsinh}(ax))}{a^2}$	43

```
[In] int(x/arcsinh(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(-1/4/arcsinh(a*x)^2*sinh(2*arcsinh(a*x))-1/2/arcsinh(a*x)*cosh(2*arc
sinh(a*x))+Shi(2*arcsinh(a*x)))
```

Fricas [F]

$$\int \frac{x}{\operatorname{arcsinh}(ax)^3}\,dx = \int \frac{x}{\operatorname{arsinh}\left(ax\right)^3}\,dx$$

```
[In] integrate(x/arcsinh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x/arcsinh(a*x)^3, x)
```

Sympy [F]

$$\int \frac{x}{\operatorname{arcsinh}(ax)^3}\,dx = \int \frac{x}{\operatorname{asinh}^3\left(ax\right)}\,dx$$

```
[In] integrate(x/asinh(a*x)**3,x)
```

```
[Out] Integral(x/asinh(a*x)**3, x)
```

Maxima [F]

$$\int \frac{x}{\operatorname{arcsinh}(ax)^3}\,dx = \int \frac{x}{\operatorname{arsinh}\left(ax\right)^3}\,dx$$

```
[In] integrate(x/arcsinh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a^8*x^8 + 3*a^6*x^6 + 3*a^4*x^4 + a^2*x^2 + (a^5*x^5 + a^3*x^3)*(a^2*
x^2 + 1)^(3/2) + (3*a^6*x^6 + 5*a^4*x^4 + 2*a^2*x^2)*(a^2*x^2 + 1) + (2*a^8
*x^8 + 6*a^6*x^6 + 6*a^4*x^4 + 2*a^2*x^2 + 2*(a^5*x^5 + a^3*x^3)*(a^2*x^2 +
 1)^(3/2) + (6*a^6*x^6 + 10*a^4*x^4 + 5*a^2*x^2 + 1)*(a^2*x^2 + 1) + (6*a^7
*x^7 + 14*a^5*x^5 + 11*a^3*x^3 + 3*a*x)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a
^2*x^2 + 1)) + (3*a^7*x^7 + 7*a^5*x^5 + 5*a^3*x^3 + a*x)*sqrt(a^2*x^2 + 1))
/((a^8*x^6 + 3*a^6*x^4 + (a^2*x^2 + 1)^(3/2)*a^5*x^3 + 3*a^4*x^2 + 3*(a^6*x
^4 + a^4*x^2)*(a^2*x^2 + 1) + a^2 + 3*(a^7*x^5 + 2*a^5*x^3 + a^3*x)*sqrt(a^
2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))^2) + integrate(1/2*(4*a^9*x^9 + 16
*a^7*x^7 + 4*(a^2*x^2 + 1)^2*a^5*x^5 + 24*a^5*x^5 + 16*a^3*x^3 + (16*a^6*x^
6 + 16*a^4*x^4 - 3)*(a^2*x^2 + 1)^(3/2) + 24*(a^7*x^7 + 2*a^5*x^5 + a^3*x^3
)*(a^2*x^2 + 1) + 4*a*x + (16*a^8*x^8 + 48*a^6*x^6 + 48*a^4*x^4 + 19*a^2*x^
2 + 3)*sqrt(a^2*x^2 + 1))/((a^9*x^8 + 4*a^7*x^6 + (a^2*x^2 + 1)^2*a^5*x^4 +
 6*a^5*x^4 + 4*a^3*x^2 + 4*(a^6*x^5 + a^4*x^3)*(a^2*x^2 + 1)^(3/2) + 6*(a^7
*x^6 + 2*a^5*x^4 + a^3*x^2)*(a^2*x^2 + 1) + 4*(a^8*x^7 + 3*a^6*x^5 + 3*a^4*
x^3 + a^2*x)*sqrt(a^2*x^2 + 1) + a)*log(a*x + sqrt(a^2*x^2 + 1))), x)
```

Giac [F]

$$\int \frac{x}{\operatorname{arcsinh}(ax)^3}\,dx = \int \frac{x}{\operatorname{arsinh}\left(ax\right)^3}\,dx$$

```
[In] integrate(x/arcsinh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x/arcsinh(a*x)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\operatorname{arcsinh}(ax)^3}\,dx = \int \frac{x}{\operatorname{asinh}\left(a\,x\right)^3}\,dx$$

```
[In] int(x/asinh(a*x)^3,x)
```

```
[Out] int(x/asinh(a*x)^3, x)
```

3.64 $\int \frac{1}{\operatorname{arcsinh}(ax)^3} \, dx$

Optimal result

Integrand size = 6, antiderivative size = 50

$$\int \frac{1}{\operatorname{arcsinh}(ax)^3} \, dx = -\frac{\sqrt{1+a^2x^2}}{2a\operatorname{arcsinh}(ax)^2} - \frac{x}{2\operatorname{arcsinh}(ax)} + \frac{\operatorname{Chi}(\operatorname{arcsinh}(ax))}{2a}$$

```
[Out] -1/2*x/arcsinh(a*x)+1/2*Chi(arcsinh(a*x))/a-1/2*(a^2*x^2+1)^(1/2)/a/arcsinh
(a*x)^2
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {5773, 5818, 5774, 3382}

$$\int \frac{1}{\operatorname{arcsinh}(ax)^3} \, dx = -\frac{\sqrt{a^2x^2+1}}{2a\operatorname{arcsinh}(ax)^2} + \frac{\operatorname{Chi}(\operatorname{arcsinh}(ax))}{2a} - \frac{x}{2\operatorname{arcsinh}(ax)}$$

```
[In] Int[ArcSinh[a*x]^(-3),x]
```

```
[Out] -1/2*Sqrt[1 + a^2*x^2]/(a*ArcSinh[a*x]^2) - x/(2*ArcSinh[a*x]) + CoshIntegr
al[ArcSinh[a*x]]/(2*a)
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{1+a^2x^2}}{2a\text{arcsinh}(ax)^2}+\frac{1}{2}a\int\frac{x}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2}\,dx\\
&= -\frac{\sqrt{1+a^2x^2}}{2a\text{arcsinh}(ax)^2}-\frac{x}{2\text{arcsinh}(ax)}+\frac{1}{2}\int\frac{1}{\text{arcsinh}(ax)}\,dx\\
&= -\frac{\sqrt{1+a^2x^2}}{2a\text{arcsinh}(ax)^2}-\frac{x}{2\text{arcsinh}(ax)}+\frac{\text{Subst}\left(\int\frac{\cosh(x)}{x}\,dx,x,\text{arcsinh}(ax)\right)}{2a}\\
&= -\frac{\sqrt{1+a^2x^2}}{2a\text{arcsinh}(ax)^2}-\frac{x}{2\text{arcsinh}(ax)}+\frac{\text{Chi}(\text{arcsinh}(ax))}{2a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

$$\int\frac{1}{\text{arcsinh}(ax)^3}\,dx=-\frac{\sqrt{1+a^2x^2}+ax\text{arcsinh}(ax)-\text{arcsinh}(ax)^2\text{Chi}(\text{arcsinh}(ax))}{2a\text{arcsinh}(ax)^2}$$

```
[In] Integrate[ArcSinh[a*x]^(-3),x]
```

```
[Out] -1/2*(Sqrt[1 + a^2*x^2] + a*x*ArcSinh[a*x] - ArcSinh[a*x]^2*CoshIntegral[Ar
cSinh[a*x]])/(a*ArcSinh[a*x]^2)
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{-\frac{\sqrt{a^{2}x^{2}+1}}{2\,\mathrm{arcsinh}(ax)^{2}}-\frac{ax}{2\,\mathrm{arcsinh}(ax)}+\frac{\mathrm{Chi}(\mathrm{arcsinh}(ax))}{2}}{a}$	42
default	$\frac{-\frac{\sqrt{a^{2}x^{2}+1}}{2\,\mathrm{arcsinh}(ax)^{2}}-\frac{ax}{2\,\mathrm{arcsinh}(ax)}+\frac{\mathrm{Chi}(\mathrm{arcsinh}(ax))}{2}}{a}$	42

```
[In] int(1/arcsinh(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(-1/2/arcsinh(a*x)^2*(a^2*x^2+1)^(1/2)-1/2*a*x/arcsinh(a*x)+1/2*Chi(arc
sinh(a*x)))
```

Fricas [F]

$$\int \frac{1}{\mathrm{arcsinh}(ax)^{3}}\,dx=\int \frac{1}{\mathrm{arsinh}\left(ax\right)^{3}}\,dx$$

```
[In] integrate(1/arcsinh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x)^(-3), x)
```

Sympy [F]

$$\int \frac{1}{\mathrm{arcsinh}(ax)^{3}}\,dx=\int \frac{1}{\mathrm{asinh}^{3}\left(ax\right)}\,dx$$

```
[In] integrate(1/asinh(a*x)**3,x)
```

```
[Out] Integral(asinh(a*x)**(-3), x)
```

Maxima [F]

$$\int \frac{1}{\mathrm{arcsinh}(ax)^{3}}\,dx=\int \frac{1}{\mathrm{arsinh}\left(ax\right)^{3}}\,dx$$

```
[In] integrate(1/arcsinh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a^7*x^7 + 3*a^5*x^5 + 3*a^3*x^3 + (a^4*x^4 + a^2*x^2)*(a^2*x^2 + 1)^(
3/2) + (3*a^5*x^5 + 5*a^3*x^3 + 2*a*x)*(a^2*x^2 + 1) + a*x + (a^7*x^7 + 3*a
```

```
^5*x^5 + 3*a^3*x^3 + (a^4*x^4 - 1)*(a^2*x^2 + 1)^(3/2) + 3*(a^5*x^5 + a^3*x
^3)*(a^2*x^2 + 1) + a*x + (3*a^6*x^6 + 6*a^4*x^4 + 4*a^2*x^2 + 1)*sqrt(a^2*
x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1)) + (3*a^6*x^6 + 7*a^4*x^4 + 5*a^2*x^2
 + 1)*sqrt(a^2*x^2 + 1))/((a^7*x^6 + 3*a^5*x^4 + (a^2*x^2 + 1)^(3/2)*a^4*x^
3 + 3*a^3*x^2 + 3*(a^5*x^4 + a^3*x^2)*(a^2*x^2 + 1) + 3*(a^6*x^5 + 2*a^4*x^
3 + a^2*x)*sqrt(a^2*x^2 + 1) + a)*log(a*x + sqrt(a^2*x^2 + 1))^2) + integra
te(1/2*(a^8*x^8 + 4*a^6*x^6 + 6*a^4*x^4 + 4*a^2*x^2 + (a^4*x^4 + 3)*(a^2*x^
2 + 1)^2 + (4*a^5*x^5 + 4*a^3*x^3 + 3*a*x)*(a^2*x^2 + 1)^(3/2) + 3*(2*a^6*x
^6 + 4*a^4*x^4 + a^2*x^2 - 1)*(a^2*x^2 + 1) + (4*a^7*x^7 + 12*a^5*x^5 + 9*a
^3*x^3 + a*x)*sqrt(a^2*x^2 + 1) + 1)/((a^8*x^8 + 4*a^6*x^6 + (a^2*x^2 + 1)^
2*a^4*x^4 + 6*a^4*x^4 + 4*a^2*x^2 + 4*(a^5*x^5 + a^3*x^3)*(a^2*x^2 + 1)^(3/
2) + 6*(a^6*x^6 + 2*a^4*x^4 + a^2*x^2)*(a^2*x^2 + 1) + 4*(a^7*x^7 + 3*a^5*x
^5 + 3*a^3*x^3 + a*x)*sqrt(a^2*x^2 + 1) + 1)*log(a*x + sqrt(a^2*x^2 + 1))),
 x)
```

Giac [F]

$$\int \frac{1}{\operatorname{arcsinh}(ax)^3}\,dx = \int \frac{1}{\operatorname{arsinh}\left(ax\right)^3}\,dx$$

```
[In] integrate(1/arcsinh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^(-3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\operatorname{arcsinh}(ax)^3}\,dx = \int \frac{1}{\operatorname{asinh}\left(a\,x\right)^3}\,dx$$

```
[In] int(1/asinh(a*x)^3,x)
```

```
[Out] int(1/asinh(a*x)^3, x)
```

3.65 $\int \frac{1}{x\mathbf{arcsinh}(ax)^3}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{1}{x\text{arcsinh}(ax)^3}\,dx = \text{Int}\left(\frac{1}{x\text{arcsinh}(ax)^3},x\right)$$

```
[Out] Unintegrable(1/x/arcsinh(a*x)^3,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\text{arcsinh}(ax)^3}\,dx = \int \frac{1}{x\text{arcsinh}(ax)^3}\,dx$$

```
[In] Int[1/(x*ArcSinh[a*x]^3),x]
```

```
[Out] Defer[Int][1/(x*ArcSinh[a*x]^3), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x\text{arcsinh}(ax)^3}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\text{arcsinh}(ax)^3}\,dx=\int \frac{1}{x\text{arcsinh}(ax)^3}\,dx$$

```
[In] Integrate[1/(x*ArcSinh[a*x]^3),x]
[Out] Integrate[1/(x*ArcSinh[a*x]^3), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\operatorname{arcsinh}\left(ax\right)^{3}}dx$$

```
[In] int(1/x/arcsinh(a*x)^3,x)
[Out] int(1/x/arcsinh(a*x)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\text{arcsinh}(ax)^3}\,dx=\int \frac{1}{x\operatorname{arsinh}\left(ax\right)^{3}}\,dx$$

```
[In] integrate(1/x/arcsinh(a*x)^3,x, algorithm="fricas")
[Out] integral(1/(x*arcsinh(a*x)^3), x)
```

Sympy [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\text{arcsinh}(ax)^3}\,dx=\int \frac{1}{x\operatorname{asinh}^{3}\left(ax\right)}\,dx$$

```
[In] integrate(1/x/asinh(a*x)**3,x)
[Out] Integral(1/(x*asinh(a*x)**3), x)
```

Maxima [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 697, normalized size of antiderivative = 69.70

$$\int \frac{1}{x\text{arcsinh}(ax)^3}\,dx = \int \frac{1}{x\,\text{arsinh}\,(ax)^3}\,dx$$

```
[In] integrate(1/x/arcsinh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a^8*x^8 + 3*a^6*x^6 + 3*a^4*x^4 + a^2*x^2 + (a^5*x^5 + a^3*x^3)*(a^2*
x^2 + 1)^(3/2) + (3*a^6*x^6 + 5*a^4*x^4 + 2*a^2*x^2)*(a^2*x^2 + 1) - (2*(a^
3*x^3 + a*x)*(a^2*x^2 + 1)^(3/2) + (4*a^4*x^4 + 5*a^2*x^2 + 1)*(a^2*x^2 + 1
) + (2*a^5*x^5 + 3*a^3*x^3 + a*x)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2
 + 1)) + (3*a^7*x^7 + 7*a^5*x^5 + 5*a^3*x^3 + a*x)*sqrt(a^2*x^2 + 1))/((a^8
*x^8 + 3*a^6*x^6 + (a^2*x^2 + 1)^(3/2)*a^5*x^5 + 3*a^4*x^4 + a^2*x^2 + 3*(a
^6*x^6 + a^4*x^4)*(a^2*x^2 + 1) + 3*(a^7*x^7 + 2*a^5*x^5 + a^3*x^3)*sqrt(a^
2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))^2) + integrate(1/2*(4*(a^4*x^4 + 2
*a^2*x^2)*(a^2*x^2 + 1)^2 + (12*a^5*x^5 + 22*a^3*x^3 + 7*a*x)*(a^2*x^2 + 1)
^(3/2) + 2*(6*a^6*x^6 + 10*a^4*x^4 + 5*a^2*x^2 + 1)*(a^2*x^2 + 1) + (4*a^7*
x^7 + 6*a^5*x^5 + 3*a^3*x^3 + a*x)*sqrt(a^2*x^2 + 1))/((a^10*x^11 + 4*a^8*x
^9 + (a^2*x^2 + 1)^2*a^6*x^7 + 6*a^6*x^7 + 4*a^4*x^5 + a^2*x^3 + 4*(a^7*x^8
 + a^5*x^6)*(a^2*x^2 + 1)^(3/2) + 6*(a^8*x^9 + 2*a^6*x^7 + a^4*x^5)*(a^2*x^
2 + 1) + 4*(a^9*x^10 + 3*a^7*x^8 + 3*a^5*x^6 + a^3*x^4)*sqrt(a^2*x^2 + 1))*
log(a*x + sqrt(a^2*x^2 + 1))), x)
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\text{arcsinh}(ax)^3}\,dx = \int \frac{1}{x\,\text{arsinh}\,(ax)^3}\,dx$$

```
[In] integrate(1/x/arcsinh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(1/(x*arcsinh(a*x)^3), x)
```

Mupad [N/A]

Not integrable

Time = 2.49 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\text{arcsinh}(ax)^3}\,dx=\int \frac{1}{x\,\mathrm{asinh}\left(a\,x\right)^3}\,dx$$

```
[In] int(1/(x*asinh(a*x)^3),x)
[Out] int(1/(x*asinh(a*x)^3), x)
```

3.66 $\int \frac{1}{x^2 \textbf{arcsinh}(ax)^3} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{1}{x^2 \text{arcsinh}(ax)^3} \, dx = \text{Int}\left(\frac{1}{x^2 \text{arcsinh}(ax)^3}, x\right)$$

```
[Out] Unintegrable(1/x^2/arcsinh(a*x)^3,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2 \text{arcsinh}(ax)^3} \, dx = \int \frac{1}{x^2 \text{arcsinh}(ax)^3} \, dx$$

```
[In] Int[1/(x^2*ArcSinh[a*x]^3),x]
```

```
[Out] Defer[Int][1/(x^2*ArcSinh[a*x]^3), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x^2 \text{arcsinh}(ax)^3} \, dx$$

Mathematica [N/A]

Not integrable

Time = 4.65 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \mathrm{arcsinh}(ax)^3}\,dx = \int \frac{1}{x^2 \mathrm{arcsinh}(ax)^3}\,dx$$

```
[In] Integrate[1/(x^2*ArcSinh[a*x]^3),x]
[Out] Integrate[1/(x^2*ArcSinh[a*x]^3), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 \operatorname{arcsinh}\left(ax\right)^3}dx$$

```
[In] int(1/x^2/arcsinh(a*x)^3,x)
[Out] int(1/x^2/arcsinh(a*x)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \mathrm{arcsinh}(ax)^3}\,dx = \int \frac{1}{x^2 \operatorname{arsinh}\left(ax\right)^3}\,dx$$

```
[In] integrate(1/x^2/arcsinh(a*x)^3,x, algorithm="fricas")
[Out] integral(1/(x^2*arcsinh(a*x)^3), x)
```

Sympy [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \mathrm{arcsinh}(ax)^3}\,dx = \int \frac{1}{x^2 \operatorname{asinh}^3\left(ax\right)}\,dx$$

```
[In] integrate(1/x**2/asinh(a*x)**3,x)
[Out] Integral(1/(x**2*asinh(a*x)**3), x)
```

Maxima [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 822, normalized size of antiderivative = 82.20

$$\int \frac{1}{x^2\text{arcsinh}(ax)^3}\,dx=\int \frac{1}{x^2\,\operatorname{arsinh}\left(ax\right)^3}\,dx$$

```
[In] integrate(1/x^2/arcsinh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a^8*x^8 + 3*a^6*x^6 + 3*a^4*x^4 + a^2*x^2 + (a^5*x^5 + a^3*x^3)*(a^2*
x^2 + 1)^(3/2) + (3*a^6*x^6 + 5*a^4*x^4 + 2*a^2*x^2)*(a^2*x^2 + 1) - (a^8*x
^8 + 3*a^6*x^6 + 3*a^4*x^4 + a^2*x^2 + (a^5*x^5 + 4*a^3*x^3 + 3*a*x)*(a^2*x
^2 + 1)^(3/2) + (3*a^6*x^6 + 11*a^4*x^4 + 10*a^2*x^2 + 2)*(a^2*x^2 + 1) + (
3*a^7*x^7 + 10*a^5*x^5 + 10*a^3*x^3 + 3*a*x)*sqrt(a^2*x^2 + 1))*log(a*x + s
qrt(a^2*x^2 + 1)) + (3*a^7*x^7 + 7*a^5*x^5 + 5*a^3*x^3 + a*x)*sqrt(a^2*x^2
+ 1))/((a^8*x^9 + 3*a^6*x^7 + (a^2*x^2 + 1)^(3/2)*a^5*x^6 + 3*a^4*x^5 + a^2
*x^3 + 3*(a^6*x^7 + a^4*x^5)*(a^2*x^2 + 1) + 3*(a^7*x^8 + 2*a^5*x^6 + a^3*x
^4)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))^2) + integrate(1/2*(a^1
0*x^10 + 4*a^8*x^8 + 6*a^6*x^6 + 4*a^4*x^4 + a^2*x^2 + (a^6*x^6 + 12*a^4*x^
4 + 15*a^2*x^2)*(a^2*x^2 + 1)^2 + (4*a^7*x^7 + 40*a^5*x^5 + 57*a^3*x^3 + 18
*a*x)*(a^2*x^2 + 1)^(3/2) + 3*(2*a^8*x^8 + 16*a^6*x^6 + 25*a^4*x^4 + 13*a^2
*x^2 + 2)*(a^2*x^2 + 1) + (4*a^9*x^9 + 24*a^7*x^7 + 39*a^5*x^5 + 25*a^3*x^3
 + 6*a*x)*sqrt(a^2*x^2 + 1))/((a^10*x^12 + 4*a^8*x^10 + (a^2*x^2 + 1)^2*a^6
*x^8 + 6*a^6*x^8 + 4*a^4*x^6 + a^2*x^4 + 4*(a^7*x^9 + a^5*x^7)*(a^2*x^2 + 1
)^(3/2) + 6*(a^8*x^10 + 2*a^6*x^8 + a^4*x^6)*(a^2*x^2 + 1) + 4*(a^9*x^11 +
3*a^7*x^9 + 3*a^5*x^7 + a^3*x^5)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2
+ 1))), x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2\text{arcsinh}(ax)^3}\,dx=\int \frac{1}{x^2\,\operatorname{arsinh}\left(ax\right)^3}\,dx$$

```
[In] integrate(1/x^2/arcsinh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(1/(x^2*arcsinh(a*x)^3), x)
```

Mupad [N/A]

Not integrable

Time = 2.51 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \operatorname{arcsinh}(ax)^3} \, dx = \int \frac{1}{x^2 \operatorname{asinh}\left(a\,x\right)^3} \, dx$$

```
[In] int(1/(x^2*asinh(a*x)^3),x)
[Out] int(1/(x^2*asinh(a*x)^3), x)
```

3.67 $\int \frac{x^4}{\operatorname{arcsinh}(ax)^4}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 155

$$\begin{aligned}\int \frac{x^4}{\operatorname{arcsinh}(ax)^4}\,dx = & -\frac{x^4\sqrt{1+a^2x^2}}{3a\operatorname{arcsinh}(ax)^3}-\frac{2x^3}{3a^2\operatorname{arcsinh}(ax)^2}-\frac{5x^5}{6\operatorname{arcsinh}(ax)^2}\\ & -\frac{2x^2\sqrt{1+a^2x^2}}{a^3\operatorname{arcsinh}(ax)}-\frac{25x^4\sqrt{1+a^2x^2}}{6a\operatorname{arcsinh}(ax)}+\frac{\operatorname{Shi}(\operatorname{arcsinh}(ax))}{48a^5}\\ & -\frac{27\operatorname{Shi}(3\operatorname{arcsinh}(ax))}{32a^5}+\frac{125\operatorname{Shi}(5\operatorname{arcsinh}(ax))}{96a^5}\end{aligned}$$

```
[Out] -2/3*x^3/a^2/arcsinh(a*x)^2-5/6*x^5/arcsinh(a*x)^2+1/48*Shi(arcsinh(a*x))/a
^5-27/32*Shi(3*arcsinh(a*x))/a^5+125/96*Shi(5*arcsinh(a*x))/a^5-1/3*x^4*(a^
2*x^2+1)^(1/2)/a/arcsinh(a*x)^3-2*x^2*(a^2*x^2+1)^(1/2)/a^3/arcsinh(a*x)-25
/6*x^4*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5779, 5818, 5778, 3379}

$$\begin{aligned}\int \frac{x^4}{\operatorname{arcsinh}(ax)^4}\,dx = & \frac{\operatorname{Shi}(\operatorname{arcsinh}(ax))}{48a^5}-\frac{27\operatorname{Shi}(3\operatorname{arcsinh}(ax))}{32a^5}\\ & +\frac{125\operatorname{Shi}(5\operatorname{arcsinh}(ax))}{96a^5}-\frac{2x^3}{3a^2\operatorname{arcsinh}(ax)^2}-\frac{25x^4\sqrt{a^2x^2+1}}{6a\operatorname{arcsinh}(ax)}\\ & -\frac{x^4\sqrt{a^2x^2+1}}{3a\operatorname{arcsinh}(ax)^3}-\frac{2x^2\sqrt{a^2x^2+1}}{a^3\operatorname{arcsinh}(ax)}-\frac{5x^5}{6\operatorname{arcsinh}(ax)^2}\end{aligned}$$

```
[In] Int[x^4/ArcSinh[a*x]^4,x]
```

```
[Out] -1/3*(x^4*Sqrt[1 + a^2*x^2])/(a*ArcSinh[a*x]^3) - (2*x^3)/(3*a^2*ArcSinh[a*
x]^2) - (5*x^5)/(6*ArcSinh[a*x]^2) - (2*x^2*Sqrt[1 + a^2*x^2])/(a^3*ArcSinh
[a*x]) - (25*x^4*Sqrt[1 + a^2*x^2])/(6*a*ArcSinh[a*x]) + SinhIntegral[ArcSi
nh[a*x]]/(48*a^5) - (27*SinhIntegral[3*ArcSinh[a*x]])/(32*a^5) + (125*SinhI
ntegral[5*ArcSinh[a*x]])/(96*a^5)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/S
qrt[1 + c^2*x^2]), x], x] - Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Sinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rubi steps

integral

$$= -\frac{x^4\sqrt{1+a^2x^2}}{3a\text{arcsinh}(ax)^3} + \frac{4\int \frac{x^3}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}\,dx}{3a} + \frac{1}{3}(5a)\int \frac{x^5}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}\,dx$$

$$= -\frac{x^4\sqrt{1+a^2x^2}}{3a\text{arcsinh}(ax)^3} - \frac{2x^3}{3a^2\text{arcsinh}(ax)^2} - \frac{5x^5}{6\text{arcsinh}(ax)^2} + \frac{25}{6}\int \frac{x^4}{\text{arcsinh}(ax)^2}\,dx + \frac{2\int \frac{x^2}{\text{arcsinh}(ax)^2}\,dx}{a^2}$$

$$= -\frac{x^4\sqrt{1+a^2x^2}}{3a\text{arcsinh}(ax)^3} - \frac{2x^3}{3a^2\text{arcsinh}(ax)^2} - \frac{5x^5}{6\text{arcsinh}(ax)^2} - \frac{2x^2\sqrt{1+a^2x^2}}{a^3\text{arcsinh}(ax)} - \frac{25x^4\sqrt{1+a^2x^2}}{6a\text{arcsinh}(ax)} + \frac{2\text{Subst}\left(\int \left(-\frac{\sinh(x)}{4x}+\frac{3\sinh(3x)}{4x}\right)dx, x, \text{arcsinh}(ax)\right)}{a^5} + \frac{25\text{Subst}\left(\int \left(\frac{\sinh(x)}{8x}-\frac{9\sinh(3x)}{16x}+\frac{5\sinh(5x)}{16x}\right)dx, x, \text{arcsinh}(ax)\right)}{6a^5}$$

$$= -\frac{x^4\sqrt{1+a^2x^2}}{3a\text{arcsinh}(ax)^3} - \frac{2x^3}{3a^2\text{arcsinh}(ax)^2} - \frac{5x^5}{6\text{arcsinh}(ax)^2} - \frac{2x^2\sqrt{1+a^2x^2}}{a^3\text{arcsinh}(ax)} - \frac{25x^4\sqrt{1+a^2x^2}}{6a\text{arcsinh}(ax)} - \frac{\text{Subst}\left(\int \frac{\sinh(x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{2a^5} + \frac{25\text{Subst}\left(\int \frac{\sinh(x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{48a^5} + \frac{125\text{Subst}\left(\int \frac{\sinh(5x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{96a^5} + \frac{3\text{Subst}\left(\int \frac{\sinh(3x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{2a^5} - \frac{75\text{Subst}\left(\int \frac{\sinh(3x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{32a^5}$$

$$= -\frac{x^4\sqrt{1+a^2x^2}}{3a\text{arcsinh}(ax)^3} - \frac{2x^3}{3a^2\text{arcsinh}(ax)^2} - \frac{5x^5}{6\text{arcsinh}(ax)^2} - \frac{2x^2\sqrt{1+a^2x^2}}{a^3\text{arcsinh}(ax)} - \frac{25x^4\sqrt{1+a^2x^2}}{6a\text{arcsinh}(ax)} + \frac{\text{Shi}(\text{arcsinh}(ax))}{48a^5} - \frac{27\text{Shi}(3\text{arcsinh}(ax))}{32a^5} + \frac{125\text{Shi}(5\text{arcsinh}(ax))}{96a^5}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.01

$$\int \frac{x^4}{\text{arcsinh}(ax)^4}\,dx =$$

$$-\frac{32a^4x^4\sqrt{1+a^2x^2}+64a^3x^3\text{arcsinh}(ax)+80a^5x^5\text{arcsinh}(ax)+192a^2x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2+400a^4x}{}$$

```
[In] Integrate[x^4/ArcSinh[a*x]^4,x]
```

```
[Out] -1/96*(32*a^4*x^4*Sqrt[1 + a^2*x^2] + 64*a^3*x^3*ArcSinh[a*x] + 80*a^5*x^5*
ArcSinh[a*x] + 192*a^2*x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^2 + 400*a^4*x^4*S
qrt[1 + a^2*x^2]*ArcSinh[a*x]^2 - 2*ArcSinh[a*x]^3*SinhIntegral[ArcSinh[a*x
]] + 81*ArcSinh[a*x]^3*SinhIntegral[3*ArcSinh[a*x]] - 125*ArcSinh[a*x]^3*Si
nhIntegral[5*ArcSinh[a*x]])/(a^5*ArcSinh[a*x]^3)
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\dfrac{-\frac{\sqrt{a^2x^2+1}}{24\operatorname{arcsinh}(ax)^3}-\frac{ax}{48\operatorname{arcsinh}(ax)^2}-\frac{\sqrt{a^2x^2+1}}{48\operatorname{arcsinh}(ax)}+\frac{\operatorname{Shi}(\operatorname{arcsinh}(ax))}{48}+\frac{\cosh(3\operatorname{arcsinh}(ax))}{16\operatorname{arcsinh}(ax)^3}+\frac{3\sinh(3\operatorname{arcsinh}(ax))}{32\operatorname{arcsinh}(ax)^2}+\frac{9\cosh(3\ \mathrm{arc}}{32\ \mathrm{arcsin}}}{a^5}$
default	$\dfrac{-\frac{\sqrt{a^2x^2+1}}{24\operatorname{arcsinh}(ax)^3}-\frac{ax}{48\operatorname{arcsinh}(ax)^2}-\frac{\sqrt{a^2x^2+1}}{48\operatorname{arcsinh}(ax)}+\frac{\operatorname{Shi}(\operatorname{arcsinh}(ax))}{48}+\frac{\cosh(3\operatorname{arcsinh}(ax))}{16\operatorname{arcsinh}(ax)^3}+\frac{3\sinh(3\operatorname{arcsinh}(ax))}{32\operatorname{arcsinh}(ax)^2}+\frac{9\cosh(3\ \mathrm{arc}}{32\ \mathrm{arcsin}}}{a^5}$

```
[In] int(x^4/arcsinh(a*x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^5*(-1/24/arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)-1/48*a*x/arcsinh(a*x)^2-1/48/
arcsinh(a*x)*(a^2*x^2+1)^(1/2)+1/48*Shi(arcsinh(a*x))+1/16/arcsinh(a*x)^3*c
osh(3*arcsinh(a*x))+3/32/arcsinh(a*x)^2*sinh(3*arcsinh(a*x))+9/32/arcsinh(a
*x)*cosh(3*arcsinh(a*x))-27/32*Shi(3*arcsinh(a*x))-1/48/arcsinh(a*x)^3*cosh
(5*arcsinh(a*x))-5/96/arcsinh(a*x)^2*sinh(5*arcsinh(a*x))-25/96/arcsinh(a*x
)*cosh(5*arcsinh(a*x))+125/96*Shi(5*arcsinh(a*x)))
```

Fricas [F]

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^4}\,dx = \int \frac{x^4}{\operatorname{arsinh}\left(ax\right)^4}\,dx$$

```
[In] integrate(x^4/arcsinh(a*x)^4,x, algorithm="fricas")
```

```
[Out] integral(x^4/arcsinh(a*x)^4, x)
```

Sympy [F]

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^4}\,dx = \int \frac{x^4}{\operatorname{asinh}^4\left(ax\right)}\,dx$$

```
[In] integrate(x**4/asinh(a*x)**4,x)
```

```
[Out] Integral(x**4/asinh(a*x)**4, x)
```

Maxima [F]

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^4}\,dx = \int \frac{x^4}{\operatorname{arsinh}\left(ax\right)^4}\,dx$$

```
[In] integrate(x^4/arcsinh(a*x)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(2*a^13*x^15 + 10*a^11*x^13 + 20*a^9*x^11 + 20*a^7*x^9 + 10*a^5*x^7 +
2*a^3*x^5 + 2*(a^8*x^10 + a^6*x^8)*(a^2*x^2 + 1)^(5/2) + 2*(5*a^9*x^11 + 9*
a^7*x^9 + 4*a^5*x^7)*(a^2*x^2 + 1)^2 + (25*a^13*x^15 + 125*a^11*x^13 + 250*
a^9*x^11 + 250*a^7*x^9 + 125*a^5*x^7 + 25*a^3*x^5 + (25*a^8*x^10 + 49*a^6*x
^8 + 27*a^4*x^6 + 3*a^2*x^4)*(a^2*x^2 + 1)^(5/2) + (125*a^9*x^11 + 321*a^7*
x^9 + 286*a^5*x^7 + 102*a^3*x^5 + 12*a*x^3)*(a^2*x^2 + 1)^2 + (250*a^10*x^1
2 + 794*a^8*x^10 + 946*a^6*x^8 + 519*a^4*x^6 + 129*a^2*x^4 + 12*x^2)*(a^2*x
^2 + 1)^(3/2) + 2*(125*a^11*x^13 + 473*a^9*x^11 + 696*a^7*x^9 + 497*a^5*x^7
 + 173*a^3*x^5 + 24*a*x^3)*(a^2*x^2 + 1) + (125*a^12*x^14 + 549*a^10*x^12 +
 955*a^8*x^10 + 824*a^6*x^8 + 354*a^4*x^6 + 61*a^2*x^4)*sqrt(a^2*x^2 + 1))*
log(a*x + sqrt(a^2*x^2 + 1))^2 + 4*(5*a^10*x^12 + 13*a^8*x^10 + 11*a^6*x^8
+ 3*a^4*x^6)*(a^2*x^2 + 1)^(3/2) + 4*(5*a^11*x^13 + 17*a^9*x^11 + 21*a^7*x^
9 + 11*a^5*x^7 + 2*a^3*x^5)*(a^2*x^2 + 1) + (5*a^13*x^15 + 25*a^11*x^13 + 5
0*a^9*x^11 + 50*a^7*x^9 + 25*a^5*x^7 + 5*a^3*x^5 + (5*a^8*x^10 + 8*a^6*x^8
+ 3*a^4*x^6)*(a^2*x^2 + 1)^(5/2) + (25*a^9*x^11 + 57*a^7*x^9 + 42*a^5*x^7 +
 10*a^3*x^5)*(a^2*x^2 + 1)^2 + (50*a^10*x^12 + 148*a^8*x^10 + 158*a^6*x^8 +
 71*a^4*x^6 + 11*a^2*x^4)*(a^2*x^2 + 1)^(3/2) + 2*(25*a^11*x^13 + 91*a^9*x^
11 + 126*a^7*x^9 + 81*a^5*x^7 + 23*a^3*x^5 + 2*a*x^3)*(a^2*x^2 + 1) + (25*a
^12*x^14 + 108*a^10*x^12 + 183*a^8*x^10 + 151*a^6*x^8 + 60*a^4*x^6 + 9*a^2*
x^4)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1)) + 2*(5*a^12*x^14 + 21*
a^10*x^12 + 34*a^8*x^10 + 26*a^6*x^8 + 9*a^4*x^6 + a^2*x^4)*sqrt(a^2*x^2 +
1))/((a^13*x^10 + 5*a^11*x^8 + (a^2*x^2 + 1)^(5/2)*a^8*x^5 + 10*a^9*x^6 + 1
0*a^7*x^4 + 5*a^5*x^2 + 5*(a^9*x^6 + a^7*x^4)*(a^2*x^2 + 1)^2 + a^3 + 10*(a
^10*x^7 + 2*a^8*x^5 + a^6*x^3)*(a^2*x^2 + 1)^(3/2) + 10*(a^11*x^8 + 3*a^9*x
^6 + 3*a^7*x^4 + a^5*x^2)*(a^2*x^2 + 1) + 5*(a^12*x^9 + 4*a^10*x^7 + 6*a^8*
x^5 + 4*a^6*x^3 + a^4*x)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))^3)
 + integrate(1/6*(125*a^15*x^16 + 750*a^13*x^14 + 1875*a^11*x^12 + 2500*a^9
*x^10 + 1875*a^7*x^8 + 750*a^5*x^6 + 125*a^3*x^4 + (125*a^9*x^10 + 147*a^7*
x^8 + 27*a^5*x^6 - 3*a^3*x^4)*(a^2*x^2 + 1)^3 + (750*a^10*x^11 + 1485*a^8*x
^9 + 901*a^6*x^7 + 147*a^4*x^5 - 12*a^2*x^3)*(a^2*x^2 + 1)^(5/2) + (1875*a^
11*x^12 + 5220*a^9*x^10 + 5209*a^7*x^8 + 2185*a^5*x^6 + 321*a^3*x^4)*(a^2*x
^2 + 1)^2 + (2500*a^12*x^13 + 8970*a^10*x^11 + 12366*a^8*x^9 + 8143*a^6*x^7
 + 2583*a^4*x^5 + 360*a^2*x^3 + 24*x)*(a^2*x^2 + 1)^(3/2) + (1875*a^13*x^14
 + 8235*a^11*x^12 + 14449*a^9*x^10 + 12834*a^7*x^8 + 6030*a^5*x^6 + 1429*a^
3*x^4 + 144*a*x^2)*(a^2*x^2 + 1) + (750*a^14*x^15 + 3897*a^12*x^13 + 8293*a
^10*x^11 + 9226*a^8*x^9 + 5655*a^6*x^7 + 1819*a^4*x^5 + 244*a^2*x^3)*sqrt(a
^2*x^2 + 1))/((a^15*x^12 + 6*a^13*x^10 + 15*a^11*x^8 + (a^2*x^2 + 1)^3*a^9*
```

```
x^6 + 20*a^9*x^6 + 15*a^7*x^4 + 6*a^5*x^2 + 6*(a^10*x^7 + a^8*x^5)*(a^2*x^2
 + 1)^(5/2) + 15*(a^11*x^8 + 2*a^9*x^6 + a^7*x^4)*(a^2*x^2 + 1)^2 + a^3 + 2
0*(a^12*x^9 + 3*a^10*x^7 + 3*a^8*x^5 + a^6*x^3)*(a^2*x^2 + 1)^(3/2) + 15*(a
^13*x^10 + 4*a^11*x^8 + 6*a^9*x^6 + 4*a^7*x^4 + a^5*x^2)*(a^2*x^2 + 1) + 6*
(a^14*x^11 + 5*a^12*x^9 + 10*a^10*x^7 + 10*a^8*x^5 + 5*a^6*x^3 + a^4*x)*sqr
t(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))), x)
```

Giac [F]

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^4}\,dx = \int \frac{x^4}{\operatorname{arsinh}\left(ax\right)^4}\,dx$$

```
[In] integrate(x^4/arcsinh(a*x)^4,x, algorithm="giac")
[Out] integrate(x^4/arcsinh(a*x)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^4}\,dx = \int \frac{x^4}{\operatorname{asinh}\left(a\,x\right)^4}\,dx$$

```
[In] int(x^4/asinh(a*x)^4,x)
[Out] int(x^4/asinh(a*x)^4, x)
```

3.68 $\int \frac{x^3}{\operatorname{arcsinh}(ax)^4}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 141

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^4}\,dx = -\frac{x^3\sqrt{1+a^2x^2}}{3a\operatorname{arcsinh}(ax)^3} - \frac{x^2}{2a^2\operatorname{arcsinh}(ax)^2} - \frac{2x^4}{3\operatorname{arcsinh}(ax)^2} - \frac{x\sqrt{1+a^2x^2}}{a^3\operatorname{arcsinh}(ax)} - \frac{8x^3\sqrt{1+a^2x^2}}{3a\operatorname{arcsinh}(ax)} - \frac{\operatorname{Chi}(2\operatorname{arcsinh}(ax))}{3a^4} + \frac{4\operatorname{Chi}(4\operatorname{arcsinh}(ax))}{3a^4}$$

```
[Out] -1/2*x^2/a^2/arcsinh(a*x)^2-2/3*x^4/arcsinh(a*x)^2-1/3*Chi(2*arcsinh(a*x))/
a^4+4/3*Chi(4*arcsinh(a*x))/a^4-1/3*x^3*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^3-
x*(a^2*x^2+1)^(1/2)/a^3/arcsinh(a*x)-8/3*x^3*(a^2*x^2+1)^(1/2)/a/arcsinh(a*
x)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5779, 5818, 5778, 3382}

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^4}\,dx = -\frac{\operatorname{Chi}(2\operatorname{arcsinh}(ax))}{3a^4} + \frac{4\operatorname{Chi}(4\operatorname{arcsinh}(ax))}{3a^4} - \frac{x^2}{2a^2\operatorname{arcsinh}(ax)^2} - \frac{8x^3\sqrt{a^2x^2+1}}{3a\operatorname{arcsinh}(ax)} - \frac{x^3\sqrt{a^2x^2+1}}{3a\operatorname{arcsinh}(ax)^3} - \frac{x\sqrt{a^2x^2+1}}{a^3\operatorname{arcsinh}(ax)} - \frac{2x^4}{3\operatorname{arcsinh}(ax)^2}$$

```
[In] Int[x^3/ArcSinh[a*x]^4,x]
```

```
[Out] -1/3*(x^3*Sqrt[1 + a^2*x^2])/(a*ArcSinh[a*x]^3) - x^2/(2*a^2*ArcSinh[a*x]^2
) - (2*x^4)/(3*ArcSinh[a*x]^2) - (x*Sqrt[1 + a^2*x^2])/(a^3*ArcSinh[a*x]) -
 (8*x^3*Sqrt[1 + a^2*x^2])/(3*a*ArcSinh[a*x]) - CoshIntegral[2*ArcSinh[a*x]
]/(3*a^4) + (4*CoshIntegral[4*ArcSinh[a*x]])/(3*a^4)
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/S
qrt[1 + c^2*x^2]), x], x] - Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Sinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{x^3\sqrt{1+a^2x^2}}{3a\operatorname{arcsinh}(ax)^3}+\frac{\int \frac{x^2}{\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}\,dx}{a}+\frac{1}{3}(4a)\int \frac{x^4}{\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^3}\,dx\\ &= -\frac{x^3\sqrt{1+a^2x^2}}{3a\operatorname{arcsinh}(ax)^3}-\frac{x^2}{2a^2\operatorname{arcsinh}(ax)^2}-\frac{2x^4}{3\operatorname{arcsinh}(ax)^2}\\ &\quad+\frac{8}{3}\int \frac{x^3}{\operatorname{arcsinh}(ax)^2}\,dx+\frac{\int \frac{x}{\operatorname{arcsinh}(ax)^2}\,dx}{a^2}\end{aligned}$$

$$
\begin{aligned}
&= -\frac{x^3\sqrt{1+a^2x^2}}{3a\text{arcsinh}(ax)^3} - \frac{x^2}{2a^2\text{arcsinh}(ax)^2} - \frac{2x^4}{3\text{arcsinh}(ax)^2} - \frac{x\sqrt{1+a^2x^2}}{a^3\text{arcsinh}(ax)} \\
&\quad - \frac{8x^3\sqrt{1+a^2x^2}}{3a\text{arcsinh}(ax)} + \frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{a^4} \\
&\quad + \frac{8\text{Subst}\left(\int \left(-\frac{\cosh(2x)}{2x} + \frac{\cosh(4x)}{2x}\right)\,dx, x, \text{arcsinh}(ax)\right)}{3a^4} \\
&= -\frac{x^3\sqrt{1+a^2x^2}}{3a\text{arcsinh}(ax)^3} - \frac{x^2}{2a^2\text{arcsinh}(ax)^2} - \frac{2x^4}{3\text{arcsinh}(ax)^2} \\
&\quad - \frac{x\sqrt{1+a^2x^2}}{a^3\text{arcsinh}(ax)} - \frac{8x^3\sqrt{1+a^2x^2}}{3a\text{arcsinh}(ax)} + \frac{\text{Chi}(2\text{arcsinh}(ax))}{a^4} \\
&\quad - \frac{4\text{Subst}\left(\int \frac{\cosh(2x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{3a^4} + \frac{4\text{Subst}\left(\int \frac{\cosh(4x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{3a^4} \\
&= -\frac{x^3\sqrt{1+a^2x^2}}{3a\text{arcsinh}(ax)^3} - \frac{x^2}{2a^2\text{arcsinh}(ax)^2} - \frac{2x^4}{3\text{arcsinh}(ax)^2} - \frac{x\sqrt{1+a^2x^2}}{a^3\text{arcsinh}(ax)} \\
&\quad - \frac{8x^3\sqrt{1+a^2x^2}}{3a\text{arcsinh}(ax)} - \frac{\text{Chi}(2\text{arcsinh}(ax))}{3a^4} + \frac{4\text{Chi}(4\text{arcsinh}(ax))}{3a^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.74

$$
\int \frac{x^3}{\text{arcsinh}(ax)^4}\,dx =
$$

$$
-\frac{\frac{ax\left(2a^2x^2\sqrt{1+a^2x^2}+ax(3+4a^2x^2)\text{arcsinh}(ax)+2\sqrt{1+a^2x^2}(3+8a^2x^2)\text{arcsinh}(ax)^2\right)}{\text{arcsinh}(ax)^3} + 2\text{Chi}(2\text{arcsinh}(ax)) - 8\text{Chi}(4\text{arcsi}}{6a^4}
$$

```
[In] Integrate[x^3/ArcSinh[a*x]^4,x]
```

```
[Out] -1/6*((a*x*(2*a^2*x^2*Sqrt[1 + a^2*x^2] + a*x*(3 + 4*a^2*x^2)*ArcSinh[a*x]
+ 2*Sqrt[1 + a^2*x^2]*(3 + 8*a^2*x^2)*ArcSinh[a*x]^2))/ArcSinh[a*x]^3 + 2*C
oshIntegral[2*ArcSinh[a*x]] - 8*CoshIntegral[4*ArcSinh[a*x]])/a^4
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.81

method	result
derivativedivides	$\frac{\frac{\sinh(2\operatorname{arcsinh}(ax))}{12\operatorname{arcsinh}(ax)^3}+\frac{\cosh(2\operatorname{arcsinh}(ax))}{12\operatorname{arcsinh}(ax)^2}+\frac{\sinh(2\operatorname{arcsinh}(ax))}{6\operatorname{arcsinh}(ax)}-\frac{\operatorname{Chi}(2\operatorname{arcsinh}(ax))}{3}-\frac{\sinh(4\operatorname{arcsinh}(ax))}{24\operatorname{arcsinh}(ax)^3}-\frac{\cosh(4\operatorname{arcsinh}(ax))}{12\operatorname{arcsinh}(ax)^2}-}{a^4}$
default	$\frac{\frac{\sinh(2\operatorname{arcsinh}(ax))}{12\operatorname{arcsinh}(ax)^3}+\frac{\cosh(2\operatorname{arcsinh}(ax))}{12\operatorname{arcsinh}(ax)^2}+\frac{\sinh(2\operatorname{arcsinh}(ax))}{6\operatorname{arcsinh}(ax)}-\frac{\operatorname{Chi}(2\operatorname{arcsinh}(ax))}{3}-\frac{\sinh(4\operatorname{arcsinh}(ax))}{24\operatorname{arcsinh}(ax)^3}-\frac{\cosh(4\operatorname{arcsinh}(ax))}{12\operatorname{arcsinh}(ax)^2}-}{a^4}$

```
[In] int(x^3/arcsinh(a*x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*(1/12/arcsinh(a*x)^3*sinh(2*arcsinh(a*x))+1/12/arcsinh(a*x)^2*cosh(2*
arcsinh(a*x))+1/6/arcsinh(a*x)*sinh(2*arcsinh(a*x))-1/3*Chi(2*arcsinh(a*x))
-1/24/arcsinh(a*x)^3*sinh(4*arcsinh(a*x))-1/12/arcsinh(a*x)^2*cosh(4*arcsin
h(a*x))-1/3/arcsinh(a*x)*sinh(4*arcsinh(a*x))+4/3*Chi(4*arcsinh(a*x)))
```

Fricas [F]

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^4}\,dx = \int \frac{x^3}{\operatorname{arsinh}\left(ax\right)^4}\,dx$$

```
[In] integrate(x^3/arcsinh(a*x)^4,x, algorithm="fricas")
```

```
[Out] integral(x^3/arcsinh(a*x)^4, x)
```

Sympy [F]

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^4}\,dx = \int \frac{x^3}{\operatorname{asinh}^4\left(ax\right)}\,dx$$

```
[In] integrate(x**3/asinh(a*x)**4,x)
```

```
[Out] Integral(x**3/asinh(a*x)**4, x)
```

Maxima [F]

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^4}\,dx = \int \frac{x^3}{\operatorname{arsinh}\left(ax\right)^4}\,dx$$

```
[In] integrate(x^3/arcsinh(a*x)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(2*a^13*x^14 + 10*a^11*x^12 + 20*a^9*x^10 + 20*a^7*x^8 + 10*a^5*x^6 +
2*a^3*x^4 + 2*(a^8*x^9 + a^6*x^7)*(a^2*x^2 + 1)^(5/2) + 2*(5*a^9*x^10 + 9*a
^7*x^8 + 4*a^5*x^6)*(a^2*x^2 + 1)^2 + (16*a^13*x^14 + 80*a^11*x^12 + 160*a^
9*x^10 + 160*a^7*x^8 + 80*a^5*x^6 + 16*a^3*x^4 + 4*(4*a^8*x^9 + 7*a^6*x^7 +
 3*a^4*x^5)*(a^2*x^2 + 1)^(5/2) + (80*a^9*x^10 + 192*a^7*x^8 + 154*a^5*x^6
+ 45*a^3*x^4 + 3*a*x^2)*(a^2*x^2 + 1)^2 + (160*a^10*x^11 + 488*a^8*x^9 + 55
0*a^6*x^7 + 279*a^4*x^5 + 63*a^2*x^3 + 6*x)*(a^2*x^2 + 1)^(3/2) + (160*a^11
*x^12 + 592*a^9*x^10 + 846*a^7*x^8 + 583*a^5*x^6 + 196*a^3*x^4 + 27*a*x^2)*
(a^2*x^2 + 1) + (80*a^12*x^13 + 348*a^10*x^11 + 598*a^8*x^9 + 509*a^6*x^7 +
 216*a^4*x^5 + 37*a^2*x^3)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))^
2 + 4*(5*a^10*x^11 + 13*a^8*x^9 + 11*a^6*x^7 + 3*a^4*x^5)*(a^2*x^2 + 1)^(3/
2) + 4*(5*a^11*x^12 + 17*a^9*x^10 + 21*a^7*x^8 + 11*a^5*x^6 + 2*a^3*x^4)*(a
^2*x^2 + 1) + (4*a^13*x^14 + 20*a^11*x^12 + 40*a^9*x^10 + 40*a^7*x^8 + 20*a
^5*x^6 + 4*a^3*x^4 + 2*(2*a^8*x^9 + 3*a^6*x^7 + a^4*x^5)*(a^2*x^2 + 1)^(5/2
) + (20*a^9*x^10 + 44*a^7*x^8 + 31*a^5*x^6 + 7*a^3*x^4)*(a^2*x^2 + 1)^2 + (
40*a^10*x^11 + 116*a^8*x^9 + 121*a^6*x^7 + 53*a^4*x^5 + 8*a^2*x^3)*(a^2*x^2
 + 1)^(3/2) + (40*a^11*x^12 + 144*a^9*x^10 + 197*a^7*x^8 + 125*a^5*x^6 + 35
*a^3*x^4 + 3*a*x^2)*(a^2*x^2 + 1) + (20*a^12*x^13 + 86*a^10*x^11 + 145*a^8*
x^9 + 119*a^6*x^7 + 47*a^4*x^5 + 7*a^2*x^3)*sqrt(a^2*x^2 + 1))*log(a*x + sq
rt(a^2*x^2 + 1)) + 2*(5*a^12*x^13 + 21*a^10*x^11 + 34*a^8*x^9 + 26*a^6*x^7
+ 9*a^4*x^5 + a^2*x^3)*sqrt(a^2*x^2 + 1))/((a^13*x^10 + 5*a^11*x^8 + (a^2*x
^2 + 1)^(5/2)*a^8*x^5 + 10*a^9*x^6 + 10*a^7*x^4 + 5*a^5*x^2 + 5*(a^9*x^6 +
a^7*x^4)*(a^2*x^2 + 1)^2 + a^3 + 10*(a^10*x^7 + 2*a^8*x^5 + a^6*x^3)*(a^2*x
^2 + 1)^(3/2) + 10*(a^11*x^8 + 3*a^9*x^6 + 3*a^7*x^4 + a^5*x^2)*(a^2*x^2 +
1) + 5*(a^12*x^9 + 4*a^10*x^7 + 6*a^8*x^5 + 4*a^6*x^3 + a^4*x)*sqrt(a^2*x^2
 + 1))*log(a*x + sqrt(a^2*x^2 + 1))^3) + integrate(1/6*(64*a^15*x^15 + 384*
a^13*x^13 + 960*a^11*x^11 + 1280*a^9*x^9 + 960*a^7*x^7 + 384*a^5*x^5 + 64*a
^3*x^3 + 8*(8*a^9*x^9 + 7*a^7*x^7)*(a^2*x^2 + 1)^3 + (384*a^10*x^10 + 664*a
^8*x^8 + 308*a^6*x^6 + 12*a^4*x^4 - 9*a^2*x^2)*(a^2*x^2 + 1)^(5/2) + 2*(480
*a^11*x^11 + 1240*a^9*x^9 + 1096*a^7*x^7 + 360*a^5*x^5 + 15*a^3*x^3 - 9*a*x
)*(a^2*x^2 + 1)^2 + 2*(640*a^12*x^12 + 2200*a^10*x^10 + 2844*a^8*x^8 + 1684
*a^6*x^6 + 433*a^4*x^4 + 36*a^2*x^2 + 3)*(a^2*x^2 + 1)^(3/2) + 2*(480*a^13*
x^13 + 2060*a^11*x^11 + 3496*a^9*x^9 + 2952*a^7*x^7 + 1283*a^5*x^5 + 274*a^
3*x^3 + 27*a*x)*(a^2*x^2 + 1) + (384*a^14*x^14 + 1976*a^12*x^12 + 4148*a^10
*x^10 + 4524*a^8*x^8 + 2699*a^6*x^6 + 842*a^4*x^4 + 111*a^2*x^2)*sqrt(a^2*x
^2 + 1))/((a^15*x^12 + 6*a^13*x^10 + 15*a^11*x^8 + (a^2*x^2 + 1)^3*a^9*x^6
+ 20*a^9*x^6 + 15*a^7*x^4 + 6*a^5*x^2 + 6*(a^10*x^7 + a^8*x^5)*(a^2*x^2 + 1
```

```
)^(5/2) + 15*(a^11*x^8 + 2*a^9*x^6 + a^7*x^4)*(a^2*x^2 + 1)^2 + a^3 + 20*(a
^12*x^9 + 3*a^10*x^7 + 3*a^8*x^5 + a^6*x^3)*(a^2*x^2 + 1)^(3/2) + 15*(a^13*
x^10 + 4*a^11*x^8 + 6*a^9*x^6 + 4*a^7*x^4 + a^5*x^2)*(a^2*x^2 + 1) + 6*(a^1
4*x^11 + 5*a^12*x^9 + 10*a^10*x^7 + 10*a^8*x^5 + 5*a^6*x^3 + a^4*x)*sqrt(a^
2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^4}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3/arcsinh(a*x)^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^4}\,dx = \int \frac{x^3}{\operatorname{asinh}\left(a\,x\right)^4}\,dx$$

```
[In] int(x^3/asinh(a*x)^4,x)
```

```
[Out] int(x^3/asinh(a*x)^4, x)
```

3.69 $\int \frac{x^2}{\operatorname{arcsinh}(ax)^4}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 138

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^4}\,dx = -\frac{x^2\sqrt{1+a^2x^2}}{3a\operatorname{arcsinh}(ax)^3} - \frac{x}{3a^2\operatorname{arcsinh}(ax)^2} - \frac{x^3}{2\operatorname{arcsinh}(ax)^2} - \frac{\sqrt{1+a^2x^2}}{3a^3\operatorname{arcsinh}(ax)} - \frac{3x^2\sqrt{1+a^2x^2}}{2a\operatorname{arcsinh}(ax)} - \frac{\operatorname{Shi}(\operatorname{arcsinh}(ax))}{24a^3} + \frac{9\operatorname{Shi}(3\operatorname{arcsinh}(ax))}{8a^3}$$

```
[Out] -1/3*x/a^2/arcsinh(a*x)^2-1/2*x^3/arcsinh(a*x)^2-1/24*Shi(arcsinh(a*x))/a^3
+9/8*Shi(3*arcsinh(a*x))/a^3-1/3*x^2*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^3-1/3
*(a^2*x^2+1)^(1/2)/a^3/arcsinh(a*x)-3/2*x^2*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x
)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5779, 5818, 5778, 3379, 5773, 5819}

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^4}\,dx = -\frac{\operatorname{Shi}(\operatorname{arcsinh}(ax))}{24a^3} + \frac{9\operatorname{Shi}(3\operatorname{arcsinh}(ax))}{8a^3} - \frac{3x^2\sqrt{a^2x^2+1}}{2a\operatorname{arcsinh}(ax)} - \frac{x^2\sqrt{a^2x^2+1}}{3a\operatorname{arcsinh}(ax)^3} - \frac{x}{3a^2\operatorname{arcsinh}(ax)^2} - \frac{\sqrt{a^2x^2+1}}{3a^3\operatorname{arcsinh}(ax)} - \frac{x^3}{2\operatorname{arcsinh}(ax)^2}$$

```
[In] Int[x^2/ArcSinh[a*x]^4,x]
```

```
[Out] -1/3*(x^2*Sqrt[1 + a^2*x^2])/(a*ArcSinh[a*x]^3) - x/(3*a^2*ArcSinh[a*x]^2)
- x^3/(2*ArcSinh[a*x]^2) - Sqrt[1 + a^2*x^2]/(3*a^3*ArcSinh[a*x]) - (3*x^2*
```

```
Sqrt[1 + a^2*x^2])/(2*a*ArcSinh[a*x]) - SinhIntegral[ArcSinh[a*x]]/(24*a^3)
 + (9*SinhIntegral[3*ArcSinh[a*x]])/(8*a^3)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/S
qrt[1 + c^2*x^2]), x], x] - Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Sinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
```

```
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{x^2\sqrt{1+a^2x^2}}{3a\text{arcsinh}(ax)^3} + \frac{2\int \frac{x}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}\,dx}{3a} + a\int \frac{x^3}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}\,dx \\
&= -\frac{x^2\sqrt{1+a^2x^2}}{3a\text{arcsinh}(ax)^3} - \frac{x}{3a^2\text{arcsinh}(ax)^2} - \frac{x^3}{2\text{arcsinh}(ax)^2} \\
&\quad + \frac{3}{2}\int \frac{x^2}{\text{arcsinh}(ax)^2}\,dx + \frac{\int \frac{1}{\text{arcsinh}(ax)^2}\,dx}{3a^2} \\
&= -\frac{x^2\sqrt{1+a^2x^2}}{3a\text{arcsinh}(ax)^3} - \frac{x}{3a^2\text{arcsinh}(ax)^2} - \frac{x^3}{2\text{arcsinh}(ax)^2} - \frac{\sqrt{1+a^2x^2}}{3a^3\text{arcsinh}(ax)} - \frac{3x^2\sqrt{1+a^2x^2}}{2a\text{arcsinh}(ax)} \\
&\quad + \frac{3\text{Subst}\left(\int \left(-\frac{\sinh(x)}{4x} + \frac{3\sinh(3x)}{4x}\right)\,dx, x, \text{arcsinh}(ax)\right)}{2a^3} + \frac{\int \frac{x}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx}{3a} \\
&= -\frac{x^2\sqrt{1+a^2x^2}}{3a\text{arcsinh}(ax)^3} - \frac{x}{3a^2\text{arcsinh}(ax)^2} - \frac{x^3}{2\text{arcsinh}(ax)^2} - \frac{\sqrt{1+a^2x^2}}{3a^3\text{arcsinh}(ax)} \\
&\quad - \frac{3x^2\sqrt{1+a^2x^2}}{2a\text{arcsinh}(ax)} + \frac{\text{Subst}\left(\int \frac{\sinh(x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{3a^3} \\
&\quad - \frac{3\text{Subst}\left(\int \frac{\sinh(x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{8a^3} + \frac{9\text{Subst}\left(\int \frac{\sinh(3x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{8a^3} \\
&= -\frac{x^2\sqrt{1+a^2x^2}}{3a\text{arcsinh}(ax)^3} - \frac{x}{3a^2\text{arcsinh}(ax)^2} - \frac{x^3}{2\text{arcsinh}(ax)^2} - \frac{\sqrt{1+a^2x^2}}{3a^3\text{arcsinh}(ax)} \\
&\quad - \frac{3x^2\sqrt{1+a^2x^2}}{2a\text{arcsinh}(ax)} - \frac{\text{Shi}(\text{arcsinh}(ax))}{24a^3} + \frac{9\text{Shi}(3\text{arcsinh}(ax))}{8a^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.72

$$\int \frac{x^2}{\text{arcsinh}(ax)^4}\,dx = \\ -\frac{\frac{4\left(2a^2x^2\sqrt{1+a^2x^2}+ax(2+3a^2x^2)\text{arcsinh}(ax)+\sqrt{1+a^2x^2}(2+9a^2x^2)\text{arcsinh}(ax)^2\right)}{\text{arcsinh}(ax)^3} + \text{Shi}(\text{arcsinh}(ax)) - 27\text{Shi}(3\text{arcsinh}(}{24a^3}$$

```
[In] Integrate[x^2/ArcSinh[a*x]^4,x]
```

```
[Out] -1/24*((4*(2*a^2*x^2*Sqrt[1 + a^2*x^2] + a*x*(2 + 3*a^2*x^2)*ArcSinh[a*x] +
 Sqrt[1 + a^2*x^2]*(2 + 9*a^2*x^2)*ArcSinh[a*x]^2))/ArcSinh[a*x]^3 + SinhIn
tegral[ArcSinh[a*x]] - 27*SinhIntegral[3*ArcSinh[a*x]])/a^3
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{\frac{\sqrt{a^2x^2+1}}{12\,\mathrm{arcsinh}(ax)^3}+\frac{ax}{24\,\mathrm{arcsinh}(ax)^2}+\frac{\sqrt{a^2x^2+1}}{24\ \mathrm{arcsinh}(ax)}-\frac{\mathrm{Shi}(\mathrm{arcsinh}(ax))}{24}-\frac{\cosh(3\ \mathrm{arcsinh}(ax))}{12\,\mathrm{arcsinh}(ax)^3}-\frac{\sinh(3\ \mathrm{arcsinh}(ax))}{8\,\mathrm{arcsinh}(ax)^2}-\frac{3\cosh(3\ \mathrm{arcsinh}}{8\ \mathrm{arcsinh}(a}}{a^3}$
default	$\frac{\frac{\sqrt{a^2x^2+1}}{12\,\mathrm{arcsinh}(ax)^3}+\frac{ax}{24\,\mathrm{arcsinh}(ax)^2}+\frac{\sqrt{a^2x^2+1}}{24\ \mathrm{arcsinh}(ax)}-\frac{\mathrm{Shi}(\mathrm{arcsinh}(ax))}{24}-\frac{\cosh(3\ \mathrm{arcsinh}(ax))}{12\,\mathrm{arcsinh}(ax)^3}-\frac{\sinh(3\ \mathrm{arcsinh}(ax))}{8\,\mathrm{arcsinh}(ax)^2}-\frac{3\cosh(3\ \mathrm{arcsinh}}{8\ \mathrm{arcsinh}(a}}{a^3}$

```
[In] int(x^2/arcsinh(a*x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(1/12/arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)+1/24*a*x/arcsinh(a*x)^2+1/24/a
rcsinh(a*x)*(a^2*x^2+1)^(1/2)-1/24*Shi(arcsinh(a*x))-1/12/arcsinh(a*x)^3*co
sh(3*arcsinh(a*x))-1/8/arcsinh(a*x)^2*sinh(3*arcsinh(a*x))-3/8/arcsinh(a*x)
*cosh(3*arcsinh(a*x))+9/8*Shi(3*arcsinh(a*x)))
```

Fricas [F]

$$\int \frac{x^2}{\mathrm{arcsinh}(ax)^4}\,dx = \int \frac{x^2}{\mathrm{arsinh}\left(ax\right)^4}\,dx$$

```
[In] integrate(x^2/arcsinh(a*x)^4,x, algorithm="fricas")
```

```
[Out] integral(x^2/arcsinh(a*x)^4, x)
```

Sympy [F]

$$\int \frac{x^2}{\mathrm{arcsinh}(ax)^4}\,dx = \int \frac{x^2}{\mathrm{asinh}^4\left(ax\right)}\,dx$$

```
[In] integrate(x**2/asinh(a*x)**4,x)
```

```
[Out] Integral(x**2/asinh(a*x)**4, x)
```

Maxima [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^4}\,dx = \int \frac{x^2}{\operatorname{arsinh}\left(ax\right)^4}\,dx$$

```
[In] integrate(x^2/arcsinh(a*x)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(2*a^13*x^13 + 10*a^11*x^11 + 20*a^9*x^9 + 20*a^7*x^7 + 10*a^5*x^5 + 2
*a^3*x^3 + 2*(a^8*x^8 + a^6*x^6)*(a^2*x^2 + 1)^(5/2) + 2*(5*a^9*x^9 + 9*a^7
*x^7 + 4*a^5*x^5)*(a^2*x^2 + 1)^2 + (9*a^13*x^13 + 45*a^11*x^11 + 90*a^9*x^
9 + 90*a^7*x^7 + 45*a^5*x^5 + 9*a^3*x^3 + (9*a^8*x^8 + 13*a^6*x^6 + 3*a^4*x
^4 - a^2*x^2)*(a^2*x^2 + 1)^(5/2) + (45*a^9*x^9 + 97*a^7*x^7 + 64*a^5*x^5 +
 10*a^3*x^3 - 2*a*x)*(a^2*x^2 + 1)^2 + (90*a^10*x^10 + 258*a^8*x^8 + 264*a^
6*x^6 + 113*a^4*x^4 + 19*a^2*x^2 + 2)*(a^2*x^2 + 1)^(3/2) + 2*(45*a^11*x^11
 + 161*a^9*x^9 + 219*a^7*x^7 + 141*a^5*x^5 + 44*a^3*x^3 + 6*a*x)*(a^2*x^2 +
 1) + (45*a^12*x^12 + 193*a^10*x^10 + 325*a^8*x^8 + 270*a^6*x^6 + 112*a^4*x
^4 + 19*a^2*x^2)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))^2 + 4*(5*a
^10*x^10 + 13*a^8*x^8 + 11*a^6*x^6 + 3*a^4*x^4)*(a^2*x^2 + 1)^(3/2) + 4*(5*
a^11*x^11 + 17*a^9*x^9 + 21*a^7*x^7 + 11*a^5*x^5 + 2*a^3*x^3)*(a^2*x^2 + 1)
 + (3*a^13*x^13 + 15*a^11*x^11 + 30*a^9*x^9 + 30*a^7*x^7 + 15*a^5*x^5 + 3*a
^3*x^3 + (3*a^8*x^8 + 4*a^6*x^6 + a^4*x^4)*(a^2*x^2 + 1)^(5/2) + (15*a^9*x^
9 + 31*a^7*x^7 + 20*a^5*x^5 + 4*a^3*x^3)*(a^2*x^2 + 1)^2 + (30*a^10*x^10 +
84*a^8*x^8 + 84*a^6*x^6 + 35*a^4*x^4 + 5*a^2*x^2)*(a^2*x^2 + 1)^(3/2) + 2*(
15*a^11*x^11 + 53*a^9*x^9 + 71*a^7*x^7 + 44*a^5*x^5 + 12*a^3*x^3 + a*x)*(a^
2*x^2 + 1) + (15*a^12*x^12 + 64*a^10*x^10 + 107*a^8*x^8 + 87*a^6*x^6 + 34*a
^4*x^4 + 5*a^2*x^2)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1)) + 2*(5*
a^12*x^12 + 21*a^10*x^10 + 34*a^8*x^8 + 26*a^6*x^6 + 9*a^4*x^4 + a^2*x^2)*s
qrt(a^2*x^2 + 1))/((a^13*x^10 + 5*a^11*x^8 + (a^2*x^2 + 1)^(5/2)*a^8*x^5 +
10*a^9*x^6 + 10*a^7*x^4 + 5*a^5*x^2 + 5*(a^9*x^6 + a^7*x^4)*(a^2*x^2 + 1)^2
 + a^3 + 10*(a^10*x^7 + 2*a^8*x^5 + a^6*x^3)*(a^2*x^2 + 1)^(3/2) + 10*(a^11
*x^8 + 3*a^9*x^6 + 3*a^7*x^4 + a^5*x^2)*(a^2*x^2 + 1) + 5*(a^12*x^9 + 4*a^1
0*x^7 + 6*a^8*x^5 + 4*a^6*x^3 + a^4*x)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^
2*x^2 + 1))^3) + integrate(1/6*(27*a^14*x^14 + 162*a^12*x^12 + 405*a^10*x^1
0 + 540*a^8*x^8 + 405*a^6*x^6 + 162*a^4*x^4 + (27*a^8*x^8 + 13*a^6*x^6 - 3*
a^4*x^4 + 3*a^2*x^2)*(a^2*x^2 + 1)^3 + 27*a^2*x^2 + (162*a^9*x^9 + 227*a^7*
x^7 + 63*a^5*x^5 - 3*a^3*x^3 + 6*a*x)*(a^2*x^2 + 1)^(5/2) + (405*a^10*x^10
+ 940*a^8*x^8 + 687*a^6*x^6 + 143*a^4*x^4 - 21*a^2*x^2 - 12)*(a^2*x^2 + 1)^
2 + (540*a^11*x^11 + 1750*a^9*x^9 + 2058*a^7*x^7 + 1017*a^5*x^5 + 145*a^3*x
^3 - 24*a*x)*(a^2*x^2 + 1)^(3/2) + (405*a^12*x^12 + 1685*a^10*x^10 + 2727*a
^8*x^8 + 2118*a^6*x^6 + 782*a^4*x^4 + 123*a^2*x^2 + 12)*(a^2*x^2 + 1) + (16
2*a^13*x^13 + 823*a^11*x^11 + 1695*a^9*x^9 + 1790*a^7*x^7 + 1015*a^5*x^5 +
297*a^3*x^3 + 38*a*x)*sqrt(a^2*x^2 + 1))/((a^14*x^12 + 6*a^12*x^10 + 15*a^1
0*x^8 + (a^2*x^2 + 1)^3*a^8*x^6 + 20*a^8*x^6 + 15*a^6*x^4 + 6*a^4*x^2 + 6*(
a^9*x^7 + a^7*x^5)*(a^2*x^2 + 1)^(5/2) + 15*(a^10*x^8 + 2*a^8*x^6 + a^6*x^4
```

```
)*(a^2*x^2 + 1)^2 + 20*(a^11*x^9 + 3*a^9*x^7 + 3*a^7*x^5 + a^5*x^3)*(a^2*x^
2 + 1)^(3/2) + 15*(a^12*x^10 + 4*a^10*x^8 + 6*a^8*x^6 + 4*a^6*x^4 + a^4*x^2
)*(a^2*x^2 + 1) + a^2 + 6*(a^13*x^11 + 5*a^11*x^9 + 10*a^9*x^7 + 10*a^7*x^5
 + 5*a^5*x^3 + a^3*x)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))), x)
```

Giac [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^4}\,dx = \int \frac{x^2}{\operatorname{arsinh}\left(ax\right)^4}\,dx$$

```
[In] integrate(x^2/arcsinh(a*x)^4,x, algorithm="giac")
```

```
[Out] integrate(x^2/arcsinh(a*x)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^4}\,dx = \int \frac{x^2}{\operatorname{asinh}\left(a\,x\right)^4}\,dx$$

```
[In] int(x^2/asinh(a*x)^4,x)
```

```
[Out] int(x^2/asinh(a*x)^4, x)
```

3.70 $\int \frac{x}{\mathbf{arcsinh}(ax)^4}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 95

$$\int \frac{x}{\operatorname{arcsinh}(ax)^4}\,dx = -\frac{x\sqrt{1+a^2x^2}}{3a\operatorname{arcsinh}(ax)^3} - \frac{1}{6a^2\operatorname{arcsinh}(ax)^2} - \frac{x^2}{3\operatorname{arcsinh}(ax)^2} - \frac{2x\sqrt{1+a^2x^2}}{3a\operatorname{arcsinh}(ax)} + \frac{2\operatorname{Chi}(2\operatorname{arcsinh}(ax))}{3a^2}$$

```
[Out] -1/6/a^2/arcsinh(a*x)^2-1/3*x^2/arcsinh(a*x)^2+2/3*Chi(2*arcsinh(a*x))/a^2-
1/3*x*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^3-2/3*x*(a^2*x^2+1)^(1/2)/a/arcsinh(
a*x)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {5779, 5818, 5778, 3382, 5783}

$$\int \frac{x}{\operatorname{arcsinh}(ax)^4}\,dx = \frac{2\operatorname{Chi}(2\operatorname{arcsinh}(ax))}{3a^2} - \frac{2x\sqrt{a^2x^2+1}}{3a\operatorname{arcsinh}(ax)} - \frac{x\sqrt{a^2x^2+1}}{3a\operatorname{arcsinh}(ax)^3} - \frac{1}{6a^2\operatorname{arcsinh}(ax)^2} - \frac{x^2}{3\operatorname{arcsinh}(ax)^2}$$

```
[In] Int[x/ArcSinh[a*x]^4,x]
```

```
[Out] -1/3*(x*Sqrt[1 + a^2*x^2])/(a*ArcSinh[a*x]^3) - 1/(6*a^2*ArcSinh[a*x]^2) -
x^2/(3*ArcSinh[a*x]^2) - (2*x*Sqrt[1 + a^2*x^2])/(3*a*ArcSinh[a*x]) + (2*Co
shIntegral[2*ArcSinh[a*x]])/(3*a^2)
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/S
qrt[1 + c^2*x^2]), x], x] - Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Sinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{x\sqrt{1+a^2x^2}}{3a\text{arcsinh}(ax)^3} + \frac{\int \frac{1}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}\,dx}{3a} + \frac{1}{3}(2a)\int \frac{x^2}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}\,dx \\
&= -\frac{x\sqrt{1+a^2x^2}}{3a\text{arcsinh}(ax)^3} - \frac{1}{6a^2\text{arcsinh}(ax)^2} - \frac{x^2}{3\text{arcsinh}(ax)^2} + \frac{2}{3}\int \frac{x}{\text{arcsinh}(ax)^2}\,dx
\end{aligned}
$$

$$= -\frac{x\sqrt{1+a^2x^2}}{3a\text{arcsinh}(ax)^3} - \frac{1}{6a^2\text{arcsinh}(ax)^2} - \frac{x^2}{3\text{arcsinh}(ax)^2} - \frac{2x\sqrt{1+a^2x^2}}{3a\text{arcsinh}(ax)} + \frac{2\text{Subst}\left(\int \frac{\cosh(2x)}{x}\,dx, x, \text{arcsinh}(ax)\right)}{3a^2}$$

$$= -\frac{x\sqrt{1+a^2x^2}}{3a\text{arcsinh}(ax)^3} - \frac{1}{6a^2\text{arcsinh}(ax)^2} - \frac{x^2}{3\text{arcsinh}(ax)^2} - \frac{2x\sqrt{1+a^2x^2}}{3a\text{arcsinh}(ax)} + \frac{2\text{Chi}(2\text{arcsinh}(ax))}{3a^2}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.88

$$\int \frac{x}{\text{arcsinh}(ax)^4}\,dx =$$

$$-\frac{2ax\sqrt{1+a^2x^2} + (1+2a^2x^2)\,\text{arcsinh}(ax) + 4ax\sqrt{1+a^2x^2}\text{arcsinh}(ax)^2 - 4\text{arcsinh}(ax)^3\text{Chi}(2\text{arcsinh}(a}{6a^2\text{arcsinh}(ax)^3}$$

```
[In] Integrate[x/ArcSinh[a*x]^4,x]
```

```
[Out] -1/6*(2*a*x*Sqrt[1 + a^2*x^2] + (1 + 2*a^2*x^2)*ArcSinh[a*x] + 4*a*x*Sqrt[1
 + a^2*x^2]*ArcSinh[a*x]^2 - 4*ArcSinh[a*x]^3*CoshIntegral[2*ArcSinh[a*x]])
/(a^2*ArcSinh[a*x]^3)
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.63

method	result	size
derivativedivides	$\frac{-\frac{\sinh(2\,\text{arcsinh}(ax))}{6\,\text{arcsinh}(ax)^3}-\frac{\cosh(2\,\text{arcsinh}(ax))}{6\,\text{arcsinh}(ax)^2}-\frac{\sinh(2\,\text{arcsinh}(ax))}{3\,\text{arcsinh}(ax)}+\frac{2\,\text{Chi}(2\,\text{arcsinh}(ax))}{3}}{a^2}$	60
default	$\frac{-\frac{\sinh(2\,\text{arcsinh}(ax))}{6\,\text{arcsinh}(ax)^3}-\frac{\cosh(2\,\text{arcsinh}(ax))}{6\,\text{arcsinh}(ax)^2}-\frac{\sinh(2\,\text{arcsinh}(ax))}{3\,\text{arcsinh}(ax)}+\frac{2\,\text{Chi}(2\,\text{arcsinh}(ax))}{3}}{a^2}$	60

```
[In] int(x/arcsinh(a*x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(-1/6/arcsinh(a*x)^3*sinh(2*arcsinh(a*x))-1/6/arcsinh(a*x)^2*cosh(2*a
rcsinh(a*x))-1/3/arcsinh(a*x)*sinh(2*arcsinh(a*x))+2/3*Chi(2*arcsinh(a*x)))
```

Fricas [F]

$$\int \frac{x}{\operatorname{arcsinh}(ax)^4}\,dx=\int \frac{x}{\operatorname{arsinh}\left(ax\right)^{4}}\,dx$$

```
[In] integrate(x/arcsinh(a*x)^4,x, algorithm="fricas")
[Out] integral(x/arcsinh(a*x)^4, x)
```

Sympy [F]

$$\int \frac{x}{\operatorname{arcsinh}(ax)^4}\,dx=\int \frac{x}{\operatorname{asinh}^{4}\left(ax\right)}\,dx$$

```
[In] integrate(x/asinh(a*x)**4,x)
[Out] Integral(x/asinh(a*x)**4, x)
```

Maxima [F]

$$\int \frac{x}{\operatorname{arcsinh}(ax)^4}\,dx=\int \frac{x}{\operatorname{arsinh}\left(ax\right)^{4}}\,dx$$

```
[In] integrate(x/arcsinh(a*x)^4,x, algorithm="maxima")
[Out] -1/6*(2*a^12*x^12 + 10*a^10*x^10 + 20*a^8*x^8 + 20*a^6*x^6 + 10*a^4*x^4 + 2
*a^2*x^2 + 2*(a^7*x^7 + a^5*x^5)*(a^2*x^2 + 1)^(5/2) + 2*(5*a^8*x^8 + 9*a^6
*x^6 + 4*a^4*x^4)*(a^2*x^2 + 1)^2 + (4*a^12*x^12 + 20*a^10*x^10 + 40*a^8*x^
8 + 40*a^6*x^6 + 20*a^4*x^4 + 4*a^2*x^2 + 4*(a^7*x^7 + a^5*x^5)*(a^2*x^2 +
1)^(5/2) + (20*a^8*x^8 + 36*a^6*x^6 + 16*a^4*x^4 - 3*a^2*x^2 - 3)*(a^2*x^2
+ 1)^2 + (40*a^9*x^9 + 104*a^7*x^7 + 88*a^5*x^5 + 21*a^3*x^3 - 3*a*x)*(a^2*
x^2 + 1)^(3/2) + (40*a^10*x^10 + 136*a^8*x^8 + 168*a^6*x^6 + 91*a^4*x^4 + 2
2*a^2*x^2 + 3)*(a^2*x^2 + 1) + (20*a^11*x^11 + 84*a^9*x^9 + 136*a^7*x^7 + 1
07*a^5*x^5 + 42*a^3*x^3 + 7*a*x)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2
+ 1))^2 + 4*(5*a^9*x^9 + 13*a^7*x^7 + 11*a^5*x^5 + 3*a^3*x^3)*(a^2*x^2 + 1)
^(3/2) + 4*(5*a^10*x^10 + 17*a^8*x^8 + 21*a^6*x^6 + 11*a^4*x^4 + 2*a^2*x^2)
*(a^2*x^2 + 1) + (2*a^12*x^12 + 10*a^10*x^10 + 20*a^8*x^8 + 20*a^6*x^6 + 10
*a^4*x^4 + 2*a^2*x^2 + 2*(a^7*x^7 + a^5*x^5)*(a^2*x^2 + 1)^(5/2) + (10*a^8*
x^8 + 18*a^6*x^6 + 9*a^4*x^4 + a^2*x^2)*(a^2*x^2 + 1)^2 + (20*a^9*x^9 + 52*
a^7*x^7 + 47*a^5*x^5 + 17*a^3*x^3 + 2*a*x)*(a^2*x^2 + 1)^(3/2) + (20*a^10*x
^10 + 68*a^8*x^8 + 87*a^6*x^6 + 51*a^4*x^4 + 13*a^2*x^2 + 1)*(a^2*x^2 + 1)
+ (10*a^11*x^11 + 42*a^9*x^9 + 69*a^7*x^7 + 55*a^5*x^5 + 21*a^3*x^3 + 3*a*x
```

```
)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1)) + 2*(5*a^11*x^11 + 21*a^9
*x^9 + 34*a^7*x^7 + 26*a^5*x^5 + 9*a^3*x^3 + a*x)*sqrt(a^2*x^2 + 1))/((a^12
*x^10 + 5*a^10*x^8 + (a^2*x^2 + 1)^(5/2)*a^7*x^5 + 10*a^8*x^6 + 10*a^6*x^4
+ 5*a^4*x^2 + 5*(a^8*x^6 + a^6*x^4)*(a^2*x^2 + 1)^2 + 10*(a^9*x^7 + 2*a^7*x
^5 + a^5*x^3)*(a^2*x^2 + 1)^(3/2) + 10*(a^10*x^8 + 3*a^8*x^6 + 3*a^6*x^4 +
a^4*x^2)*(a^2*x^2 + 1) + a^2 + 5*(a^11*x^9 + 4*a^9*x^7 + 6*a^7*x^5 + 4*a^5*
x^3 + a^3*x)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))^3) + integrate
(1/6*(8*a^13*x^13 + 48*a^11*x^11 + 120*a^9*x^9 + 8*(a^2*x^2 + 1)^3*a^7*x^7
+ 160*a^7*x^7 + 120*a^5*x^5 + 48*a^3*x^3 + (48*a^8*x^8 + 48*a^6*x^6 + 4*a^4
*x^4 + 12*a^2*x^2 + 15)*(a^2*x^2 + 1)^(5/2) + 8*(15*a^9*x^9 + 30*a^7*x^7 +
17*a^5*x^5 + 5*a^3*x^3 + 3*a*x)*(a^2*x^2 + 1)^2 + 2*(80*a^10*x^10 + 240*a^8
*x^8 + 252*a^6*x^6 + 104*a^4*x^4 + 3*a^2*x^2 - 9)*(a^2*x^2 + 1)^(3/2) + 8*(
15*a^11*x^11 + 60*a^9*x^9 + 92*a^7*x^7 + 63*a^5*x^5 + 15*a^3*x^3 - a*x)*(a^
2*x^2 + 1) + 8*a*x + (48*a^12*x^12 + 240*a^10*x^10 + 484*a^8*x^8 + 484*a^6*
x^6 + 243*a^4*x^4 + 58*a^2*x^2 + 7)*sqrt(a^2*x^2 + 1))/((a^13*x^12 + 6*a^11
*x^10 + 15*a^9*x^8 + (a^2*x^2 + 1)^3*a^7*x^6 + 20*a^7*x^6 + 15*a^5*x^4 + 6*
a^3*x^2 + 6*(a^8*x^7 + a^6*x^5)*(a^2*x^2 + 1)^(5/2) + 15*(a^9*x^8 + 2*a^7*x
^6 + a^5*x^4)*(a^2*x^2 + 1)^2 + 20*(a^10*x^9 + 3*a^8*x^7 + 3*a^6*x^5 + a^4*
x^3)*(a^2*x^2 + 1)^(3/2) + 15*(a^11*x^10 + 4*a^9*x^8 + 6*a^7*x^6 + 4*a^5*x^
4 + a^3*x^2)*(a^2*x^2 + 1) + 6*(a^12*x^11 + 5*a^10*x^9 + 10*a^8*x^7 + 10*a^
6*x^5 + 5*a^4*x^3 + a^2*x)*sqrt(a^2*x^2 + 1) + a)*log(a*x + sqrt(a^2*x^2 +
1))), x)
```

Giac [F]

$$\int \frac{x}{\operatorname{arcsinh}(ax)^4}\,dx = \int \frac{x}{\operatorname{arsinh}\left(ax\right)^4}\,dx$$

```
[In] integrate(x/arcsinh(a*x)^4,x, algorithm="giac")
```

```
[Out] integrate(x/arcsinh(a*x)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\operatorname{arcsinh}(ax)^4}\,dx = \int \frac{x}{\operatorname{asinh}\left(a\,x\right)^4}\,dx$$

```
[In] int(x/asinh(a*x)^4,x)
```

```
[Out] int(x/asinh(a*x)^4, x)
```

3.71 $\int \frac{1}{\operatorname{arcsinh}(ax)^4}\,dx$

Optimal result

Integrand size = 6, antiderivative size = 76

$$\int \frac{1}{\operatorname{arcsinh}(ax)^4}\,dx = -\frac{\sqrt{1+a^2x^2}}{3a\operatorname{arcsinh}(ax)^3} - \frac{x}{6\operatorname{arcsinh}(ax)^2} - \frac{\sqrt{1+a^2x^2}}{6a\operatorname{arcsinh}(ax)} + \frac{\operatorname{Shi}(\operatorname{arcsinh}(ax))}{6a}$$

```
[Out] -1/6*x/arcsinh(a*x)^2+1/6*Shi(arcsinh(a*x))/a-1/3*(a^2*x^2+1)^(1/2)/a/arcsi
nh(a*x)^3-1/6*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5773, 5818, 5819, 3379}

$$\int \frac{1}{\operatorname{arcsinh}(ax)^4}\,dx = -\frac{\sqrt{a^2x^2+1}}{6a\operatorname{arcsinh}(ax)} - \frac{\sqrt{a^2x^2+1}}{3a\operatorname{arcsinh}(ax)^3} + \frac{\operatorname{Shi}(\operatorname{arcsinh}(ax))}{6a} - \frac{x}{6\operatorname{arcsinh}(ax)^2}$$

```
[In] Int[ArcSinh[a*x]^(-4),x]
```

```
[Out] -1/3*Sqrt[1 + a^2*x^2]/(a*ArcSinh[a*x]^3) - x/(6*ArcSinh[a*x]^2) - Sqrt[1 +
 a^2*x^2]/(6*a*ArcSinh[a*x]) + SinhIntegral[ArcSinh[a*x]]/(6*a)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{1+a^2x^2}}{3a\text{arcsinh}(ax)^3}+\frac{1}{3}a\int\frac{x}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^3}\,dx\\
&= -\frac{\sqrt{1+a^2x^2}}{3a\text{arcsinh}(ax)^3}-\frac{x}{6\text{arcsinh}(ax)^2}+\frac{1}{6}\int\frac{1}{\text{arcsinh}(ax)^2}\,dx\\
&= -\frac{\sqrt{1+a^2x^2}}{3a\text{arcsinh}(ax)^3}-\frac{x}{6\text{arcsinh}(ax)^2}-\frac{\sqrt{1+a^2x^2}}{6a\text{arcsinh}(ax)}+\frac{1}{6}a\int\frac{x}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)}\,dx\\
&= -\frac{\sqrt{1+a^2x^2}}{3a\text{arcsinh}(ax)^3}-\frac{x}{6\text{arcsinh}(ax)^2}-\frac{\sqrt{1+a^2x^2}}{6a\text{arcsinh}(ax)}+\frac{\text{Subst}\left(\int\frac{\sinh(x)}{x}\,dx,x,\text{arcsinh}(ax)\right)}{6a}\\
&= -\frac{\sqrt{1+a^2x^2}}{3a\text{arcsinh}(ax)^3}-\frac{x}{6\text{arcsinh}(ax)^2}-\frac{\sqrt{1+a^2x^2}}{6a\text{arcsinh}(ax)}+\frac{\text{Shi}(\text{arcsinh}(ax))}{6a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.91

$$\int \frac{1}{\operatorname{arcsinh}(ax)^4}\,dx$$
$$= -\frac{2\sqrt{1+a^2x^2}+ax\operatorname{arcsinh}(ax)+\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^2-\operatorname{arcsinh}(ax)^3\operatorname{Shi}(\operatorname{arcsinh}(ax))}{6a\operatorname{arcsinh}(ax)^3}$$

```
[In] Integrate[ArcSinh[a*x]^(-4),x]
```

```
[Out] -1/6*(2*Sqrt[1 + a^2*x^2] + a*x*ArcSinh[a*x] + Sqrt[1 + a^2*x^2]*ArcSinh[a*
x]^2 - ArcSinh[a*x]^3*SinhIntegral[ArcSinh[a*x]])/(a*ArcSinh[a*x]^3)
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{-\frac{\sqrt{a^2x^2+1}}{3\operatorname{arcsinh}(ax)^3}-\frac{ax}{6\operatorname{arcsinh}(ax)^2}-\frac{\sqrt{a^2x^2+1}}{6\ \operatorname{arcsinh}(ax)}+\frac{\operatorname{Shi}(\operatorname{arcsinh}(ax))}{6}}{a}$	61
default	$\frac{-\frac{\sqrt{a^2x^2+1}}{3\operatorname{arcsinh}(ax)^3}-\frac{ax}{6\operatorname{arcsinh}(ax)^2}-\frac{\sqrt{a^2x^2+1}}{6\ \operatorname{arcsinh}(ax)}+\frac{\operatorname{Shi}(\operatorname{arcsinh}(ax))}{6}}{a}$	61

```
[In] int(1/arcsinh(a*x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(-1/3/arcsinh(a*x)^3*(a^2*x^2+1)^(1/2)-1/6*a*x/arcsinh(a*x)^2-1/6/arcsi
nh(a*x)*(a^2*x^2+1)^(1/2)+1/6*Shi(arcsinh(a*x)))
```

Fricas [F]

$$\int \frac{1}{\operatorname{arcsinh}(ax)^4}\,dx = \int \frac{1}{\operatorname{arsinh}\left(ax\right)^4}\,dx$$

```
[In] integrate(1/arcsinh(a*x)^4,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x)^(-4), x)
```

Sympy [F]

$$\int \frac{1}{\operatorname{arcsinh}(ax)^4}\,dx = \int \frac{1}{\operatorname{asinh}^4(ax)}\,dx$$

```
[In] integrate(1/asinh(a*x)**4,x)
```

```
[Out] Integral(asinh(a*x)**(-4), x)
```

Maxima [F]

$$\int \frac{1}{\operatorname{arcsinh}(ax)^4}\,dx = \int \frac{1}{\operatorname{arsinh}\left(ax\right)^4}\,dx$$

```
[In] integrate(1/arcsinh(a*x)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(2*a^11*x^11 + 10*a^9*x^9 + 20*a^7*x^7 + 20*a^5*x^5 + 10*a^3*x^3 + 2*(
a^6*x^6 + a^4*x^4)*(a^2*x^2 + 1)^(5/2) + 2*(5*a^7*x^7 + 9*a^5*x^5 + 4*a^3*x
^3)*(a^2*x^2 + 1)^2 + (a^11*x^11 + 5*a^9*x^9 + 10*a^7*x^7 + 10*a^5*x^5 + 5*
a^3*x^3 + (a^6*x^6 + a^4*x^4 + 3*a^2*x^2 + 3)*(a^2*x^2 + 1)^(5/2) + (5*a^7*
x^7 + 9*a^5*x^5 + 10*a^3*x^3 + 6*a*x)*(a^2*x^2 + 1)^2 + (10*a^8*x^8 + 26*a^
6*x^6 + 22*a^4*x^4 + 3*a^2*x^2 - 3)*(a^2*x^2 + 1)^(3/2) + 2*(5*a^9*x^9 + 17
*a^7*x^7 + 18*a^5*x^5 + 5*a^3*x^3 - a*x)*(a^2*x^2 + 1) + a*x + (5*a^10*x^10
 + 21*a^8*x^8 + 31*a^6*x^6 + 20*a^4*x^4 + 6*a^2*x^2 + 1)*sqrt(a^2*x^2 + 1))
*log(a*x + sqrt(a^2*x^2 + 1))^2 + 4*(5*a^8*x^8 + 13*a^6*x^6 + 11*a^4*x^4 +
3*a^2*x^2)*(a^2*x^2 + 1)^(3/2) + 4*(5*a^9*x^9 + 17*a^7*x^7 + 21*a^5*x^5 + 1
1*a^3*x^3 + 2*a*x)*(a^2*x^2 + 1) + 2*a*x + (a^11*x^11 + 5*a^9*x^9 + 10*a^7*
x^7 + 10*a^5*x^5 + 5*a^3*x^3 + (a^6*x^6 - a^2*x^2)*(a^2*x^2 + 1)^(5/2) + (5
*a^7*x^7 + 5*a^5*x^5 - 2*a^3*x^3 - 2*a*x)*(a^2*x^2 + 1)^2 + (10*a^8*x^8 + 2
0*a^6*x^6 + 10*a^4*x^4 - a^2*x^2 - 1)*(a^2*x^2 + 1)^(3/2) + 2*(5*a^9*x^9 +
15*a^7*x^7 + 16*a^5*x^5 + 7*a^3*x^3 + a*x)*(a^2*x^2 + 1) + a*x + (5*a^10*x^
10 + 20*a^8*x^8 + 31*a^6*x^6 + 23*a^4*x^4 + 8*a^2*x^2 + 1)*sqrt(a^2*x^2 + 1
))*log(a*x + sqrt(a^2*x^2 + 1)) + 2*(5*a^10*x^10 + 21*a^8*x^8 + 34*a^6*x^6
 + 26*a^4*x^4 + 9*a^2*x^2 + 1)*sqrt(a^2*x^2 + 1))/((a^11*x^10 + 5*a^9*x^8 +
(a^2*x^2 + 1)^(5/2)*a^6*x^5 + 10*a^7*x^6 + 10*a^5*x^4 + 5*a^3*x^2 + 5*(a^7*
x^6 + a^5*x^4)*(a^2*x^2 + 1)^2 + 10*(a^8*x^7 + 2*a^6*x^5 + a^4*x^3)*(a^2*x^
2 + 1)^(3/2) + 10*(a^9*x^8 + 3*a^7*x^6 + 3*a^5*x^4 + a^3*x^2)*(a^2*x^2 + 1)
 + 5*(a^10*x^9 + 4*a^8*x^7 + 6*a^6*x^5 + 4*a^4*x^3 + a^2*x)*sqrt(a^2*x^2 +
1) + a)*log(a*x + sqrt(a^2*x^2 + 1))^3) + integrate(1/6*(a^12*x^12 + 6*a^10
*x^10 + 15*a^8*x^8 + 20*a^6*x^6 + 15*a^4*x^4 + (a^6*x^6 - a^4*x^4 - 9*a^2*x
^2 - 15)*(a^2*x^2 + 1)^3 + 6*a^2*x^2 + (6*a^7*x^7 + a^5*x^5 - 31*a^3*x^3 -
33*a*x)*(a^2*x^2 + 1)^(5/2) + (15*a^8*x^8 + 20*a^6*x^6 - 19*a^4*x^4 - 3*a^2
*x^2 + 21)*(a^2*x^2 + 1)^2 + (20*a^9*x^9 + 50*a^7*x^7 + 54*a^5*x^5 + 59*a^3
```

```
*x^3 + 35*a*x)*(a^2*x^2 + 1)^(3/2) + (15*a^10*x^10 + 55*a^8*x^8 + 101*a^6*x
^6 + 90*a^4*x^4 + 22*a^2*x^2 - 7)*(a^2*x^2 + 1) + (6*a^11*x^11 + 29*a^9*x^9
 + 65*a^7*x^7 + 66*a^5*x^5 + 23*a^3*x^3 - a*x)*sqrt(a^2*x^2 + 1) + 1)/((a^1
2*x^12 + 6*a^10*x^10 + 15*a^8*x^8 + (a^2*x^2 + 1)^3*a^6*x^6 + 20*a^6*x^6 +
15*a^4*x^4 + 6*a^2*x^2 + 6*(a^7*x^7 + a^5*x^5)*(a^2*x^2 + 1)^(5/2) + 15*(a^
8*x^8 + 2*a^6*x^6 + a^4*x^4)*(a^2*x^2 + 1)^2 + 20*(a^9*x^9 + 3*a^7*x^7 + 3*
a^5*x^5 + a^3*x^3)*(a^2*x^2 + 1)^(3/2) + 15*(a^10*x^10 + 4*a^8*x^8 + 6*a^6*
x^6 + 4*a^4*x^4 + a^2*x^2)*(a^2*x^2 + 1) + 6*(a^11*x^11 + 5*a^9*x^9 + 10*a^
7*x^7 + 10*a^5*x^5 + 5*a^3*x^3 + a*x)*sqrt(a^2*x^2 + 1) + 1)*log(a*x + sqrt
(a^2*x^2 + 1))), x)
```

Giac [F]

$$\int \frac{1}{\operatorname{arcsinh}(ax)^4}\,dx = \int \frac{1}{\operatorname{arsinh}\left(ax\right)^4}\,dx$$

```
[In] integrate(1/arcsinh(a*x)^4,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^(-4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\operatorname{arcsinh}(ax)^4}\,dx = \int \frac{1}{\operatorname{asinh}\left(a\,x\right)^4}\,dx$$

```
[In] int(1/asinh(a*x)^4,x)
```

```
[Out] int(1/asinh(a*x)^4, x)
```

3.72 $\int \frac{1}{x\mathbf{arcsinh}(ax)^4}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{1}{x\operatorname{arcsinh}(ax)^4}\,dx = \operatorname{Int}\left(\frac{1}{x\operatorname{arcsinh}(ax)^4},x\right)$$

```
[Out] Unintegrable(1/x/arcsinh(a*x)^4,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\operatorname{arcsinh}(ax)^4}\,dx = \int \frac{1}{x\operatorname{arcsinh}(ax)^4}\,dx$$

```
[In] Int[1/(x*ArcSinh[a*x]^4),x]
```

```
[Out] Defer[Int][1/(x*ArcSinh[a*x]^4), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x\operatorname{arcsinh}(ax)^4}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.99 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\text{arcsinh}(ax)^4} \, dx = \int \frac{1}{x\text{arcsinh}(ax)^4} \, dx$$

```
[In] Integrate[1/(x*ArcSinh[a*x]^4),x]
[Out] Integrate[1/(x*ArcSinh[a*x]^4), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \operatorname{arcsinh}\left(ax\right)^{4}}dx$$

```
[In] int(1/x/arcsinh(a*x)^4,x)
[Out] int(1/x/arcsinh(a*x)^4,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\text{arcsinh}(ax)^4} \, dx = \int \frac{1}{x \operatorname{arsinh}\left(ax\right)^{4}} \, dx$$

```
[In] integrate(1/x/arcsinh(a*x)^4,x, algorithm="fricas")
[Out] integral(1/(x*arcsinh(a*x)^4), x)
```

Sympy [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\text{arcsinh}(ax)^4} \, dx = \int \frac{1}{x \operatorname{asinh}^{4}\left(ax\right)} \, dx$$

```
[In] integrate(1/x/asinh(a*x)**4,x)
[Out] Integral(1/(x*asinh(a*x)**4), x)
```

Maxima [N/A]

Not integrable

Time = 1.29 (sec) , antiderivative size = 1611, normalized size of antiderivative = 161.10

$$\int \frac{1}{x\text{arcsinh}(ax)^4}\,dx = \int \frac{1}{x\,\text{arsinh}\,(ax)^4}\,dx$$

```
[In] integrate(1/x/arcsinh(a*x)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(2*a^13*x^13 + 10*a^11*x^11 + 20*a^9*x^9 + 20*a^7*x^7 + 10*a^5*x^5 + 2
*a^3*x^3 + 2*(a^8*x^8 + a^6*x^6)*(a^2*x^2 + 1)^(5/2) + 2*(5*a^9*x^9 + 9*a^7
*x^7 + 4*a^5*x^5)*(a^2*x^2 + 1)^2 + (4*(a^6*x^6 + 3*a^4*x^4 + 2*a^2*x^2)*(a
^2*x^2 + 1)^(5/2) + (16*a^7*x^7 + 46*a^5*x^5 + 37*a^3*x^3 + 7*a*x)*(a^2*x^2
 + 1)^2 + (24*a^8*x^8 + 66*a^6*x^6 + 59*a^4*x^4 + 19*a^2*x^2 + 2)*(a^2*x^2
+ 1)^(3/2) + (16*a^9*x^9 + 42*a^7*x^7 + 39*a^5*x^5 + 16*a^3*x^3 + 3*a*x)*(a
^2*x^2 + 1) + (4*a^10*x^10 + 10*a^8*x^8 + 9*a^6*x^6 + 4*a^4*x^4 + a^2*x^2)*
sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))^2 + 4*(5*a^10*x^10 + 13*a^8
*x^8 + 11*a^6*x^6 + 3*a^4*x^4)*(a^2*x^2 + 1)^(3/2) + 4*(5*a^11*x^11 + 17*a^
9*x^9 + 21*a^7*x^7 + 11*a^5*x^5 + 2*a^3*x^3)*(a^2*x^2 + 1) - (2*(a^6*x^6 +
a^4*x^4)*(a^2*x^2 + 1)^(5/2) + (8*a^7*x^7 + 13*a^5*x^5 + 5*a^3*x^3)*(a^2*x^
2 + 1)^2 + (12*a^8*x^8 + 27*a^6*x^6 + 19*a^4*x^4 + 4*a^2*x^2)*(a^2*x^2 + 1)
^(3/2) + (8*a^9*x^9 + 23*a^7*x^7 + 23*a^5*x^5 + 9*a^3*x^3 + a*x)*(a^2*x^2 +
 1) + (2*a^10*x^10 + 7*a^8*x^8 + 9*a^6*x^6 + 5*a^4*x^4 + a^2*x^2)*sqrt(a^2*
x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1)) + 2*(5*a^12*x^12 + 21*a^10*x^10 + 34
*a^8*x^8 + 26*a^6*x^6 + 9*a^4*x^4 + a^2*x^2)*sqrt(a^2*x^2 + 1))/((a^13*x^13
 + 5*a^11*x^11 + (a^2*x^2 + 1)^(5/2)*a^8*x^8 + 10*a^9*x^9 + 10*a^7*x^7 + 5*
a^5*x^5 + a^3*x^3 + 5*(a^9*x^9 + a^7*x^7)*(a^2*x^2 + 1)^2 + 10*(a^10*x^10 +
 2*a^8*x^8 + a^6*x^6)*(a^2*x^2 + 1)^(3/2) + 10*(a^11*x^11 + 3*a^9*x^9 + 3*a
^7*x^7 + a^5*x^5)*(a^2*x^2 + 1) + 5*(a^12*x^12 + 4*a^10*x^10 + 6*a^8*x^8 +
4*a^6*x^6 + a^4*x^4)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))^3) - i
ntegrate(1/6*(8*(a^7*x^7 + 6*a^5*x^5 + 6*a^3*x^3)*(a^2*x^2 + 1)^3 + (40*a^8
*x^8 + 204*a^6*x^6 + 228*a^4*x^4 + 57*a^2*x^2)*(a^2*x^2 + 1)^(5/2) + 2*(40*
a^9*x^9 + 168*a^7*x^7 + 200*a^5*x^5 + 87*a^3*x^3 + 15*a*x)*(a^2*x^2 + 1)^2
+ 2*(40*a^10*x^10 + 132*a^8*x^8 + 156*a^6*x^6 + 91*a^4*x^4 + 30*a^2*x^2 + 3
)*(a^2*x^2 + 1)^(3/2) + 2*(20*a^11*x^11 + 48*a^9*x^9 + 48*a^7*x^7 + 35*a^5*
x^5 + 18*a^3*x^3 + 3*a*x)*(a^2*x^2 + 1) + (8*a^12*x^12 + 12*a^10*x^10 + 4*a
^8*x^8 + 5*a^6*x^6 + 6*a^4*x^4 + a^2*x^2)*sqrt(a^2*x^2 + 1))/((a^15*x^16 +
6*a^13*x^14 + 15*a^11*x^12 + (a^2*x^2 + 1)^3*a^9*x^10 + 20*a^9*x^10 + 15*a^
7*x^8 + 6*a^5*x^6 + a^3*x^4 + 6*(a^10*x^11 + a^8*x^9)*(a^2*x^2 + 1)^(5/2) +
 15*(a^11*x^12 + 2*a^9*x^10 + a^7*x^8)*(a^2*x^2 + 1)^2 + 20*(a^12*x^13 + 3*
a^10*x^11 + 3*a^8*x^9 + a^6*x^7)*(a^2*x^2 + 1)^(3/2) + 15*(a^13*x^14 + 4*a^
11*x^12 + 6*a^9*x^10 + 4*a^7*x^8 + a^5*x^6)*(a^2*x^2 + 1) + 6*(a^14*x^15 +
5*a^12*x^13 + 10*a^10*x^11 + 10*a^8*x^9 + 5*a^6*x^7 + a^4*x^5)*sqrt(a^2*x^2
 + 1))*log(a*x + sqrt(a^2*x^2 + 1))), x)
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\text{arcsinh}(ax)^4}\,dx = \int \frac{1}{x\,\text{arsinh}\left(ax\right)^4}\,dx$$

```
[In] integrate(1/x/arcsinh(a*x)^4,x, algorithm="giac")
[Out] integrate(1/(x*arcsinh(a*x)^4), x)
```

Mupad [N/A]

Not integrable

Time = 2.60 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\text{arcsinh}(ax)^4}\,dx = \int \frac{1}{x\,\mathrm{asinh}\left(a\,x\right)^4}\,dx$$

```
[In] int(1/(x*asinh(a*x)^4),x)
[Out] int(1/(x*asinh(a*x)^4), x)
```

3.73 $\int \frac{1}{x^2\textbf{arcsinh}(ax)^4}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{1}{x^2\text{arcsinh}(ax)^4}\,dx = \text{Int}\left(\frac{1}{x^2\text{arcsinh}(ax)^4},x\right)$$

```
[Out] Unintegrable(1/x^2/arcsinh(a*x)^4,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2\text{arcsinh}(ax)^4}\,dx = \int \frac{1}{x^2\text{arcsinh}(ax)^4}\,dx$$

```
[In] Int[1/(x^2*ArcSinh[a*x]^4),x]
```

```
[Out] Defer[Int][1/(x^2*ArcSinh[a*x]^4), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x^2\text{arcsinh}(ax)^4}\,dx$$

Mathematica [N/A]

Not integrable

Time = 7.51 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2\text{arcsinh}(ax)^4}\,dx = \int \frac{1}{x^2\text{arcsinh}(ax)^4}\,dx$$

```
[In] Integrate[1/(x^2*ArcSinh[a*x]^4),x]
[Out] Integrate[1/(x^2*ArcSinh[a*x]^4), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{2} \operatorname{arcsinh}\left(a x \right)^{4}}d x$$

```
[In] int(1/x^2/arcsinh(a*x)^4,x)
[Out] int(1/x^2/arcsinh(a*x)^4,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2\text{arcsinh}(ax)^4}\,dx = \int \frac{1}{x^{2} \operatorname{arsinh}\left(a x\right)^{4}}\,{d x}$$

```
[In] integrate(1/x^2/arcsinh(a*x)^4,x, algorithm="fricas")
[Out] integral(1/(x^2*arcsinh(a*x)^4), x)
```

Sympy [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2\text{arcsinh}(ax)^4}\,dx = \int \frac{1}{x^{2} \operatorname{asinh}^{4}{\left(a x \right)}}\, dx$$

```
[In] integrate(1/x**2/asinh(a*x)**4,x)
[Out] Integral(1/(x**2*asinh(a*x)**4), x)
```

Maxima [N/A]

Not integrable

Time = 1.51 (sec) , antiderivative size = 1885, normalized size of antiderivative = 188.50

$$\int \frac{1}{x^2\text{arcsinh}(ax)^4}\,dx = \int \frac{1}{x^2\,\text{arsinh}\left(ax\right)^4}\,dx$$

```
[In] integrate(1/x^2/arcsinh(a*x)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(2*a^13*x^13 + 10*a^11*x^11 + 20*a^9*x^9 + 20*a^7*x^7 + 10*a^5*x^5 + 2
*a^3*x^3 + 2*(a^8*x^8 + a^6*x^6)*(a^2*x^2 + 1)^(5/2) + 2*(5*a^9*x^9 + 9*a^7
*x^7 + 4*a^5*x^5)*(a^2*x^2 + 1)^2 + (a^13*x^13 + 5*a^11*x^11 + 10*a^9*x^9 +
 10*a^7*x^7 + 5*a^5*x^5 + a^3*x^3 + (a^8*x^8 + 13*a^6*x^6 + 27*a^4*x^4 + 15
*a^2*x^2)*(a^2*x^2 + 1)^(5/2) + (5*a^9*x^9 + 57*a^7*x^7 + 124*a^5*x^5 + 90*
a^3*x^3 + 18*a*x)*(a^2*x^2 + 1)^2 + (10*a^10*x^10 + 98*a^8*x^8 + 220*a^6*x^
6 + 189*a^4*x^4 + 63*a^2*x^2 + 6)*(a^2*x^2 + 1)^(3/2) + 2*(5*a^11*x^11 + 41
*a^9*x^9 + 93*a^7*x^7 + 89*a^5*x^5 + 38*a^3*x^3 + 6*a*x)*(a^2*x^2 + 1) + (5
*a^12*x^12 + 33*a^10*x^10 + 73*a^8*x^8 + 74*a^6*x^6 + 36*a^4*x^4 + 7*a^2*x^
2)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))^2 + 4*(5*a^10*x^10 + 13*
a^8*x^8 + 11*a^6*x^6 + 3*a^4*x^4)*(a^2*x^2 + 1)^(3/2) + 4*(5*a^11*x^11 + 17
*a^9*x^9 + 21*a^7*x^7 + 11*a^5*x^5 + 2*a^3*x^3)*(a^2*x^2 + 1) - (a^13*x^13
+ 5*a^11*x^11 + 10*a^9*x^9 + 10*a^7*x^7 + 5*a^5*x^5 + a^3*x^3 + (a^8*x^8 +
4*a^6*x^6 + 3*a^4*x^4)*(a^2*x^2 + 1)^(5/2) + (5*a^9*x^9 + 21*a^7*x^7 + 24*a
^5*x^5 + 8*a^3*x^3)*(a^2*x^2 + 1)^2 + (10*a^10*x^10 + 44*a^8*x^8 + 64*a^6*x
^6 + 37*a^4*x^4 + 7*a^2*x^2)*(a^2*x^2 + 1)^(3/2) + 2*(5*a^11*x^11 + 23*a^9*
x^9 + 39*a^7*x^7 + 30*a^5*x^5 + 10*a^3*x^3 + a*x)*(a^2*x^2 + 1) + (5*a^12*x
^12 + 24*a^10*x^10 + 45*a^8*x^8 + 41*a^6*x^6 + 18*a^4*x^4 + 3*a^2*x^2)*sqrt
(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1)) + 2*(5*a^12*x^12 + 21*a^10*x^10
 + 34*a^8*x^8 + 26*a^6*x^6 + 9*a^4*x^4 + a^2*x^2)*sqrt(a^2*x^2 + 1))/((a^13
*x^14 + 5*a^11*x^12 + (a^2*x^2 + 1)^(5/2)*a^8*x^9 + 10*a^9*x^10 + 10*a^7*x^
8 + 5*a^5*x^6 + a^3*x^4 + 5*(a^9*x^10 + a^7*x^8)*(a^2*x^2 + 1)^2 + 10*(a^10
*x^11 + 2*a^8*x^9 + a^6*x^7)*(a^2*x^2 + 1)^(3/2) + 10*(a^11*x^12 + 3*a^9*x^
10 + 3*a^7*x^8 + a^5*x^6)*(a^2*x^2 + 1) + 5*(a^12*x^13 + 4*a^10*x^11 + 6*a^
8*x^9 + 4*a^6*x^7 + a^4*x^5)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1)
)^3) - integrate(1/6*(a^15*x^15 + 6*a^13*x^13 + 15*a^11*x^11 + 20*a^9*x^9 +
 15*a^7*x^7 + 6*a^5*x^5 + a^3*x^3 + (a^9*x^9 + 39*a^7*x^7 + 135*a^5*x^5 + 1
05*a^3*x^3)*(a^2*x^2 + 1)^3 + (6*a^10*x^10 + 201*a^8*x^8 + 677*a^6*x^6 + 66
3*a^4*x^4 + 174*a^2*x^2)*(a^2*x^2 + 1)^(5/2) + (15*a^11*x^11 + 420*a^9*x^9
+ 1373*a^7*x^7 + 1565*a^5*x^5 + 705*a^3*x^3 + 108*a*x)*(a^2*x^2 + 1)^2 + (2
0*a^12*x^12 + 450*a^10*x^10 + 1422*a^8*x^8 + 1787*a^6*x^6 + 1059*a^4*x^4 +
288*a^2*x^2 + 24)*(a^2*x^2 + 1)^(3/2) + (15*a^13*x^13 + 255*a^11*x^11 + 773
*a^9*x^9 + 1026*a^7*x^7 + 714*a^5*x^5 + 257*a^3*x^3 + 36*a*x)*(a^2*x^2 + 1)
 + (6*a^14*x^14 + 69*a^12*x^12 + 197*a^10*x^10 + 266*a^8*x^8 + 201*a^6*x^6
+ 83*a^4*x^4 + 14*a^2*x^2)*sqrt(a^2*x^2 + 1))/((a^15*x^17 + 6*a^13*x^15 + 1
```

```
5*a^11*x^13 + (a^2*x^2 + 1)^3*a^9*x^11 + 20*a^9*x^11 + 15*a^7*x^9 + 6*a^5*x
^7 + a^3*x^5 + 6*(a^10*x^12 + a^8*x^10)*(a^2*x^2 + 1)^(5/2) + 15*(a^11*x^13
 + 2*a^9*x^11 + a^7*x^9)*(a^2*x^2 + 1)^2 + 20*(a^12*x^14 + 3*a^10*x^12 + 3*
a^8*x^10 + a^6*x^8)*(a^2*x^2 + 1)^(3/2) + 15*(a^13*x^15 + 4*a^11*x^13 + 6*a
^9*x^11 + 4*a^7*x^9 + a^5*x^7)*(a^2*x^2 + 1) + 6*(a^14*x^16 + 5*a^12*x^14 +
 10*a^10*x^12 + 10*a^8*x^10 + 5*a^6*x^8 + a^4*x^6)*sqrt(a^2*x^2 + 1))*log(a
*x + sqrt(a^2*x^2 + 1))), x)
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2\text{arcsinh}(ax)^4}\,dx = \int \frac{1}{x^2\,\text{arsinh}\,(ax)^4}\,dx$$

```
[In] integrate(1/x^2/arcsinh(a*x)^4,x, algorithm="giac")
```

```
[Out] integrate(1/(x^2*arcsinh(a*x)^4), x)
```

Mupad [N/A]

Not integrable

Time = 2.58 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2\text{arcsinh}(ax)^4}\,dx = \int \frac{1}{x^2\,\text{asinh}\,(a\,x)^4}\,dx$$

```
[In] int(1/(x^2*asinh(a*x)^4),x)
```

```
[Out] int(1/(x^2*asinh(a*x)^4), x)
```

3.74 $\int x^4 \sqrt{\operatorname{arcsinh}(ax)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 182

$$\begin{aligned}\int x^4\sqrt{\operatorname{arcsinh}(ax)}\,dx =\ & \frac{1}{5}x^5\sqrt{\operatorname{arcsinh}(ax)}+\frac{\sqrt{\pi}\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{32a^5} \\ & -\frac{\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{64a^5}+\frac{\sqrt{\frac{\pi}{5}}\operatorname{erf}\left(\sqrt{5}\sqrt{\operatorname{arcsinh}(ax)}\right)}{320a^5} \\ & -\frac{\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{32a^5}+\frac{\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{64a^5} \\ & -\frac{\sqrt{\frac{\pi}{5}}\operatorname{erfi}\left(\sqrt{5}\sqrt{\operatorname{arcsinh}(ax)}\right)}{320a^5}\end{aligned}$$

```
[Out] 1/1600*erf(5^(1/2)*arcsinh(a*x)^(1/2))*5^(1/2)*Pi^(1/2)/a^5-1/1600*erfi(5^(
1/2)*arcsinh(a*x)^(1/2))*5^(1/2)*Pi^(1/2)/a^5-1/192*erf(3^(1/2)*arcsinh(a*x
)^(1/2))*3^(1/2)*Pi^(1/2)/a^5+1/192*erfi(3^(1/2)*arcsinh(a*x)^(1/2))*3^(1/2
)*Pi^(1/2)/a^5+1/32*erf(arcsinh(a*x)^(1/2))*Pi^(1/2)/a^5-1/32*erfi(arcsinh(
a*x)^(1/2))*Pi^(1/2)/a^5+1/5*x^5*arcsinh(a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used

$= \{5777, 5819, 3393, 3389, 2211, 2235, 2236\}$

$$\int x^4\sqrt{\operatorname{arcsinh}(ax)}\,dx = \frac{\sqrt{\pi}\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{32a^5} - \frac{\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{64a^5} + \frac{\sqrt{\frac{\pi}{5}}\operatorname{erf}\left(\sqrt{5}\sqrt{\operatorname{arcsinh}(ax)}\right)}{320a^5} - \frac{\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{32a^5} + \frac{\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{64a^5} - \frac{\sqrt{\frac{\pi}{5}}\operatorname{erfi}\left(\sqrt{5}\sqrt{\operatorname{arcsinh}(ax)}\right)}{320a^5} + \frac{1}{5}x^5\sqrt{\operatorname{arcsinh}(ax)}$$

```
[In] Int[x^4*Sqrt[ArcSinh[a*x]],x]
```

```
[Out] (x^5*Sqrt[ArcSinh[a*x]])/5 + (Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]])/(32*a^5) -
(Sqrt[Pi/3]*Erf[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(64*a^5) + (Sqrt[Pi/5]*Erf[Sqr
t[5]*Sqrt[ArcSinh[a*x]]])/(320*a^5) - (Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x]]])/(
32*a^5) + (Sqrt[Pi/3]*Erfi[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(64*a^5) - (Sqrt[Pi
/5]*Erfi[Sqrt[5]*Sqrt[ArcSinh[a*x]]])/(320*a^5)
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{5}x^5\sqrt{\operatorname{arcsinh}(ax)} - \frac{1}{10}a\int \frac{x^5}{\sqrt{1+a^2x^2}\sqrt{\operatorname{arcsinh}(ax)}}\,dx \\
&= \frac{1}{5}x^5\sqrt{\operatorname{arcsinh}(ax)} - \frac{\text{Subst}\left(\int \frac{\sinh^5(x)}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{10a^5} \\
&= \frac{1}{5}x^5\sqrt{\operatorname{arcsinh}(ax)} + \frac{i\text{Subst}\left(\int \left(\frac{5i\sinh(x)}{8\sqrt{x}} - \frac{5i\sinh(3x)}{16\sqrt{x}} + \frac{i\sinh(5x)}{16\sqrt{x}}\right)\,dx, x, \operatorname{arcsinh}(ax)\right)}{10a^5} \\
&= \frac{1}{5}x^5\sqrt{\operatorname{arcsinh}(ax)} - \frac{\text{Subst}\left(\int \frac{\sinh(5x)}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{160a^5} \\
&\quad + \frac{\text{Subst}\left(\int \frac{\sinh(3x)}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{32a^5} - \frac{\text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{16a^5} \\
&= \frac{1}{5}x^5\sqrt{\operatorname{arcsinh}(ax)} + \frac{\text{Subst}\left(\int \frac{e^{-5x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{320a^5} \\
&\quad - \frac{\text{Subst}\left(\int \frac{e^{5x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{320a^5} \\
&\quad - \frac{\text{Subst}\left(\int \frac{e^{-3x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{64a^5} + \frac{\text{Subst}\left(\int \frac{e^{3x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{64a^5} \\
&\quad + \frac{\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{32a^5} - \frac{\text{Subst}\left(\int \frac{e^{x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{32a^5}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{5}x^5\sqrt{\operatorname{arcsinh}(ax)} + \frac{\operatorname{Subst}\left(\int e^{-5x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{160a^5} \\
&\quad - \frac{\operatorname{Subst}\left(\int e^{5x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{160a^5} \\
&\quad - \frac{\operatorname{Subst}\left(\int e^{-3x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{32a^5} + \frac{\operatorname{Subst}\left(\int e^{3x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{32a^5} \\
&\quad + \frac{\operatorname{Subst}\left(\int e^{-x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{16a^5} - \frac{\operatorname{Subst}\left(\int e^{x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{16a^5}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1}{5}x^5\sqrt{\operatorname{arcsinh}(ax)} + \frac{\sqrt{\pi}\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{32a^5} - \frac{\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{64a^5} \\
&\quad + \frac{\sqrt{\frac{\pi}{5}}\operatorname{erf}\left(\sqrt{5}\sqrt{\operatorname{arcsinh}(ax)}\right)}{320a^5} - \frac{\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{32a^5} \\
&\quad + \frac{\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{64a^5} - \frac{\sqrt{\frac{\pi}{5}}\operatorname{erfi}\left(\sqrt{5}\sqrt{\operatorname{arcsinh}(ax)}\right)}{320a^5}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.84

$$
\int x^4\sqrt{\operatorname{arcsinh}(ax)}\,dx
$$

$$
= \frac{\frac{3\sqrt{5}\sqrt{\operatorname{arcsinh}(ax)}\Gamma\left(\frac{3}{2},-5\operatorname{arcsinh}(ax)\right)}{\sqrt{-\operatorname{arcsinh}(ax)}} + \frac{25\sqrt{3}\sqrt{-\operatorname{arcsinh}(ax)}\Gamma\left(\frac{3}{2},-3\operatorname{arcsinh}(ax)\right)}{\sqrt{\operatorname{arcsinh}(ax)}} + \frac{150\sqrt{\operatorname{arcsinh}(ax)}\Gamma\left(\frac{3}{2},-\operatorname{arcsinh}(ax\right.}{\sqrt{-\operatorname{arcsinh}(ax)}}}{2400a^5}
$$

```
[In] Integrate[x^4*Sqrt[ArcSinh[a*x]],x]
```

```
[Out] ((3*Sqrt[5]*Sqrt[ArcSinh[a*x]]*Gamma[3/2, -5*ArcSinh[a*x]])/Sqrt[-ArcSinh[a
*x]] + (25*Sqrt[3]*Sqrt[-ArcSinh[a*x]]*Gamma[3/2, -3*ArcSinh[a*x]])/Sqrt[Ar
cSinh[a*x]] + (150*Sqrt[ArcSinh[a*x]]*Gamma[3/2, -ArcSinh[a*x]])/Sqrt[-ArcS
inh[a*x]] - 150*Gamma[3/2, ArcSinh[a*x]] + 25*Sqrt[3]*Gamma[3/2, 3*ArcSinh[
a*x]] - 3*Sqrt[5]*Gamma[3/2, 5*ArcSinh[a*x]])/(2400*a^5)
```

Maple [F]

$$\int x^4 \sqrt{\operatorname{arcsinh}(ax)}dx$$

```
[In] int(x^4*arcsinh(a*x)^(1/2),x)
```

```
[Out] int(x^4*arcsinh(a*x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^4 \sqrt{\operatorname{arcsinh}(ax)}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^4*arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x^4 \sqrt{\operatorname{arcsinh}(ax)}\, dx = \int x^4 \sqrt{\operatorname{asinh}(ax)}\, dx$$

```
[In] integrate(x**4*asinh(a*x)**(1/2),x)
```

```
[Out] Integral(x**4*sqrt(asinh(a*x)), x)
```

Maxima [F]

$$\int x^4 \sqrt{\operatorname{arcsinh}(ax)}\, dx = \int x^4 \sqrt{\operatorname{arsinh}(ax)}\, dx$$

```
[In] integrate(x^4*arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4*sqrt(arcsinh(a*x)), x)
```

Giac [F]

$$\int x^4\sqrt{\text{arcsinh}(ax)}\,dx = \int x^4\sqrt{\text{arsinh}\,(ax)}\,dx$$

```
[In] integrate(x^4*arcsinh(a*x)^(1/2),x, algorithm="giac")
[Out] integrate(x^4*sqrt(arcsinh(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4\sqrt{\text{arcsinh}(ax)}\,dx = \int x^4\,\sqrt{\text{asinh}\,(a\,x)}\,dx$$

```
[In] int(x^4*asinh(a*x)^(1/2),x)
[Out] int(x^4*asinh(a*x)^(1/2), x)
```

3.75 $\int x^3 \sqrt{\text{arcsinh}(ax)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 139

$$\int x^3 \sqrt{\text{arcsinh}(ax)}\,dx = -\frac{3\sqrt{\text{arcsinh}(ax)}}{32a^4} + \frac{1}{4}x^4\sqrt{\text{arcsinh}(ax)} - \frac{\sqrt{\pi}\,\text{erf}\left(2\sqrt{\text{arcsinh}(ax)}\right)}{256a^4} + \frac{\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{32a^4} - \frac{\sqrt{\pi}\,\text{erfi}\left(2\sqrt{\text{arcsinh}(ax)}\right)}{256a^4} + \frac{\sqrt{\frac{\pi}{2}}\,\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{32a^4}$$

```
[Out] 1/64*erf(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^4+1/64*erfi(2^(1/2)
*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^4-1/256*erf(2*arcsinh(a*x)^(1/2))*P
i^(1/2)/a^4-1/256*erfi(2*arcsinh(a*x)^(1/2))*Pi^(1/2)/a^4-3/32*arcsinh(a*x)
^(1/2)/a^4+1/4*x^4*arcsinh(a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5777, 5819, 3393, 3388, 2211, 2235, 2236}

$$\int x^3 \sqrt{\text{arcsinh}(ax)}\,dx = -\frac{\sqrt{\pi}\,\text{erf}\left(2\sqrt{\text{arcsinh}(ax)}\right)}{256a^4} + \frac{\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{32a^4} - \frac{\sqrt{\pi}\,\text{erfi}\left(2\sqrt{\text{arcsinh}(ax)}\right)}{256a^4} + \frac{\sqrt{\frac{\pi}{2}}\,\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{32a^4} - \frac{3\sqrt{\text{arcsinh}(ax)}}{32a^4} + \frac{1}{4}x^4\sqrt{\text{arcsinh}(ax)}$$

```
[In] Int[x^3*Sqrt[ArcSinh[a*x]],x]
```

```
[Out] (-3*Sqrt[ArcSinh[a*x]])/(32*a^4) + (x^4*Sqrt[ArcSinh[a*x]])/4 - (Sqrt[Pi]*E
rf[2*Sqrt[ArcSinh[a*x]]])/(256*a^4) + (Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcSinh[
a*x]]])/(32*a^4) - (Sqrt[Pi]*Erfi[2*Sqrt[ArcSinh[a*x]]])/(256*a^4) + (Sqrt[
Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(32*a^4)
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
```

```
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}x^4\sqrt{\operatorname{arcsinh}(ax)} - \frac{1}{8}a\int \frac{x^4}{\sqrt{1+a^2x^2}\sqrt{\operatorname{arcsinh}(ax)}}\,dx \\
&= \frac{1}{4}x^4\sqrt{\operatorname{arcsinh}(ax)} - \frac{\operatorname{Subst}\left(\int \frac{\sinh^4(x)}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{8a^4} \\
&= \frac{1}{4}x^4\sqrt{\operatorname{arcsinh}(ax)} - \frac{\operatorname{Subst}\left(\int \left(\frac{3}{8\sqrt{x}} - \frac{\cosh(2x)}{2\sqrt{x}} + \frac{\cosh(4x)}{8\sqrt{x}}\right)\,dx, x, \operatorname{arcsinh}(ax)\right)}{8a^4} \\
&= -\frac{3\sqrt{\operatorname{arcsinh}(ax)}}{32a^4} + \frac{1}{4}x^4\sqrt{\operatorname{arcsinh}(ax)} - \frac{\operatorname{Subst}\left(\int \frac{\cosh(4x)}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{64a^4} \\
&\quad + \frac{\operatorname{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{16a^4} \\
&= -\frac{3\sqrt{\operatorname{arcsinh}(ax)}}{32a^4} + \frac{1}{4}x^4\sqrt{\operatorname{arcsinh}(ax)} \\
&\quad - \frac{\operatorname{Subst}\left(\int \frac{e^{-4x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{128a^4} - \frac{\operatorname{Subst}\left(\int \frac{e^{4x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{128a^4} \\
&\quad + \frac{\operatorname{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{32a^4} + \frac{\operatorname{Subst}\left(\int \frac{e^{2x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{32a^4} \\
&= -\frac{3\sqrt{\operatorname{arcsinh}(ax)}}{32a^4} + \frac{1}{4}x^4\sqrt{\operatorname{arcsinh}(ax)} \\
&\quad - \frac{\operatorname{Subst}\left(\int e^{-4x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{64a^4} - \frac{\operatorname{Subst}\left(\int e^{4x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{64a^4} \\
&\quad + \frac{\operatorname{Subst}\left(\int e^{-2x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{16a^4} + \frac{\operatorname{Subst}\left(\int e^{2x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{16a^4} \\
&= -\frac{3\sqrt{\operatorname{arcsinh}(ax)}}{32a^4} + \frac{1}{4}x^4\sqrt{\operatorname{arcsinh}(ax)} \\
&\quad - \frac{\sqrt{\pi}\operatorname{erf}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{256a^4} + \frac{\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{32a^4} \\
&\quad - \frac{\sqrt{\pi}\operatorname{erfi}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{256a^4} + \frac{\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{32a^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.71

$$\int x^3\sqrt{\operatorname{arcsinh}(ax)}\,dx = \frac{\frac{\sqrt{\operatorname{arcsinh}(ax)}\Gamma\left(\frac{3}{2},-4\operatorname{arcsinh}(ax)\right)}{\sqrt{-\operatorname{arcsinh}(ax)}}+\frac{4\sqrt{2}\sqrt{-\operatorname{arcsinh}(ax)}\Gamma\left(\frac{3}{2},-2\operatorname{arcsinh}(ax)\right)}{\sqrt{\operatorname{arcsinh}(ax)}}-4\sqrt{2}\Gamma\left(\frac{3}{2},2\operatorname{arcsinh}(ax)\right)+\Gamma(\frac{3}{2},4\mathrm{ar}}{128a^4}$$

```
[In] Integrate[x^3*Sqrt[ArcSinh[a*x]],x]
```

```
[Out] ((Sqrt[ArcSinh[a*x]]*Gamma[3/2, -4*ArcSinh[a*x]])/Sqrt[-ArcSinh[a*x]] + (4*
Sqrt[2]*Sqrt[-ArcSinh[a*x]]*Gamma[3/2, -2*ArcSinh[a*x]])/Sqrt[ArcSinh[a*x]]
 - 4*Sqrt[2]*Gamma[3/2, 2*ArcSinh[a*x]] + Gamma[3/2, 4*ArcSinh[a*x]])/(128*
a^4)
```

Maple [F]

$$\int x^3\sqrt{\operatorname{arcsinh}\left(ax\right)}dx$$

```
[In] int(x^3*arcsinh(a*x)^(1/2),x)
```

```
[Out] int(x^3*arcsinh(a*x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^3\sqrt{\operatorname{arcsinh}(ax)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3*arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x^3\sqrt{\operatorname{arcsinh}(ax)}\,dx = \int x^3\sqrt{\operatorname{asinh}(ax)}\,dx$$

```
[In] integrate(x**3*asinh(a*x)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(asinh(a*x)), x)
```

Maxima [F]

$$\int x^3\sqrt{\operatorname{arcsinh}(ax)}\,dx = \int x^3\sqrt{\operatorname{arsinh}(ax)}\,dx$$

```
[In] integrate(x^3*arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3*sqrt(arcsinh(a*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int x^3\sqrt{\operatorname{arcsinh}(ax)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3*arcsinh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{\operatorname{arcsinh}(ax)}\,dx = \int x^3\,\sqrt{\operatorname{asinh}(a\,x)}\,dx$$

```
[In] int(x^3*asinh(a*x)^(1/2),x)
```

```
[Out] int(x^3*asinh(a*x)^(1/2), x)
```

3.76 $\int x^2\sqrt{\text{arcsinh}(ax)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 120

$$\int x^2\sqrt{\text{arcsinh}(ax)}\,dx=\frac{1}{3}x^3\sqrt{\text{arcsinh}(ax)}-\frac{\sqrt{\pi}\,\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)}{16a^3}+\frac{\sqrt{\frac{\pi}{3}}\,\text{erf}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)}{48a^3}+\frac{\sqrt{\pi}\,\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)}{16a^3}-\frac{\sqrt{\frac{\pi}{3}}\,\text{erfi}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)}{48a^3}$$

```
[Out] 1/144*erf(3^(1/2)*arcsinh(a*x)^(1/2))*3^(1/2)*Pi^(1/2)/a^3-1/144*erfi(3^(1/
2)*arcsinh(a*x)^(1/2))*3^(1/2)*Pi^(1/2)/a^3-1/16*erf(arcsinh(a*x)^(1/2))*Pi
^(1/2)/a^3+1/16*erfi(arcsinh(a*x)^(1/2))*Pi^(1/2)/a^3+1/3*x^3*arcsinh(a*x)^
(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5777, 5819, 3393, 3389, 2211, 2235, 2236}

$$\int x^2\sqrt{\text{arcsinh}(ax)}\,dx=-\frac{\sqrt{\pi}\,\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)}{16a^3}+\frac{\sqrt{\frac{\pi}{3}}\,\text{erf}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)}{48a^3}+\frac{\sqrt{\pi}\,\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)}{16a^3}-\frac{\sqrt{\frac{\pi}{3}}\,\text{erfi}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)}{48a^3}+\frac{1}{3}x^3\sqrt{\text{arcsinh}(ax)}$$

```
[In] Int[x^2*Sqrt[ArcSinh[a*x]],x]
```

```
[Out] (x^3*Sqrt[ArcSinh[a*x]])/3 - (Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]])/(16*a^3) +
(Sqrt[Pi/3]*Erf[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(48*a^3) + (Sqrt[Pi]*Erfi[Sqrt
[ArcSinh[a*x]]])/(16*a^3) - (Sqrt[Pi/3]*Erfi[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(
48*a^3)
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
```

```
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\sqrt{\operatorname{arcsinh}(ax)} - \frac{1}{6}a\int \frac{x^3}{\sqrt{1+a^2x^2}\sqrt{\operatorname{arcsinh}(ax)}}\,dx \\
&= \frac{1}{3}x^3\sqrt{\operatorname{arcsinh}(ax)} - \frac{\text{Subst}\left(\int \frac{\sinh^3(x)}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{6a^3} \\
&= \frac{1}{3}x^3\sqrt{\operatorname{arcsinh}(ax)} - \frac{i\text{Subst}\left(\int \left(\frac{3i\sinh(x)}{4\sqrt{x}} - \frac{i\sinh(3x)}{4\sqrt{x}}\right)\,dx, x, \operatorname{arcsinh}(ax)\right)}{6a^3} \\
&= \frac{1}{3}x^3\sqrt{\operatorname{arcsinh}(ax)} - \frac{\text{Subst}\left(\int \frac{\sinh(3x)}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{24a^3} \\
&\quad + \frac{\text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{8a^3} \\
&= \frac{1}{3}x^3\sqrt{\operatorname{arcsinh}(ax)} + \frac{\text{Subst}\left(\int \frac{e^{-3x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{48a^3} \\
&\quad - \frac{\text{Subst}\left(\int \frac{e^{3x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{48a^3} \\
&\quad - \frac{\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{16a^3} + \frac{\text{Subst}\left(\int \frac{e^{x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{16a^3} \\
&= \frac{1}{3}x^3\sqrt{\operatorname{arcsinh}(ax)} + \frac{\text{Subst}\left(\int e^{-3x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{24a^3} \\
&\quad - \frac{\text{Subst}\left(\int e^{3x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{24a^3} \\
&\quad - \frac{\text{Subst}\left(\int e^{-x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{8a^3} + \frac{\text{Subst}\left(\int e^{x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{8a^3} \\
&= \frac{1}{3}x^3\sqrt{\operatorname{arcsinh}(ax)} - \frac{\sqrt{\pi}\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{16a^3} + \frac{\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{48a^3} \\
&\quad + \frac{\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{16a^3} - \frac{\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{48a^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.82

$$\int x^2\sqrt{\text{arcsinh}(ax)}\,dx = \frac{\frac{\sqrt{3}\sqrt{\text{arcsinh}(ax)}\Gamma\left(\frac{3}{2},-3\text{arcsinh}(ax)\right)}{\sqrt{-\text{arcsinh}(ax)}}+\frac{9\sqrt{-\text{arcsinh}(ax)}\Gamma\left(\frac{3}{2},-\text{arcsinh}(ax)\right)}{\sqrt{\text{arcsinh}(ax)}}+9\Gamma\left(\frac{3}{2},\text{arcsinh}(ax)\right)-\sqrt{3}\Gamma\left(\frac{3}{2},3\text{arcsi}\right.}{72a^3}$$

```
[In] Integrate[x^2*Sqrt[ArcSinh[a*x]],x]
```

```
[Out] ((Sqrt[3]*Sqrt[ArcSinh[a*x]]*Gamma[3/2, -3*ArcSinh[a*x]])/Sqrt[-ArcSinh[a*x
]] + (9*Sqrt[-ArcSinh[a*x]]*Gamma[3/2, -ArcSinh[a*x]])/Sqrt[ArcSinh[a*x]] +
 9*Gamma[3/2, ArcSinh[a*x]] - Sqrt[3]*Gamma[3/2, 3*ArcSinh[a*x]])/(72*a^3)
```

Maple [F]

$$\int x^2\sqrt{\text{arcsinh}\,(ax)}dx$$

```
[In] int(x^2*arcsinh(a*x)^(1/2),x)
```

```
[Out] int(x^2*arcsinh(a*x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^2\sqrt{\text{arcsinh}(ax)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2*arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x^2\sqrt{\text{arcsinh}(ax)}\,dx = \int x^2\sqrt{\text{asinh}\,(ax)}\,dx$$

```
[In] integrate(x**2*asinh(a*x)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(asinh(a*x)), x)
```

Maxima [F]

$$\int x^2\sqrt{\operatorname{arcsinh}(ax)}\,dx = \int x^2\sqrt{\operatorname{arsinh}\left(ax\right)}\,dx$$

```
[In] integrate(x^2*arcsinh(a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(x^2*sqrt(arcsinh(a*x)), x)
```

Giac [F]

$$\int x^2\sqrt{\operatorname{arcsinh}(ax)}\,dx = \int x^2\sqrt{\operatorname{arsinh}\left(ax\right)}\,dx$$

```
[In] integrate(x^2*arcsinh(a*x)^(1/2),x, algorithm="giac")
[Out] integrate(x^2*sqrt(arcsinh(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{\operatorname{arcsinh}(ax)}\,dx = \int x^2\,\sqrt{\operatorname{asinh}\left(a\,x\right)}\,dx$$

```
[In] int(x^2*asinh(a*x)^(1/2),x)
[Out] int(x^2*asinh(a*x)^(1/2), x)
```

3.77 $\int x\sqrt{\operatorname{arcsinh}(ax)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 93

$$\int x\sqrt{\operatorname{arcsinh}(ax)}\,dx=\frac{\sqrt{\operatorname{arcsinh}(ax)}}{4a^2}+\frac{1}{2}x^2\sqrt{\operatorname{arcsinh}(ax)}-\frac{\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{16a^2}-\frac{\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{16a^2}$$

```
[Out] -1/32*erf(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^2-1/32*erfi(2^(1/2
)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^2+1/4*arcsinh(a*x)^(1/2)/a^2+1/2*x
^2*arcsinh(a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {5777, 5819, 3393, 3388, 2211, 2235, 2236}

$$\int x\sqrt{\operatorname{arcsinh}(ax)}\,dx=-\frac{\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{16a^2}-\frac{\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{16a^2}+\frac{\sqrt{\operatorname{arcsinh}(ax)}}{4a^2}+\frac{1}{2}x^2\sqrt{\operatorname{arcsinh}(ax)}$$

```
[In] Int[x*Sqrt[ArcSinh[a*x]],x]
```

```
[Out] Sqrt[ArcSinh[a*x]]/(4*a^2) + (x^2*Sqrt[ArcSinh[a*x]])/2 - (Sqrt[Pi/2]*Erf[S
qrt[2]*Sqrt[ArcSinh[a*x]]])/(16*a^2) - (Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcSin
h[a*x]]])/(16*a^2)
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}x^2\sqrt{\operatorname{arcsinh}(ax)} - \frac{1}{4}a\int \frac{x^2}{\sqrt{1+a^2x^2}\sqrt{\operatorname{arcsinh}(ax)}}\,dx \\
&= \frac{1}{2}x^2\sqrt{\operatorname{arcsinh}(ax)} - \frac{\operatorname{Subst}\left(\int \frac{\sinh^2(x)}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{4a^2} \\
&= \frac{1}{2}x^2\sqrt{\operatorname{arcsinh}(ax)} + \frac{\operatorname{Subst}\left(\int \left(\frac{1}{2\sqrt{x}} - \frac{\cosh(2x)}{2\sqrt{x}}\right)\,dx, x, \operatorname{arcsinh}(ax)\right)}{4a^2} \\
&= \frac{\sqrt{\operatorname{arcsinh}(ax)}}{4a^2} + \frac{1}{2}x^2\sqrt{\operatorname{arcsinh}(ax)} - \frac{\operatorname{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{8a^2} \\
&= \frac{\sqrt{\operatorname{arcsinh}(ax)}}{4a^2} + \frac{1}{2}x^2\sqrt{\operatorname{arcsinh}(ax)} \\
&\quad - \frac{\operatorname{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{16a^2} - \frac{\operatorname{Subst}\left(\int \frac{e^{2x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{16a^2} \\
&= \frac{\sqrt{\operatorname{arcsinh}(ax)}}{4a^2} + \frac{1}{2}x^2\sqrt{\operatorname{arcsinh}(ax)} - \frac{\operatorname{Subst}\left(\int e^{-2x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{8a^2} \\
&\quad - \frac{\operatorname{Subst}\left(\int e^{2x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{8a^2} \\
&= \frac{\sqrt{\operatorname{arcsinh}(ax)}}{4a^2} + \frac{1}{2}x^2\sqrt{\operatorname{arcsinh}(ax)} \\
&\quad - \frac{\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{16a^2} - \frac{\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{16a^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.56

$$\int x\sqrt{\operatorname{arcsinh}(ax)}\,dx = \frac{\frac{\sqrt{\operatorname{arcsinh}(ax)}\Gamma\left(\frac{3}{2},-2\operatorname{arcsinh}(ax)\right)}{\sqrt{-\operatorname{arcsinh}(ax)}} + \Gamma\left(\frac{3}{2}, 2\operatorname{arcsinh}(ax)\right)}{8\sqrt{2}a^2}$$

```
[In] Integrate[x*Sqrt[ArcSinh[a*x]],x]
```

```
[Out] ((Sqrt[ArcSinh[a*x]]*Gamma[3/2, -2*ArcSinh[a*x]])/Sqrt[-ArcSinh[a*x]] + Gam
ma[3/2, 2*ArcSinh[a*x]])/(8*Sqrt[2]*a^2)
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{\sqrt{2}\,\left(8 \sqrt{2}\, \sqrt{\operatorname{arcsinh}\left(a x \right)}\, \sqrt{\pi}\, a^{2} x^{2}+4 \sqrt{2}\, \sqrt{\operatorname{arcsinh}\left(a x \right)}\, \sqrt{\pi}-\pi \,\operatorname{erf}\left(\sqrt{2}\, \sqrt{\operatorname{arcsinh}\left(a x \right)}\right)-\pi \,\operatorname{erfi}\left(\sqrt{2}\, \sqrt{\operatorname{arcsinh}\left(a x \right)}\right)\right)}{32 \sqrt{\pi}\, a^{2}}$	75

```
[In] int(x*arcsinh(a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/32*2^(1/2)*(8*2^(1/2)*arcsinh(a*x)^(1/2)*Pi^(1/2)*a^2*x^2+4*2^(1/2)*arcsi
nh(a*x)^(1/2)*Pi^(1/2)-Pi*erf(2^(1/2)*arcsinh(a*x)^(1/2))-Pi*erfi(2^(1/2)*a
rcsinh(a*x)^(1/2)))/Pi^(1/2)/a^2
```

Fricas [F(-2)]

Exception generated.

$$\int x\sqrt{\operatorname{arcsinh}(ax)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x\sqrt{\operatorname{arcsinh}(ax)}\,dx = \int x\sqrt{\operatorname{asinh}\,(ax)}\,dx$$

```
[In] integrate(x*asinh(a*x)**(1/2),x)
```

```
[Out] Integral(x*sqrt(asinh(a*x)), x)
```

Maxima [F]

$$\int x\sqrt{\operatorname{arcsinh}(ax)}\,dx = \int x\sqrt{\operatorname{arsinh}\,(ax)}\,dx$$

```
[In] integrate(x*arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sqrt(arcsinh(a*x)), x)
```

Giac [F]

$$\int x\sqrt{\operatorname{arcsinh}(ax)}\,dx = \int x\sqrt{\operatorname{arsinh}\left(ax\right)}\,dx$$

```
[In] integrate(x*arcsinh(a*x)^(1/2),x, algorithm="giac")
[Out] integrate(x*sqrt(arcsinh(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{\operatorname{arcsinh}(ax)}\,dx = \int x\,\sqrt{\operatorname{asinh}\left(a\,x\right)}\,dx$$

```
[In] int(x*asinh(a*x)^(1/2),x)
[Out] int(x*asinh(a*x)^(1/2), x)
```

3.78 $\int \sqrt{\text{arcsinh}(ax)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 53

$$\int \sqrt{\text{arcsinh}(ax)}\,dx = x\sqrt{\text{arcsinh}(ax)} + \frac{\sqrt{\pi}\,\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)}{4a} - \frac{\sqrt{\pi}\,\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)}{4a}$$

```
[Out] 1/4*erf(arcsinh(a*x)^(1/2))*Pi^(1/2)/a-1/4*erfi(arcsinh(a*x)^(1/2))*Pi^(1/2
)/a+x*arcsinh(a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {5772, 5819, 3389, 2211, 2235, 2236}

$$\int \sqrt{\text{arcsinh}(ax)}\,dx = \frac{\sqrt{\pi}\,\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)}{4a} - \frac{\sqrt{\pi}\,\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)}{4a} + x\sqrt{\text{arcsinh}(ax)}$$

```
[In] Int[Sqrt[ArcSinh[a*x]],x]
```

```
[Out] x*Sqrt[ArcSinh[a*x]] + (Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]])/(4*a) - (Sqrt[Pi]
*Erfi[Sqrt[ArcSinh[a*x]]])/(4*a)
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= x\sqrt{\operatorname{arcsinh}(ax)} - \frac{1}{2}a\int \frac{x}{\sqrt{1+a^2x^2}\sqrt{\operatorname{arcsinh}(ax)}}\,dx \\
&= x\sqrt{\operatorname{arcsinh}(ax)} - \frac{\operatorname{Subst}\left(\int \frac{\sinh(x)}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{2a} \\
&= x\sqrt{\operatorname{arcsinh}(ax)} + \frac{\operatorname{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{4a} - \frac{\operatorname{Subst}\left(\int \frac{e^{x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{4a} \\
&= x\sqrt{\operatorname{arcsinh}(ax)} + \frac{\operatorname{Subst}\left(\int e^{-x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{2a} - \frac{\operatorname{Subst}\left(\int e^{x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{2a} \\
&= x\sqrt{\operatorname{arcsinh}(ax)} + \frac{\sqrt{\pi}\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{4a} - \frac{\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{4a}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int \sqrt{\operatorname{arcsinh}(ax)}\,dx = -\frac{\frac{\sqrt{-\operatorname{arcsinh}(ax)}\,\Gamma\left(\frac{3}{2},-\operatorname{arcsinh}(ax)\right)}{\sqrt{\operatorname{arcsinh}(ax)}}+\Gamma\left(\frac{3}{2},\operatorname{arcsinh}(ax)\right)}{2a}$$

```
[In] Integrate[Sqrt[ArcSinh[a*x]],x]
```

```
[Out] -1/2*((Sqrt[-ArcSinh[a*x]]*Gamma[3/2, -ArcSinh[a*x]])/Sqrt[ArcSinh[a*x]] +
Gamma[3/2, ArcSinh[a*x]])/a
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{4\sqrt{\operatorname{arcsinh}(ax)}\,\sqrt{\pi}\,ax+\pi\,\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)-\pi\,\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{4\sqrt{\pi}\,a}$	42

```
[In] int(arcsinh(a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(4*arcsinh(a*x)^(1/2)*Pi^(1/2)*a*x+Pi*erf(arcsinh(a*x)^(1/2))-Pi*erfi(a
rcsinh(a*x)^(1/2)))/Pi^(1/2)/a
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{\operatorname{arcsinh}(ax)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \sqrt{\operatorname{arcsinh}(ax)}\,dx = \int \sqrt{\operatorname{asinh}{\left(ax \right)}}\,dx$$

```
[In] integrate(asinh(a*x)**(1/2),x)
[Out] Integral(sqrt(asinh(a*x)), x)
```

Maxima [F]

$$\int \sqrt{\operatorname{arcsinh}(ax)}\,dx = \int \sqrt{\operatorname{arsinh}\left(ax\right)}\,dx$$

```
[In] integrate(arcsinh(a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(arcsinh(a*x)), x)
```

Giac [F]

$$\int \sqrt{\operatorname{arcsinh}(ax)}\,dx = \int \sqrt{\operatorname{arsinh}\left(ax\right)}\,dx$$

```
[In] integrate(arcsinh(a*x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(arcsinh(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\operatorname{arcsinh}(ax)}\,dx = \int \sqrt{\operatorname{asinh}\left(a\,x\right)}\,dx$$

```
[In] int(asinh(a*x)^(1/2),x)
[Out] int(asinh(a*x)^(1/2), x)
```

3.79 $\int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{x}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{x}\,dx = \operatorname{Int}\left(\frac{\sqrt{\operatorname{arcsinh}(ax)}}{x},x\right)$$

```
[Out] Unintegrable(arcsinh(a*x)^(1/2)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{x}\,dx = \int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{x}\,dx$$

```
[In] Int[Sqrt[ArcSinh[a*x]]/x,x]
```

```
[Out] Defer[Int][Sqrt[ArcSinh[a*x]]/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{x}\,dx = \int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{x}\,dx$$

```
[In] Integrate[Sqrt[ArcSinh[a*x]]/x,x]
[Out] Integrate[Sqrt[ArcSinh[a*x]]/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{\operatorname{arcsinh}\left(ax\right)}}{x}dx$$

```
[In] int(arcsinh(a*x)^(1/2)/x,x)
[Out] int(arcsinh(a*x)^(1/2)/x,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{x}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(arcsinh(a*x)^(1/2)/x,x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{x}\,dx = \int \frac{\sqrt{\operatorname{asinh}\left(ax\right)}}{x}\,dx$$

```
[In] integrate(asinh(a*x)**(1/2)/x,x)
[Out] Integral(sqrt(asinh(a*x))/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{x}\,dx = \int \frac{\sqrt{\operatorname{arsinh}\left(ax\right)}}{x}\,dx$$

```
[In] integrate(arcsinh(a*x)^(1/2)/x,x, algorithm="maxima")
[Out] integrate(sqrt(arcsinh(a*x))/x, x)
```

Giac [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{x}\,dx = \int \frac{\sqrt{\operatorname{arsinh}\left(ax\right)}}{x}\,dx$$

```
[In] integrate(arcsinh(a*x)^(1/2)/x,x, algorithm="giac")
[Out] integrate(sqrt(arcsinh(a*x))/x, x)
```

Mupad [N/A]

Not integrable

Time = 2.50 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\operatorname{arcsinh}(ax)}}{x}\,dx = \int \frac{\sqrt{\operatorname{asinh}\left(a\,x\right)}}{x}\,dx$$

```
[In] int(asinh(a*x)^(1/2)/x,x)
[Out] int(asinh(a*x)^(1/2)/x, x)
```

3.80 $\int x^4 \text{arcsinh}(ax)^{3/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 330

$$
\begin{aligned}
\int x^4 \text{arcsinh}(ax)^{3/2}\,dx &= -\frac{4\sqrt{1+a^2x^2}\sqrt{\text{arcsinh}(ax)}}{25a^5} \\
&+ \frac{2x^2\sqrt{1+a^2x^2}\sqrt{\text{arcsinh}(ax)}}{25a^3} - \frac{3x^4\sqrt{1+a^2x^2}\sqrt{\text{arcsinh}(ax)}}{50a} \\
&+ \frac{1}{5}x^5\text{arcsinh}(ax)^{3/2} + \frac{3\sqrt{\pi}\,\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)}{64a^5} - \frac{\sqrt{\frac{\pi}{3}}\,\text{erf}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)}{200a^5} \\
&- \frac{3\sqrt{3\pi}\,\text{erf}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)}{3200a^5} + \frac{3\sqrt{\frac{\pi}{5}}\,\text{erf}\left(\sqrt{5}\sqrt{\text{arcsinh}(ax)}\right)}{3200a^5} \\
&+ \frac{3\sqrt{\pi}\,\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)}{64a^5} - \frac{\sqrt{\frac{\pi}{3}}\,\text{erfi}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)}{200a^5} \\
&- \frac{3\sqrt{3\pi}\,\text{erfi}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)}{3200a^5} + \frac{3\sqrt{\frac{\pi}{5}}\,\text{erfi}\left(\sqrt{5}\sqrt{\text{arcsinh}(ax)}\right)}{3200a^5}
\end{aligned}
$$

```
[Out] 1/5*x^5*arcsinh(a*x)^(3/2)+3/16000*erf(5^(1/2)*arcsinh(a*x)^(1/2))*5^(1/2)*
Pi^(1/2)/a^5+3/16000*erfi(5^(1/2)*arcsinh(a*x)^(1/2))*5^(1/2)*Pi^(1/2)/a^5-
1/384*erf(3^(1/2)*arcsinh(a*x)^(1/2))*3^(1/2)*Pi^(1/2)/a^5-1/384*erfi(3^(1/
2)*arcsinh(a*x)^(1/2))*3^(1/2)*Pi^(1/2)/a^5+3/64*erf(arcsinh(a*x)^(1/2))*Pi
^(1/2)/a^5+3/64*erfi(arcsinh(a*x)^(1/2))*Pi^(1/2)/a^5-4/25*(a^2*x^2+1)^(1/2
)*arcsinh(a*x)^(1/2)/a^5+2/25*x^2*(a^2*x^2+1)^(1/2)*arcsinh(a*x)^(1/2)/a^3-
3/50*x^4*(a^2*x^2+1)^(1/2)*arcsinh(a*x)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 41, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {5777, 5812, 5798, 5774, 3388, 2211, 2235, 2236, 5780, 5556}

$$\begin{aligned}\int x^4\text{arcsinh}(ax)^{3/2}\,dx =& \frac{3\sqrt{\pi}\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)}{64a^5} - \frac{3\sqrt{3\pi}\text{erf}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)}{3200a^5} \\ &- \frac{\sqrt{\frac{\pi}{3}}\text{erf}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)}{200a^5} + \frac{3\sqrt{\frac{\pi}{5}}\text{erf}\left(\sqrt{5}\sqrt{\text{arcsinh}(ax)}\right)}{3200a^5} \\ &+ \frac{3\sqrt{\pi}\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)}{64a^5} - \frac{3\sqrt{3\pi}\text{erfi}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)}{3200a^5} \\ &- \frac{\sqrt{\frac{\pi}{3}}\text{erfi}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)}{200a^5} + \frac{3\sqrt{\frac{\pi}{5}}\text{erfi}\left(\sqrt{5}\sqrt{\text{arcsinh}(ax)}\right)}{3200a^5} \\ &- \frac{3x^4\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}{50a} - \frac{4\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}{25a^5} \\ &+ \frac{2x^2\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}{25a^3} + \frac{1}{5}x^5\text{arcsinh}(ax)^{3/2}\end{aligned}$$

```
[In] Int[x^4*ArcSinh[a*x]^(3/2),x]
```

```
[Out] (-4*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])/(25*a^5) + (2*x^2*Sqrt[1 + a^2*x^
2]*Sqrt[ArcSinh[a*x]])/(25*a^3) - (3*x^4*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x
]])/(50*a) + (x^5*ArcSinh[a*x]^(3/2))/5 + (3*Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]
]])/(64*a^5) - (Sqrt[Pi/3]*Erf[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(200*a^5) - (3*
Sqrt[3*Pi]*Erf[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(3200*a^5) + (3*Sqrt[Pi/5]*Erf[
Sqrt[5]*Sqrt[ArcSinh[a*x]]])/(3200*a^5) + (3*Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x
]]])/(64*a^5) - (Sqrt[Pi/3]*Erfi[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(200*a^5) - (
3*Sqrt[3*Pi]*Erfi[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(3200*a^5) + (3*Sqrt[Pi/5]*E
rfi[Sqrt[5]*Sqrt[ArcSinh[a*x]]])/(3200*a^5)
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{5}x^5\text{arcsinh}(ax)^{3/2} - \frac{1}{10}(3a)\int \frac{x^5\sqrt{\text{arcsinh}(ax)}}{\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{3x^4\sqrt{1+a^2x^2}\sqrt{\text{arcsinh}(ax)}}{50a} + \frac{1}{5}x^5\text{arcsinh}(ax)^{3/2} \\
&\quad + \frac{3}{100}\int \frac{x^4}{\sqrt{\text{arcsinh}(ax)}}\,dx + \frac{6\int \frac{x^3\sqrt{\text{arcsinh}(ax)}}{\sqrt{1+a^2x^2}}\,dx}{25a} \\
&= \frac{2x^2\sqrt{1+a^2x^2}\sqrt{\text{arcsinh}(ax)}}{25a^3} - \frac{3x^4\sqrt{1+a^2x^2}\sqrt{\text{arcsinh}(ax)}}{50a} + \frac{1}{5}x^5\text{arcsinh}(ax)^{3/2} \\
&\quad + \frac{3\text{Subst}\left(\int \frac{\cosh(x)\sinh^4(x)}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{100a^5} - \frac{4\int \frac{x\sqrt{\text{arcsinh}(ax)}}{\sqrt{1+a^2x^2}}\,dx}{25a^3} - \frac{\int \frac{x^2}{\sqrt{\text{arcsinh}(ax)}}\,dx}{25a^2} \\
&= -\frac{4\sqrt{1+a^2x^2}\sqrt{\text{arcsinh}(ax)}}{25a^5} + \frac{2x^2\sqrt{1+a^2x^2}\sqrt{\text{arcsinh}(ax)}}{25a^3} \\
&\quad - \frac{3x^4\sqrt{1+a^2x^2}\sqrt{\text{arcsinh}(ax)}}{50a} + \frac{1}{5}x^5\text{arcsinh}(ax)^{3/2} \\
&\quad + \frac{3\text{Subst}\left(\int \left(\frac{\cosh(x)}{8\sqrt{x}} - \frac{3\cosh(3x)}{16\sqrt{x}} + \frac{\cosh(5x)}{16\sqrt{x}}\right)\,dx, x, \text{arcsinh}(ax)\right)}{100a^5} \\
&\quad - \frac{\text{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{25a^5} + \frac{2\int \frac{1}{\sqrt{\text{arcsinh}(ax)}}\,dx}{25a^4}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{4\sqrt{1+a^2x^2}\sqrt{\operatorname{arcsinh}(ax)}}{25a^5} + \frac{2x^2\sqrt{1+a^2x^2}\sqrt{\operatorname{arcsinh}(ax)}}{25a^3} \\
&\quad - \frac{3x^4\sqrt{1+a^2x^2}\sqrt{\operatorname{arcsinh}(ax)}}{50a} + \frac{1}{5}x^5\operatorname{arcsinh}(ax)^{3/2} \\
&\quad + \frac{3\text{Subst}\left(\int \frac{\cosh(5x)}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{1600a^5} \\
&\quad + \frac{3\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{800a^5} - \frac{9\text{Subst}\left(\int \frac{\cosh(3x)}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{1600a^5} \\
&\quad - \frac{\text{Subst}\left(\int \left(-\frac{\cosh(x)}{4\sqrt{x}} + \frac{\cosh(3x)}{4\sqrt{x}}\right)\,dx, x, \operatorname{arcsinh}(ax)\right)}{25a^5} \\
&\quad + \frac{2\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{25a^5}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{4\sqrt{1+a^2x^2}\sqrt{\operatorname{arcsinh}(ax)}}{25a^5} + \frac{2x^2\sqrt{1+a^2x^2}\sqrt{\operatorname{arcsinh}(ax)}}{25a^3} \\
&\quad - \frac{3x^4\sqrt{1+a^2x^2}\sqrt{\operatorname{arcsinh}(ax)}}{50a} + \frac{1}{5}x^5\operatorname{arcsinh}(ax)^{3/2} \\
&\quad + \frac{3\text{Subst}\left(\int \frac{e^{-5x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{3200a^5} + \frac{3\text{Subst}\left(\int \frac{e^{5x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{3200a^5} \\
&\quad + \frac{3\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{1600a^5} + \frac{3\text{Subst}\left(\int \frac{e^{x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{1600a^5} \\
&\quad - \frac{9\text{Subst}\left(\int \frac{e^{-3x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{3200a^5} - \frac{9\text{Subst}\left(\int \frac{e^{3x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{3200a^5} \\
&\quad + \frac{\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{100a^5} - \frac{\text{Subst}\left(\int \frac{\cosh(3x)}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{100a^5} \\
&\quad + \frac{\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{25a^5} + \frac{\text{Subst}\left(\int \frac{e^{x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{25a^5}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{4\sqrt{1+a^2x^2}\sqrt{\operatorname{arcsinh}(ax)}}{25a^5}+\frac{2x^2\sqrt{1+a^2x^2}\sqrt{\operatorname{arcsinh}(ax)}}{25a^3} \\
&\quad -\frac{3x^4\sqrt{1+a^2x^2}\sqrt{\operatorname{arcsinh}(ax)}}{50a}+\frac{1}{5}x^5\operatorname{arcsinh}(ax)^{3/2} \\
&\quad +\frac{3\text{Subst}\left(\int e^{-5x^2}\,dx,x,\sqrt{\operatorname{arcsinh}(ax)}\right)}{1600a^5}+\frac{3\text{Subst}\left(\int e^{5x^2}\,dx,x,\sqrt{\operatorname{arcsinh}(ax)}\right)}{1600a^5} \\
&\quad +\frac{3\text{Subst}\left(\int e^{-x^2}\,dx,x,\sqrt{\operatorname{arcsinh}(ax)}\right)}{800a^5}+\frac{3\text{Subst}\left(\int e^{x^2}\,dx,x,\sqrt{\operatorname{arcsinh}(ax)}\right)}{800a^5} \\
&\quad -\frac{\text{Subst}\left(\int \frac{e^{-3x}}{\sqrt{x}}\,dx,x,\operatorname{arcsinh}(ax)\right)}{200a^5}+\frac{\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}}\,dx,x,\operatorname{arcsinh}(ax)\right)}{200a^5} \\
&\quad +\frac{\text{Subst}\left(\int \frac{e^{x}}{\sqrt{x}}\,dx,x,\operatorname{arcsinh}(ax)\right)}{200a^5}-\frac{\text{Subst}\left(\int \frac{e^{3x}}{\sqrt{x}}\,dx,x,\operatorname{arcsinh}(ax)\right)}{200a^5} \\
&\quad -\frac{9\text{Subst}\left(\int e^{-3x^2}\,dx,x,\sqrt{\operatorname{arcsinh}(ax)}\right)}{1600a^5}-\frac{9\text{Subst}\left(\int e^{3x^2}\,dx,x,\sqrt{\operatorname{arcsinh}(ax)}\right)}{1600a^5} \\
&\quad +\frac{2\text{Subst}\left(\int e^{-x^2}\,dx,x,\sqrt{\operatorname{arcsinh}(ax)}\right)}{25a^5}+\frac{2\text{Subst}\left(\int e^{x^2}\,dx,x,\sqrt{\operatorname{arcsinh}(ax)}\right)}{25a^5}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{4\sqrt{1+a^2x^2}\sqrt{\operatorname{arcsinh}(ax)}}{25a^5}+\frac{2x^2\sqrt{1+a^2x^2}\sqrt{\operatorname{arcsinh}(ax)}}{25a^3} \\
&\quad -\frac{3x^4\sqrt{1+a^2x^2}\sqrt{\operatorname{arcsinh}(ax)}}{50a}+\frac{1}{5}x^5\operatorname{arcsinh}(ax)^{3/2} \\
&\quad +\frac{67\sqrt{\pi}\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{1600a^5}-\frac{3\sqrt{3\pi}\operatorname{erf}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{3200a^5} \\
&\quad +\frac{3\sqrt{\frac{\pi}{5}}\operatorname{erf}\left(\sqrt{5}\sqrt{\operatorname{arcsinh}(ax)}\right)}{3200a^5}+\frac{67\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{1600a^5} \\
&\quad -\frac{3\sqrt{3\pi}\operatorname{erfi}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{3200a^5}+\frac{3\sqrt{\frac{\pi}{5}}\operatorname{erfi}\left(\sqrt{5}\sqrt{\operatorname{arcsinh}(ax)}\right)}{3200a^5} \\
&\quad -\frac{\text{Subst}\left(\int e^{-3x^2}\,dx,x,\sqrt{\operatorname{arcsinh}(ax)}\right)}{100a^5}+\frac{\text{Subst}\left(\int e^{-x^2}\,dx,x,\sqrt{\operatorname{arcsinh}(ax)}\right)}{100a^5} \\
&\quad +\frac{\text{Subst}\left(\int e^{x^2}\,dx,x,\sqrt{\operatorname{arcsinh}(ax)}\right)}{100a^5}-\frac{\text{Subst}\left(\int e^{3x^2}\,dx,x,\sqrt{\operatorname{arcsinh}(ax)}\right)}{100a^5}
\end{aligned}
$$

$$= -\frac{4\sqrt{1+a^2x^2}\sqrt{\operatorname{arcsinh}(ax)}}{25a^5} + \frac{2x^2\sqrt{1+a^2x^2}\sqrt{\operatorname{arcsinh}(ax)}}{25a^3} - \frac{3x^4\sqrt{1+a^2x^2}\sqrt{\operatorname{arcsinh}(ax)}}{50a} + \frac{1}{5}x^5\operatorname{arcsinh}(ax)^{3/2} + \frac{3\sqrt{\pi}\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{64a^5} - \frac{\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{200a^5} - \frac{3\sqrt{3\pi}\operatorname{erf}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{3200a^5} + \frac{3\sqrt{\frac{\pi}{5}}\operatorname{erf}\left(\sqrt{5}\sqrt{\operatorname{arcsinh}(ax)}\right)}{3200a^5} + \frac{3\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{64a^5} - \frac{\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{200a^5} - \frac{3\sqrt{3\pi}\operatorname{erfi}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{3200a^5} + \frac{3\sqrt{\frac{\pi}{5}}\operatorname{erfi}\left(\sqrt{5}\sqrt{\operatorname{arcsinh}(ax)}\right)}{3200a^5}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.46

$$\int x^4\operatorname{arcsinh}(ax)^{3/2}\,dx = \frac{\frac{9\sqrt{5}\sqrt{-\operatorname{arcsinh}(ax)}\,\Gamma\left(\frac{5}{2},-5\operatorname{arcsinh}(ax)\right)}{\sqrt{\operatorname{arcsinh}(ax)}} + \frac{125\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\,\Gamma\left(\frac{5}{2},-3\operatorname{arcsinh}(ax)\right)}{\sqrt{-\operatorname{arcsinh}(ax)}} + 2250\sqrt{-\mathrm{a}}}{}$$

```
[In] Integrate[x^4*ArcSinh[a*x]^(3/2),x]
```

```
[Out] ((9*Sqrt[5]*Sqrt[-ArcSinh[a*x]]*Gamma[5/2, -5*ArcSinh[a*x]])/Sqrt[ArcSinh[a
*x]] + (125*Sqrt[3]*Sqrt[ArcSinh[a*x]]*Gamma[5/2, -3*ArcSinh[a*x]])/Sqrt[-A
rcSinh[a*x]] + (2250*Sqrt[-ArcSinh[a*x]]*Gamma[5/2, -ArcSinh[a*x]])/Sqrt[Ar
cSinh[a*x]] - 2250*Gamma[5/2, ArcSinh[a*x]] + 125*Sqrt[3]*Gamma[5/2, 3*ArcS
inh[a*x]] - 9*Sqrt[5]*Gamma[5/2, 5*ArcSinh[a*x]])/(36000*a^5)
```

Maple [F]

$$\int x^{4} \operatorname{arcsinh}\left(a x\right)^{\frac{3}{2}}d x$$

```
[In] int(x^4*arcsinh(a*x)^(3/2),x)
```

```
[Out] int(x^4*arcsinh(a*x)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^4 \text{arcsinh}(ax)^{3/2} \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^4*arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x^4 \text{arcsinh}(ax)^{3/2} \, dx = \int x^4 \operatorname{asinh}^{\frac{3}{2}}(ax) \, dx$$

```
[In] integrate(x**4*asinh(a*x)**(3/2),x)
```

```
[Out] Integral(x**4*asinh(a*x)**(3/2), x)
```

Maxima [F]

$$\int x^4 \text{arcsinh}(ax)^{3/2} \, dx = \int x^4 \operatorname{arsinh}(ax)^{\frac{3}{2}} \, dx$$

```
[In] integrate(x^4*arcsinh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4*arcsinh(a*x)^(3/2), x)
```

Giac [F]

$$\int x^4 \text{arcsinh}(ax)^{3/2} \, dx = \int x^4 \operatorname{arsinh}(ax)^{\frac{3}{2}} \, dx$$

```
[In] integrate(x^4*arcsinh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^4*arcsinh(a*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \text{arcsinh}(ax)^{3/2}\,dx = \int x^4\,\text{asinh}(a\,x)^{3/2}\,dx$$

```
[In] int(x^4*asinh(a*x)^(3/2),x)
[Out] int(x^4*asinh(a*x)^(3/2), x)
```

3.81 $\int x^3\text{arcsinh}(ax)^{3/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 199

$$\begin{aligned}\int x^3\text{arcsinh}(ax)^{3/2}\,dx &= \frac{9x\sqrt{1+a^2x^2}\sqrt{\text{arcsinh}(ax)}}{64a^3} \\ &- \frac{3x^3\sqrt{1+a^2x^2}\sqrt{\text{arcsinh}(ax)}}{32a} - \frac{3\text{arcsinh}(ax)^{3/2}}{32a^4} \\ &+ \frac{1}{4}x^4\text{arcsinh}(ax)^{3/2} - \frac{3\sqrt{\pi}\,\text{erf}\left(2\sqrt{\text{arcsinh}(ax)}\right)}{2048a^4} + \frac{3\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{128a^4} \\ &+ \frac{3\sqrt{\pi}\,\text{erfi}\left(2\sqrt{\text{arcsinh}(ax)}\right)}{2048a^4} - \frac{3\sqrt{\frac{\pi}{2}}\,\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{128a^4}\end{aligned}$$

```
[Out] -3/32*arcsinh(a*x)^(3/2)/a^4+1/4*x^4*arcsinh(a*x)^(3/2)+3/256*erf(2^(1/2)*a
rcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^4-3/256*erfi(2^(1/2)*arcsinh(a*x)^(1/
2))*2^(1/2)*Pi^(1/2)/a^4-3/2048*erf(2*arcsinh(a*x)^(1/2))*Pi^(1/2)/a^4+3/20
48*erfi(2*arcsinh(a*x)^(1/2))*Pi^(1/2)/a^4+9/64*x*(a^2*x^2+1)^(1/2)*arcsinh
(a*x)^(1/2)/a^3-3/32*x^3*(a^2*x^2+1)^(1/2)*arcsinh(a*x)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules

used = {5777, 5812, 5783, 5780, 5556, 12, 3389, 2211, 2235, 2236}

$$\int x^3\text{arcsinh}(ax)^{3/2}\,dx = -\frac{3\sqrt{\pi}\text{erf}\left(2\sqrt{\text{arcsinh}(ax)}\right)}{2048a^4}+\frac{3\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{128a^4}$$
$$+\frac{3\sqrt{\pi}\text{erfi}\left(2\sqrt{\text{arcsinh}(ax)}\right)}{2048a^4}-\frac{3\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{128a^4}-\frac{3\text{arcsinh}(ax)^{3/2}}{32a^4}$$
$$-\frac{3x^3\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}{32a}+\frac{9x\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}{64a^3}+\frac{1}{4}x^4\text{arcsinh}(ax)^{3/2}$$

```
[In] Int[x^3*ArcSinh[a*x]^(3/2),x]
```

```
[Out] (9*x*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])/(64*a^3) - (3*x^3*Sqrt[1 + a^2*x
^2]*Sqrt[ArcSinh[a*x]])/(32*a) - (3*ArcSinh[a*x]^(3/2))/(32*a^4) + (x^4*Arc
Sinh[a*x]^(3/2))/4 - (3*Sqrt[Pi]*Erf[2*Sqrt[ArcSinh[a*x]]])/(2048*a^4) + (3
*Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(128*a^4) + (3*Sqrt[Pi]*Erfi[2
*Sqrt[ArcSinh[a*x]]])/(2048*a^4) - (3*Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[
a*x]]])/(128*a^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
```

```
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{4}x^4\text{arcsinh}(ax)^{3/2} - \frac{1}{8}(3a)\int \frac{x^4\sqrt{\text{arcsinh}(ax)}}{\sqrt{1+a^2x^2}}\,dx$$

$$= -\frac{3x^3\sqrt{1+a^2x^2}\sqrt{\text{arcsinh}(ax)}}{32a}+\frac{1}{4}x^4\text{arcsinh}(ax)^{3/2} + \frac{3}{64}\int \frac{x^3}{\sqrt{\text{arcsinh}(ax)}}\,dx + \frac{9\int \frac{x^2\sqrt{\text{arcsinh}(ax)}}{\sqrt{1+a^2x^2}}\,dx}{32a}$$

$$= \frac{9x\sqrt{1+a^2x^2}\sqrt{\text{arcsinh}(ax)}}{64a^3} - \frac{3x^3\sqrt{1+a^2x^2}\sqrt{\text{arcsinh}(ax)}}{32a} + \frac{1}{4}x^4\text{arcsinh}(ax)^{3/2} + \frac{3\text{Subst}\left(\int \frac{\cosh(x)\sinh^3(x)}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{64a^4} - \frac{9\int \frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{1+a^2x^2}}\,dx}{64a^3} - \frac{9\int \frac{x}{\sqrt{\text{arcsinh}(ax)}}\,dx}{128a^2}$$

$$= \frac{9x\sqrt{1+a^2x^2}\sqrt{\text{arcsinh}(ax)}}{64a^3} - \frac{3x^3\sqrt{1+a^2x^2}\sqrt{\text{arcsinh}(ax)}}{32a} - \frac{3\text{arcsinh}(ax)^{3/2}}{32a^4} + \frac{1}{4}x^4\text{arcsinh}(ax)^{3/2} + \frac{3\text{Subst}\left(\int \left(-\frac{\sinh(2x)}{4\sqrt{x}}+\frac{\sinh(4x)}{8\sqrt{x}}\right)\,dx, x, \text{arcsinh}(ax)\right)}{64a^4} - \frac{9\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{128a^4}$$

$$= \frac{9x\sqrt{1+a^2x^2}\sqrt{\text{arcsinh}(ax)}}{64a^3} - \frac{3x^3\sqrt{1+a^2x^2}\sqrt{\text{arcsinh}(ax)}}{32a} - \frac{3\text{arcsinh}(ax)^{3/2}}{32a^4} + \frac{1}{4}x^4\text{arcsinh}(ax)^{3/2} + \frac{3\text{Subst}\left(\int \frac{\sinh(4x)}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{512a^4} - \frac{3\text{Subst}\left(\int \frac{\sinh(2x)}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{256a^4} - \frac{9\text{Subst}\left(\int \frac{\sinh(2x)}{2\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{128a^4}$$

$$= \frac{9x\sqrt{1+a^2x^2}\sqrt{\text{arcsinh}(ax)}}{64a^3} - \frac{3x^3\sqrt{1+a^2x^2}\sqrt{\text{arcsinh}(ax)}}{32a} - \frac{3\text{arcsinh}(ax)^{3/2}}{32a^4} + \frac{1}{4}x^4\text{arcsinh}(ax)^{3/2} - \frac{3\text{Subst}\left(\int \frac{e^{-4x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{1024a^4} + \frac{3\text{Subst}\left(\int \frac{e^{4x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{1024a^4} + \frac{3\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{512a^4} - \frac{3\text{Subst}\left(\int \frac{e^{2x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{512a^4} - \frac{9\text{Subst}\left(\int \frac{\sinh(2x)}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{256a^4}$$

$$
\begin{aligned}
&= \frac{9x\sqrt{1+a^2x^2}\sqrt{\operatorname{arcsinh}(ax)}}{64a^3} - \frac{3x^3\sqrt{1+a^2x^2}\sqrt{\operatorname{arcsinh}(ax)}}{32a} - \frac{3\operatorname{arcsinh}(ax)^{3/2}}{32a^4} \\
&\quad + \frac{1}{4}x^4\operatorname{arcsinh}(ax)^{3/2} - \frac{3\text{Subst}\left(\int e^{-4x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{512a^4} \\
&\quad + \frac{3\text{Subst}\left(\int e^{4x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{512a^4} + \frac{3\text{Subst}\left(\int e^{-2x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{256a^4} \\
&\quad - \frac{3\text{Subst}\left(\int e^{2x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{256a^4} + \frac{9\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{512a^4} \\
&\quad - \frac{9\text{Subst}\left(\int \frac{e^{2x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{512a^4}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{9x\sqrt{1+a^2x^2}\sqrt{\operatorname{arcsinh}(ax)}}{64a^3} - \frac{3x^3\sqrt{1+a^2x^2}\sqrt{\operatorname{arcsinh}(ax)}}{32a} - \frac{3\operatorname{arcsinh}(ax)^{3/2}}{32a^4} \\
&\quad + \frac{1}{4}x^4\operatorname{arcsinh}(ax)^{3/2} - \frac{3\sqrt{\pi}\operatorname{erf}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{2048a^4} + \frac{3\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{512a^4} \\
&\quad + \frac{3\sqrt{\pi}\operatorname{erfi}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{2048a^4} - \frac{3\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{512a^4} \\
&\quad + \frac{9\text{Subst}\left(\int e^{-2x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{256a^4} - \frac{9\text{Subst}\left(\int e^{2x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{256a^4}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{9x\sqrt{1+a^2x^2}\sqrt{\operatorname{arcsinh}(ax)}}{64a^3} - \frac{3x^3\sqrt{1+a^2x^2}\sqrt{\operatorname{arcsinh}(ax)}}{32a} - \frac{3\operatorname{arcsinh}(ax)^{3/2}}{32a^4} \\
&\quad + \frac{1}{4}x^4\operatorname{arcsinh}(ax)^{3/2} - \frac{3\sqrt{\pi}\operatorname{erf}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{2048a^4} + \frac{3\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{128a^4} \\
&\quad + \frac{3\sqrt{\pi}\operatorname{erfi}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{2048a^4} - \frac{3\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{128a^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.50

$$
\int x^3\operatorname{arcsinh}(ax)^{3/2}\,dx = \frac{\frac{\sqrt{-\operatorname{arcsinh}(ax)}\Gamma\left(\frac{5}{2},-4\operatorname{arcsinh}(ax)\right)}{\sqrt{\operatorname{arcsinh}(ax)}} + \frac{8\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\Gamma\left(\frac{5}{2},-2\operatorname{arcsinh}(ax)\right)}{\sqrt{-\operatorname{arcsinh}(ax)}} - 8\sqrt{2}\Gamma\left(\frac{5}{2},2a\right.}{512a^4}
$$

```
[In] Integrate[x^3*ArcSinh[a*x]^(3/2),x]
```

```
[Out] ((Sqrt[-ArcSinh[a*x]]*Gamma[5/2, -4*ArcSinh[a*x]])/Sqrt[ArcSinh[a*x]] + (8*
Sqrt[2]*Sqrt[ArcSinh[a*x]]*Gamma[5/2, -2*ArcSinh[a*x]])/Sqrt[-ArcSinh[a*x]]
 - 8*Sqrt[2]*Gamma[5/2, 2*ArcSinh[a*x]] + Gamma[5/2, 4*ArcSinh[a*x]])/(512*
a^4)
```

Maple [F]

$$\int x^{3} \operatorname{arcsinh}\left(a x\right)^{\frac{3}{2}}d x$$

```
[In] int(x^3*arcsinh(a*x)^(3/2),x)
```

```
[Out] int(x^3*arcsinh(a*x)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^3 \text{arcsinh}(ax)^{3/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3*arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x^3 \text{arcsinh}(ax)^{3/2}\,dx = \int x^{3} \operatorname{asinh}^{\frac{3}{2}}{\left(a x \right)}\, dx$$

```
[In] integrate(x**3*asinh(a*x)**(3/2),x)
```

```
[Out] Integral(x**3*asinh(a*x)**(3/2), x)
```

Maxima [F]

$$\int x^3 \text{arcsinh}(ax)^{3/2}\,dx = \int { x^{3} \operatorname{arsinh}\left(a x\right)^{\frac{3}{2}}\,{d x}}$$

```
[In] integrate(x^3*arcsinh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3*arcsinh(a*x)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int x^3 \text{arcsinh}(ax)^{3/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3*arcsinh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \text{arcsinh}(ax)^{3/2}\,dx = \int x^3\,\text{asinh}(a\,x)^{3/2}\,dx$$

```
[In] int(x^3*asinh(a*x)^(3/2),x)
```

```
[Out] int(x^3*asinh(a*x)^(3/2), x)
```

3.82 $\int x^2\textbf{arcsinh}(ax)^{3/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 179

$$\begin{aligned}\int x^2\operatorname{arcsinh}(ax)^{3/2}\,dx &= \frac{\sqrt{1+a^2x^2}\sqrt{\operatorname{arcsinh}(ax)}}{3a^3}-\frac{x^2\sqrt{1+a^2x^2}\sqrt{\operatorname{arcsinh}(ax)}}{6a} \\ &+\frac{1}{3}x^3\operatorname{arcsinh}(ax)^{3/2}-\frac{3\sqrt{\pi}\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{32a^3}+\frac{\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{96a^3} \\ &-\frac{3\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{32a^3}+\frac{\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{96a^3}\end{aligned}$$

```
[Out] 1/3*x^3*arcsinh(a*x)^(3/2)+1/288*erf(3^(1/2)*arcsinh(a*x)^(1/2))*3^(1/2)*Pi
^(1/2)/a^3+1/288*erfi(3^(1/2)*arcsinh(a*x)^(1/2))*3^(1/2)*Pi^(1/2)/a^3-3/32
*erf(arcsinh(a*x)^(1/2))*Pi^(1/2)/a^3-3/32*erfi(arcsinh(a*x)^(1/2))*Pi^(1/2
)/a^3+1/3*(a^2*x^2+1)^(1/2)*arcsinh(a*x)^(1/2)/a^3-1/6*x^2*(a^2*x^2+1)^(1/2
)*arcsinh(a*x)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {5777, 5812, 5798, 5774, 3388, 2211, 2235, 2236, 5780, 5556}

$$\begin{aligned}\int x^2\operatorname{arcsinh}(ax)^{3/2}\,dx &= -\frac{3\sqrt{\pi}\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{32a^3}+\frac{\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{96a^3} \\ &-\frac{3\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{32a^3}+\frac{\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{96a^3} \\ &-\frac{x^2\sqrt{a^2x^2+1}\sqrt{\operatorname{arcsinh}(ax)}}{6a}+\frac{\sqrt{a^2x^2+1}\sqrt{\operatorname{arcsinh}(ax)}}{3a^3}+\frac{1}{3}x^3\operatorname{arcsinh}(ax)^{3/2}\end{aligned}$$

```
[In] Int[x^2*ArcSinh[a*x]^(3/2),x]
```

```
[Out] (Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])/(3*a^3) - (x^2*Sqrt[1 + a^2*x^2]*Sqr
t[ArcSinh[a*x]])/(6*a) + (x^3*ArcSinh[a*x]^(3/2))/3 - (3*Sqrt[Pi]*Erf[Sqrt[
ArcSinh[a*x]]])/(32*a^3) + (Sqrt[Pi/3]*Erf[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(96
*a^3) - (3*Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x]]])/(32*a^3) + (Sqrt[Pi/3]*Erfi[S
qrt[3]*Sqrt[ArcSinh[a*x]]])/(96*a^3)
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\text{arcsinh}(ax)^{3/2} - \frac{1}{2}a\int \frac{x^3\sqrt{\text{arcsinh}(ax)}}{\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{x^2\sqrt{1+a^2x^2}\sqrt{\text{arcsinh}(ax)}}{6a} + \frac{1}{3}x^3\text{arcsinh}(ax)^{3/2} \\
&\quad + \frac{1}{12}\int \frac{x^2}{\sqrt{\text{arcsinh}(ax)}}\,dx + \frac{\int \frac{x\sqrt{\text{arcsinh}(ax)}}{\sqrt{1+a^2x^2}}\,dx}{3a} \\
&= \frac{\sqrt{1+a^2x^2}\sqrt{\text{arcsinh}(ax)}}{3a^3} - \frac{x^2\sqrt{1+a^2x^2}\sqrt{\text{arcsinh}(ax)}}{6a} + \frac{1}{3}x^3\text{arcsinh}(ax)^{3/2} \\
&\quad + \frac{\text{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{12a^3} - \frac{\int \frac{1}{\sqrt{\text{arcsinh}(ax)}}\,dx}{6a^2}
\end{aligned}
$$

$$= \frac{\sqrt{1+a^2x^2}\sqrt{\operatorname{arcsinh}(ax)}}{3a^3} - \frac{x^2\sqrt{1+a^2x^2}\sqrt{\operatorname{arcsinh}(ax)}}{6a} + \frac{1}{3}x^3\operatorname{arcsinh}(ax)^{3/2} + \frac{\text{Subst}\left(\int \left(-\frac{\cosh(x)}{4\sqrt{x}} + \frac{\cosh(3x)}{4\sqrt{x}}\right) dx, x, \operatorname{arcsinh}(ax)\right)}{12a^3} - \frac{\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{x}}\, dx, x, \operatorname{arcsinh}(ax)\right)}{6a^3}$$

$$= \frac{\sqrt{1+a^2x^2}\sqrt{\operatorname{arcsinh}(ax)}}{3a^3} - \frac{x^2\sqrt{1+a^2x^2}\sqrt{\operatorname{arcsinh}(ax)}}{6a} + \frac{1}{3}x^3\operatorname{arcsinh}(ax)^{3/2} - \frac{\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{x}}\, dx, x, \operatorname{arcsinh}(ax)\right)}{48a^3} + \frac{\text{Subst}\left(\int \frac{\cosh(3x)}{\sqrt{x}}\, dx, x, \operatorname{arcsinh}(ax)\right)}{48a^3} - \frac{\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}}\, dx, x, \operatorname{arcsinh}(ax)\right)}{12a^3} - \frac{\text{Subst}\left(\int \frac{e^{x}}{\sqrt{x}}\, dx, x, \operatorname{arcsinh}(ax)\right)}{12a^3}$$

$$= \frac{\sqrt{1+a^2x^2}\sqrt{\operatorname{arcsinh}(ax)}}{3a^3} - \frac{x^2\sqrt{1+a^2x^2}\sqrt{\operatorname{arcsinh}(ax)}}{6a} + \frac{1}{3}x^3\operatorname{arcsinh}(ax)^{3/2} + \frac{\text{Subst}\left(\int \frac{e^{-3x}}{\sqrt{x}}\, dx, x, \operatorname{arcsinh}(ax)\right)}{96a^3} - \frac{\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}}\, dx, x, \operatorname{arcsinh}(ax)\right)}{96a^3} - \frac{\text{Subst}\left(\int \frac{e^{x}}{\sqrt{x}}\, dx, x, \operatorname{arcsinh}(ax)\right)}{96a^3} + \frac{\text{Subst}\left(\int \frac{e^{3x}}{\sqrt{x}}\, dx, x, \operatorname{arcsinh}(ax)\right)}{96a^3} - \frac{\text{Subst}\left(\int e^{-x^2}\, dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{6a^3} - \frac{\text{Subst}\left(\int e^{x^2}\, dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{6a^3}$$

$$= \frac{\sqrt{1+a^2x^2}\sqrt{\operatorname{arcsinh}(ax)}}{3a^3} - \frac{x^2\sqrt{1+a^2x^2}\sqrt{\operatorname{arcsinh}(ax)}}{6a} + \frac{1}{3}x^3\operatorname{arcsinh}(ax)^{3/2} - \frac{\sqrt{\pi}\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{12a^3} - \frac{\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{12a^3} + \frac{\text{Subst}\left(\int e^{-3x^2}\, dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{48a^3} - \frac{\text{Subst}\left(\int e^{-x^2}\, dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{48a^3} - \frac{\text{Subst}\left(\int e^{x^2}\, dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{48a^3} + \frac{\text{Subst}\left(\int e^{3x^2}\, dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{48a^3}$$

$$= \frac{\sqrt{1+a^2x^2}\sqrt{\operatorname{arcsinh}(ax)}}{3a^3} - \frac{x^2\sqrt{1+a^2x^2}\sqrt{\operatorname{arcsinh}(ax)}}{6a} + \frac{1}{3}x^3\operatorname{arcsinh}(ax)^{3/2} - \frac{3\sqrt{\pi}\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{32a^3} + \frac{\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{96a^3} - \frac{3\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{32a^3} + \frac{\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{96a^3}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.55

$$\int x^2\text{arcsinh}(ax)^{3/2}\,dx = \frac{\frac{\sqrt{3}\sqrt{-\text{arcsinh}(ax)}\Gamma\left(\frac{5}{2},-3\text{arcsinh}(ax)\right)}{\sqrt{\text{arcsinh}(ax)}}+\frac{27\sqrt{\text{arcsinh}(ax)}\Gamma\left(\frac{5}{2},-\text{arcsinh}(ax)\right)}{\sqrt{-\text{arcsinh}(ax)}}+27\Gamma\left(\frac{5}{2},\text{arcsin}\right.}{216a^3}$$

```
[In] Integrate[x^2*ArcSinh[a*x]^(3/2),x]
```

```
[Out] ((Sqrt[3]*Sqrt[-ArcSinh[a*x]]*Gamma[5/2, -3*ArcSinh[a*x]])/Sqrt[ArcSinh[a*x
]] + (27*Sqrt[ArcSinh[a*x]]*Gamma[5/2, -ArcSinh[a*x]])/Sqrt[-ArcSinh[a*x]]
+ 27*Gamma[5/2, ArcSinh[a*x]] - Sqrt[3]*Gamma[5/2, 3*ArcSinh[a*x]])/(216*a^
3)
```

Maple [F]

$$\int x^2 \operatorname{arcsinh}(ax)^{\frac{3}{2}}\,dx$$

```
[In] int(x^2*arcsinh(a*x)^(3/2),x)
```

```
[Out] int(x^2*arcsinh(a*x)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^2\text{arcsinh}(ax)^{3/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2*arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x^2\text{arcsinh}(ax)^{3/2}\,dx = \int x^2 \operatorname{asinh}^{\frac{3}{2}}(ax)\,dx$$

```
[In] integrate(x**2*asinh(a*x)**(3/2),x)
```

```
[Out] Integral(x**2*asinh(a*x)**(3/2), x)
```

Maxima [F]

$$\int x^2\text{arcsinh}(ax)^{3/2}\,dx = \int x^2\,\text{arsinh}\,(ax)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x^2*arcsinh(a*x)^(3/2),x, algorithm="maxima")
[Out] integrate(x^2*arcsinh(a*x)^(3/2), x)
```

Giac [F]

$$\int x^2\text{arcsinh}(ax)^{3/2}\,dx = \int x^2\,\text{arsinh}\,(ax)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x^2*arcsinh(a*x)^(3/2),x, algorithm="giac")
[Out] integrate(x^2*arcsinh(a*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\text{arcsinh}(ax)^{3/2}\,dx = \int x^2\,\text{asinh}(a\,x)^{3/2}\,dx$$

```
[In] int(x^2*asinh(a*x)^(3/2),x)
[Out] int(x^2*asinh(a*x)^(3/2), x)
```

3.83 $\int x \text{arcsinh}(ax)^{3/2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 122

$$\int x \text{arcsinh}(ax)^{3/2}\,dx = -\frac{3x\sqrt{1+a^2x^2}\sqrt{\text{arcsinh}(ax)}}{8a}+\frac{\text{arcsinh}(ax)^{3/2}}{4a^2} + \frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}-\frac{3\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{64a^2}+\frac{3\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{64a^2}$$

```
[Out] 1/4*arcsinh(a*x)^(3/2)/a^2+1/2*x^2*arcsinh(a*x)^(3/2)-3/128*erf(2^(1/2)*arc
sinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^2+3/128*erfi(2^(1/2)*arcsinh(a*x)^(1/2)
)*2^(1/2)*Pi^(1/2)/a^2-3/8*x*(a^2*x^2+1)^(1/2)*arcsinh(a*x)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5777, 5812, 5783, 5780, 5556, 12, 3389, 2211, 2235, 2236}

$$\int x \text{arcsinh}(ax)^{3/2}\,dx = -\frac{3\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{64a^2}+\frac{3\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{64a^2} - \frac{3x\sqrt{a^2x^2+1}\sqrt{\text{arcsinh}(ax)}}{8a}+\frac{\text{arcsinh}(ax)^{3/2}}{4a^2}+\frac{1}{2}x^2\text{arcsinh}(ax)^{3/2}$$

```
[In] Int[x*ArcSinh[a*x]^(3/2),x]
```

```
[Out] (-3*x*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])/(8*a) + ArcSinh[a*x]^(3/2)/(4*a
^2) + (x^2*ArcSinh[a*x]^(3/2))/2 - (3*Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a
*x]]])/(64*a^2) + (3*Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(64*a^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}x^2\text{arcsinh}(ax)^{3/2} - \frac{1}{4}(3a)\int \frac{x^2\sqrt{\text{arcsinh}(ax)}}{\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{3x\sqrt{1+a^2x^2}\sqrt{\text{arcsinh}(ax)}}{8a} + \frac{1}{2}x^2\text{arcsinh}(ax)^{3/2} \\
&\quad + \frac{3}{16}\int \frac{x}{\sqrt{\text{arcsinh}(ax)}}\,dx + \frac{3\int \frac{\sqrt{\text{arcsinh}(ax)}}{\sqrt{1+a^2x^2}}\,dx}{8a} \\
&= -\frac{3x\sqrt{1+a^2x^2}\sqrt{\text{arcsinh}(ax)}}{8a} + \frac{\text{arcsinh}(ax)^{3/2}}{4a^2} \\
&\quad + \frac{1}{2}x^2\text{arcsinh}(ax)^{3/2} + \frac{3\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{16a^2} \\
&= -\frac{3x\sqrt{1+a^2x^2}\sqrt{\text{arcsinh}(ax)}}{8a} + \frac{\text{arcsinh}(ax)^{3/2}}{4a^2} \\
&\quad + \frac{1}{2}x^2\text{arcsinh}(ax)^{3/2} + \frac{3\text{Subst}\left(\int \frac{\sinh(2x)}{2\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{16a^2} \\
&= -\frac{3x\sqrt{1+a^2x^2}\sqrt{\text{arcsinh}(ax)}}{8a} + \frac{\text{arcsinh}(ax)^{3/2}}{4a^2} \\
&\quad + \frac{1}{2}x^2\text{arcsinh}(ax)^{3/2} + \frac{3\text{Subst}\left(\int \frac{\sinh(2x)}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{32a^2}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{3x\sqrt{1+a^2x^2}\sqrt{\operatorname{arcsinh}(ax)}}{8a} + \frac{\operatorname{arcsinh}(ax)^{3/2}}{4a^2} + \frac{1}{2}x^2\operatorname{arcsinh}(ax)^{3/2} \\
&\quad - \frac{3\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{64a^2} + \frac{3\text{Subst}\left(\int \frac{e^{2x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{64a^2} \\
&= -\frac{3x\sqrt{1+a^2x^2}\sqrt{\operatorname{arcsinh}(ax)}}{8a} + \frac{\operatorname{arcsinh}(ax)^{3/2}}{4a^2} + \frac{1}{2}x^2\operatorname{arcsinh}(ax)^{3/2} \\
&\quad - \frac{3\text{Subst}\left(\int e^{-2x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{32a^2} + \frac{3\text{Subst}\left(\int e^{2x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{32a^2} \\
&= -\frac{3x\sqrt{1+a^2x^2}\sqrt{\operatorname{arcsinh}(ax)}}{8a} + \frac{\operatorname{arcsinh}(ax)^{3/2}}{4a^2} \\
&\quad + \frac{1}{2}x^2\operatorname{arcsinh}(ax)^{3/2} - \frac{3\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{64a^2} + \frac{3\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{64a^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.43

$$
\int x\operatorname{arcsinh}(ax)^{3/2}\,dx = \frac{\frac{\sqrt{-\operatorname{arcsinh}(ax)}\Gamma\left(\frac{5}{2}, -2\operatorname{arcsinh}(ax)\right)}{\sqrt{\operatorname{arcsinh}(ax)}} + \Gamma\left(\frac{5}{2}, 2\operatorname{arcsinh}(ax)\right)}{16\sqrt{2}a^2}
$$

```
[In] Integrate[x*ArcSinh[a*x]^(3/2),x]
```

```
[Out] ((Sqrt[-ArcSinh[a*x]]*Gamma[5/2, -2*ArcSinh[a*x]])/Sqrt[ArcSinh[a*x]] + Gam
ma[5/2, 2*ArcSinh[a*x]])/(16*Sqrt[2]*a^2)
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.84

method	result
default	$-\frac{\sqrt{2}\,\Big(-32\operatorname{arcsinh}(ax)^{\frac{3}{2}}\sqrt{2}\,\sqrt{\pi}\,a^2x^2+24\sqrt{2}\,\sqrt{\operatorname{arcsinh}(ax)}\,\sqrt{\pi}\,\sqrt{a^2x^2+1}\,ax-16\operatorname{arcsinh}(ax)^{\frac{3}{2}}\sqrt{2}\,\sqrt{\pi}+3\pi\,\operatorname{erf}\left(\sqrt{2}\,\sqrt{\operatorname{arcsinh}(ax)}\right)-}{128\sqrt{\pi}\,a^2}$

```
[In] int(x*arcsinh(a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/128*2^(1/2)*(-32*arcsinh(a*x)^(3/2)*2^(1/2)*Pi^(1/2)*a^2*x^2+24*2^(1/2)*
arcsinh(a*x)^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*a*x-16*arcsinh(a*x)^(3/2)*2^(
1/2)*Pi^(1/2)+3*Pi*erf(2^(1/2)*arcsinh(a*x)^(1/2))-3*Pi*erfi(2^(1/2)*arcsin
h(a*x)^(1/2)))/Pi^(1/2)/a^2
```

Fricas [F(-2)]

Exception generated.

$$\int x\text{arcsinh}(ax)^{3/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x\text{arcsinh}(ax)^{3/2}\,dx = \int x\,\text{asinh}^{\frac{3}{2}}\,(ax)\,dx$$

```
[In] integrate(x*asinh(a*x)**(3/2),x)
```

```
[Out] Integral(x*asinh(a*x)**(3/2), x)
```

Maxima [F]

$$\int x\text{arcsinh}(ax)^{3/2}\,dx = \int x\,\text{arsinh}\,(ax)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x*arcsinh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x*arcsinh(a*x)^(3/2), x)
```

Giac [F]

$$\int x\text{arcsinh}(ax)^{3/2}\,dx = \int x\,\text{arsinh}\,(ax)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x*arcsinh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x*arcsinh(a*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\text{arcsinh}(ax)^{3/2}\,dx=\int x\,\mathrm{asinh}(a\,x)^{3/2}\,dx$$

```
[In] int(x*asinh(a*x)^(3/2),x)
```

```
[Out] int(x*asinh(a*x)^(3/2), x)
```

3.84 $\int \operatorname{arcsinh}(ax)^{3/2}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 81

$$\int \operatorname{arcsinh}(ax)^{3/2}\,dx = -\frac{3\sqrt{1+a^2x^2}\sqrt{\operatorname{arcsinh}(ax)}}{2a} + x\operatorname{arcsinh}(ax)^{3/2} + \frac{3\sqrt{\pi}\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{8a} + \frac{3\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{8a}$$

```
[Out] x*arcsinh(a*x)^(3/2)+3/8*erf(arcsinh(a*x)^(1/2))*Pi^(1/2)/a+3/8*erfi(arcsin
h(a*x)^(1/2))*Pi^(1/2)/a-3/2*(a^2*x^2+1)^(1/2)*arcsinh(a*x)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {5772, 5798, 5774, 3388, 2211, 2235, 2236}

$$\int \operatorname{arcsinh}(ax)^{3/2}\,dx = -\frac{3\sqrt{a^2x^2+1}\sqrt{\operatorname{arcsinh}(ax)}}{2a} + \frac{3\sqrt{\pi}\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{8a} + \frac{3\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{8a} + x\operatorname{arcsinh}(ax)^{3/2}$$

```
[In] Int[ArcSinh[a*x]^(3/2),x]
```

```
[Out] (-3*Sqrt[1 + a^2*x^2]*Sqrt[ArcSinh[a*x]])/(2*a) + x*ArcSinh[a*x]^(3/2) + (3
*Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]])/(8*a) + (3*Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*
x]]])/(8*a)
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= x\text{arcsinh}(ax)^{3/2} - \frac{1}{2}(3a)\int \frac{x\sqrt{\text{arcsinh}(ax)}}{\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{3\sqrt{1+a^2x^2}\sqrt{\text{arcsinh}(ax)}}{2a} + x\text{arcsinh}(ax)^{3/2} + \frac{3}{4}\int \frac{1}{\sqrt{\text{arcsinh}(ax)}}\,dx \\
&= -\frac{3\sqrt{1+a^2x^2}\sqrt{\text{arcsinh}(ax)}}{2a} + x\text{arcsinh}(ax)^{3/2} + \frac{3\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{4a} \\
&= -\frac{3\sqrt{1+a^2x^2}\sqrt{\text{arcsinh}(ax)}}{2a} + x\text{arcsinh}(ax)^{3/2} \\
&\quad + \frac{3\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{8a} + \frac{3\text{Subst}\left(\int \frac{e^{x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{8a} \\
&= -\frac{3\sqrt{1+a^2x^2}\sqrt{\text{arcsinh}(ax)}}{2a} + x\text{arcsinh}(ax)^{3/2} \\
&\quad + \frac{3\text{Subst}\left(\int e^{-x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{4a} + \frac{3\text{Subst}\left(\int e^{x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{4a} \\
&= -\frac{3\sqrt{1+a^2x^2}\sqrt{\text{arcsinh}(ax)}}{2a} + x\text{arcsinh}(ax)^{3/2} \\
&\quad + \frac{3\sqrt{\pi}\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)}{8a} + \frac{3\sqrt{\pi}\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)}{8a}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.58

$$
\int \text{arcsinh}(ax)^{3/2}\,dx = \frac{\frac{\sqrt{-\text{arcsinh}(ax)}\Gamma\left(\frac{5}{2},-\text{arcsinh}(ax)\right)}{\sqrt{\text{arcsinh}(ax)}} - \Gamma\left(\frac{5}{2},\text{arcsinh}(ax)\right)}{2a}
$$

```
[In] Integrate[ArcSinh[a*x]^(3/2),x]
```

```
[Out] ((Sqrt[-ArcSinh[a*x]]*Gamma[5/2, -ArcSinh[a*x]])/Sqrt[ArcSinh[a*x]] - Gamma
[5/2, ArcSinh[a*x]])/(2*a)
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{-8\operatorname{arcsinh}(ax)^{\frac{3}{2}}\sqrt{\pi}\,ax+12\sqrt{\operatorname{arcsinh}(ax)}\,\sqrt{\pi}\,\sqrt{a^{2}x^{2}+1}-3\pi\,\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)-3\pi\,\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{8\sqrt{\pi}\,a}$	65

```
[In] int(arcsinh(a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(-8*arcsinh(a*x)^(3/2)*Pi^(1/2)*a*x+12*arcsinh(a*x)^(1/2)*Pi^(1/2)*(a^
2*x^2+1)^(1/2)-3*Pi*erf(arcsinh(a*x)^(1/2))-3*Pi*erfi(arcsinh(a*x)^(1/2)))/
Pi^(1/2)/a
```

Fricas [F(-2)]

Exception generated.

$$\int \operatorname{arcsinh}(ax)^{3/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \operatorname{arcsinh}(ax)^{3/2}\,dx = \int \operatorname{asinh}^{\frac{3}{2}}(ax)\,dx$$

```
[In] integrate(asinh(a*x)**(3/2),x)
```

```
[Out] Integral(asinh(a*x)**(3/2), x)
```

Maxima [F]

$$\int \operatorname{arcsinh}(ax)^{3/2}\,dx = \int \operatorname{arsinh}(ax)^{\frac{3}{2}}\,dx$$

```
[In] integrate(arcsinh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^(3/2), x)
```

Giac [F]

$$\int \operatorname{arcsinh}(ax)^{3/2}\,dx = \int \operatorname{arsinh}\left(ax\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(arcsinh(a*x)^(3/2),x, algorithm="giac")
[Out] integrate(arcsinh(a*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{arcsinh}(ax)^{3/2}\,dx = \int \operatorname{asinh}(a\,x)^{3/2}\,dx$$

```
[In] int(asinh(a*x)^(3/2),x)
[Out] int(asinh(a*x)^(3/2), x)
```

3.85 $\int \frac{\text{arcsinh}(ax)^{3/2}}{x} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\text{arcsinh}(ax)^{3/2}}{x} \, dx = \text{Int}\left(\frac{\text{arcsinh}(ax)^{3/2}}{x}, x\right)$$

```
[Out] Unintegrable(arcsinh(a*x)^(3/2)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{arcsinh}(ax)^{3/2}}{x} \, dx = \int \frac{\text{arcsinh}(ax)^{3/2}}{x} \, dx$$

```
[In] Int[ArcSinh[a*x]^(3/2)/x,x]
```

```
[Out] Defer[Int][ArcSinh[a*x]^(3/2)/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\text{arcsinh}(ax)^{3/2}}{x} \, dx$$

Mathematica [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\operatorname{arcsinh}(ax)^{3/2}}{x}\,dx = \int \frac{\operatorname{arcsinh}(ax)^{3/2}}{x}\,dx$$

```
[In] Integrate[ArcSinh[a*x]^(3/2)/x,x]
[Out] Integrate[ArcSinh[a*x]^(3/2)/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\operatorname{arcsinh}\left(ax\right)^{\frac{3}{2}}}{x}dx$$

```
[In] int(arcsinh(a*x)^(3/2)/x,x)
[Out] int(arcsinh(a*x)^(3/2)/x,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\operatorname{arcsinh}(ax)^{3/2}}{x}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(arcsinh(a*x)^(3/2)/x,x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 1.67 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\operatorname{arcsinh}(ax)^{3/2}}{x}\,dx = \int \frac{\operatorname{asinh}^{\frac{3}{2}}\left(ax\right)}{x}\,dx$$

```
[In] integrate(asinh(a*x)**(3/2)/x,x)
[Out] Integral(asinh(a*x)**(3/2)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}(ax)^{3/2}}{x}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^{\frac{3}{2}}}{x}\,dx$$

```
[In] integrate(arcsinh(a*x)^(3/2)/x,x, algorithm="maxima")
[Out] integrate(arcsinh(a*x)^(3/2)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}(ax)^{3/2}}{x}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^{\frac{3}{2}}}{x}\,dx$$

```
[In] integrate(arcsinh(a*x)^(3/2)/x,x, algorithm="giac")
[Out] integrate(arcsinh(a*x)^(3/2)/x, x)
```

Mupad [N/A]

Not integrable

Time = 2.46 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}(ax)^{3/2}}{x}\,dx = \int \frac{{\mathrm{asinh}\left(a\,x\right)}^{3/2}}{x}\,dx$$

```
[In] int(asinh(a*x)^(3/2)/x,x)
[Out] int(asinh(a*x)^(3/2)/x, x)
```

3.86 $\int x^4 \textbf{arcsinh}(ax)^{5/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 379

$$\int x^4 \operatorname{arcsinh}(ax)^{5/2}\,dx = \frac{2x\sqrt{\operatorname{arcsinh}(ax)}}{5a^4} - \frac{x^3\sqrt{\operatorname{arcsinh}(ax)}}{15a^2} + \frac{3}{100}x^5\sqrt{\operatorname{arcsinh}(ax)}$$
$$- \frac{4\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^{3/2}}{15a^5} + \frac{2x^2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^{3/2}}{15a^3} - \frac{x^4\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^{3/2}}{10a}$$
$$+\frac{1}{5}x^5\operatorname{arcsinh}(ax)^{5/2} + \frac{15\sqrt{\pi}\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{128a^5} - \frac{\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{240a^5} - \frac{\sqrt{3\pi}\operatorname{erf}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{1280a^5} +$$

```
[Out] 1/5*x^5*arcsinh(a*x)^(5/2)+3/32000*erf(5^(1/2)*arcsinh(a*x)^(1/2))*5^(1/2)*
Pi^(1/2)/a^5-3/32000*erfi(5^(1/2)*arcsinh(a*x)^(1/2))*5^(1/2)*Pi^(1/2)/a^5-
5/2304*erf(3^(1/2)*arcsinh(a*x)^(1/2))*3^(1/2)*Pi^(1/2)/a^5+5/2304*erfi(3^(
1/2)*arcsinh(a*x)^(1/2))*3^(1/2)*Pi^(1/2)/a^5+15/128*erf(arcsinh(a*x)^(1/2)
)*Pi^(1/2)/a^5-15/128*erfi(arcsinh(a*x)^(1/2))*Pi^(1/2)/a^5-4/15*arcsinh(a*
x)^(3/2)*(a^2*x^2+1)^(1/2)/a^5+2/15*x^2*arcsinh(a*x)^(3/2)*(a^2*x^2+1)^(1/2
)/a^3-1/10*x^4*arcsinh(a*x)^(3/2)*(a^2*x^2+1)^(1/2)/a+2/5*x*arcsinh(a*x)^(1
/2)/a^4-1/15*x^3*arcsinh(a*x)^(1/2)/a^2+3/100*x^5*arcsinh(a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.00, number of steps used = 44, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules

used = {5777, 5812, 5798, 5772, 5819, 3389, 2211, 2235, 2236, 3393}

$$\begin{aligned}\int x^4\text{arcsinh}(ax)^{5/2}\,dx =& \frac{15\sqrt{\pi}\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)}{128a^5} - \frac{\sqrt{3\pi}\text{erf}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)}{1280a^5} \\ &- \frac{\sqrt{\frac{\pi}{3}}\text{erf}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)}{240a^5} + \frac{3\sqrt{\frac{\pi}{5}}\text{erf}\left(\sqrt{5}\sqrt{\text{arcsinh}(ax)}\right)}{6400a^5} \\ &- \frac{15\sqrt{\pi}\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)}{128a^5} + \frac{\sqrt{3\pi}\text{erfi}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)}{1280a^5} \\ &+ \frac{\sqrt{\frac{\pi}{3}}\text{erfi}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)}{240a^5} - \frac{3\sqrt{\frac{\pi}{5}}\text{erfi}\left(\sqrt{5}\sqrt{\text{arcsinh}(ax)}\right)}{6400a^5} + \frac{2x\sqrt{\text{arcsinh}(ax)}}{5a^4} \\ &- \frac{x^3\sqrt{\text{arcsinh}(ax)}}{15a^2} - \frac{x^4\sqrt{a^2x^2+1}\text{arcsinh}(ax)^{3/2}}{10a} - \frac{4\sqrt{a^2x^2+1}\text{arcsinh}(ax)^{3/2}}{15a^5} \\ &+ \frac{2x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)^{3/2}}{15a^3} + \frac{1}{5}x^5\text{arcsinh}(ax)^{5/2} + \frac{3}{100}x^5\sqrt{\text{arcsinh}(ax)}\end{aligned}$$

```
[In] Int[x^4*ArcSinh[a*x]^(5/2),x]
```

```
[Out] (2*x*Sqrt[ArcSinh[a*x]])/(5*a^4) - (x^3*Sqrt[ArcSinh[a*x]])/(15*a^2) + (3*x
^5*Sqrt[ArcSinh[a*x]])/100 - (4*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(3/2))/(15*a
^5) + (2*x^2*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(3/2))/(15*a^3) - (x^4*Sqrt[1 +
 a^2*x^2]*ArcSinh[a*x]^(3/2))/(10*a) + (x^5*ArcSinh[a*x]^(5/2))/5 + (15*Sqr
t[Pi]*Erf[Sqrt[ArcSinh[a*x]]])/(128*a^5) - (Sqrt[Pi/3]*Erf[Sqrt[3]*Sqrt[Arc
Sinh[a*x]]])/(240*a^5) - (Sqrt[3*Pi]*Erf[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(1280
*a^5) + (3*Sqrt[Pi/5]*Erf[Sqrt[5]*Sqrt[ArcSinh[a*x]]])/(6400*a^5) - (15*Sqr
t[Pi]*Erfi[Sqrt[ArcSinh[a*x]]])/(128*a^5) + (Sqrt[Pi/3]*Erfi[Sqrt[3]*Sqrt[A
rcSinh[a*x]]])/(240*a^5) + (Sqrt[3*Pi]*Erfi[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(1
280*a^5) - (3*Sqrt[Pi/5]*Erfi[Sqrt[5]*Sqrt[ArcSinh[a*x]]])/(6400*a^5)
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{5}x^5\text{arcsinh}(ax)^{5/2} - \frac{1}{2}a\int \frac{x^5\text{arcsinh}(ax)^{3/2}}{\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{x^4\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}{10a} \\
&\quad + \frac{1}{5}x^5\text{arcsinh}(ax)^{5/2} + \frac{3}{20}\int x^4\sqrt{\text{arcsinh}(ax)}\,dx + \frac{2\int \frac{x^3\text{arcsinh}(ax)^{3/2}}{\sqrt{1+a^2x^2}}\,dx}{5a} \\
&= \frac{3}{100}x^5\sqrt{\text{arcsinh}(ax)} + \frac{2x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}{15a^3} \\
&\quad - \frac{x^4\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}{10a} + \frac{1}{5}x^5\text{arcsinh}(ax)^{5/2} - \frac{4\int \frac{x\text{arcsinh}(ax)^{3/2}}{\sqrt{1+a^2x^2}}\,dx}{15a^3} \\
&\quad - \frac{\int x^2\sqrt{\text{arcsinh}(ax)}\,dx}{5a^2} - \frac{1}{200}(3a)\int \frac{x^5}{\sqrt{1+a^2x^2}\sqrt{\text{arcsinh}(ax)}}\,dx \\
&= -\frac{x^3\sqrt{\text{arcsinh}(ax)}}{15a^2} + \frac{3}{100}x^5\sqrt{\text{arcsinh}(ax)} - \frac{4\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}{15a^5} \\
&\quad + \frac{2x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}{15a^3} - \frac{x^4\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}{10a} \\
&\quad + \frac{1}{5}x^5\text{arcsinh}(ax)^{5/2} - \frac{3\text{Subst}\left(\int \frac{\sinh^5(x)}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{200a^5} + \frac{2\int \sqrt{\text{arcsinh}(ax)}\,dx}{5a^4} + \frac{\int \frac{x}{\sqrt{1+a^2x^2}\sqrt{a}}}{3} \\
&= \frac{2x\sqrt{\text{arcsinh}(ax)}}{5a^4} - \frac{x^3\sqrt{\text{arcsinh}(ax)}}{15a^2} \\
&\quad + \frac{3}{100}x^5\sqrt{\text{arcsinh}(ax)} - \frac{4\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}{15a^5} \\
&\quad + \frac{2x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}{15a^3} - \frac{x^4\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}{10a} \\
&\quad + \frac{1}{5}x^5\text{arcsinh}(ax)^{5/2} + \frac{(3i)\text{Subst}\left(\int \left(\frac{5i\sinh(x)}{8\sqrt{x}} - \frac{5i\sinh(3x)}{16\sqrt{x}} + \frac{i\sinh(5x)}{16\sqrt{x}}\right)dx, x, \text{arcsinh}(ax)\right)}{200a^5} + \text{Subst}\Big(
\end{aligned}$$

$$
\begin{aligned}
&= \frac{2x\sqrt{\operatorname{arcsinh}(ax)}}{5a^4} - \frac{x^3\sqrt{\operatorname{arcsinh}(ax)}}{15a^2} \\
&\quad + \frac{3}{100}x^5\sqrt{\operatorname{arcsinh}(ax)} - \frac{4\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^{3/2}}{15a^5} \\
&\quad + \frac{2x^2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^{3/2}}{15a^3} - \frac{x^4\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^{3/2}}{10a} \\
&\quad + \frac{1}{5}x^5\operatorname{arcsinh}(ax)^{5/2} + \frac{i\text{Subst}\left(\int \left(\frac{3i\sinh(x)}{4\sqrt{x}} - \frac{i\sinh(3x)}{4\sqrt{x}}\right)\,dx, x, \operatorname{arcsinh}(ax)\right)}{30a^5} - \frac{3\text{Subst}\left(\int \frac{\sinh(5x)}{\sqrt{x}}\,dx, x\right.}{3200a^5}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2x\sqrt{\operatorname{arcsinh}(ax)}}{5a^4} - \frac{x^3\sqrt{\operatorname{arcsinh}(ax)}}{15a^2} \\
&\quad + \frac{3}{100}x^5\sqrt{\operatorname{arcsinh}(ax)} - \frac{4\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^{3/2}}{15a^5} \\
&\quad + \frac{2x^2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^{3/2}}{15a^3} - \frac{x^4\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^{3/2}}{10a} \\
&\quad + \frac{1}{5}x^5\operatorname{arcsinh}(ax)^{5/2} + \frac{3\text{Subst}\left(\int \frac{e^{-5x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{6400a^5} - \frac{3\text{Subst}\left(\int \frac{e^{5x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{6400a^5} - \frac{3\text{Su}}{}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2x\sqrt{\operatorname{arcsinh}(ax)}}{5a^4} - \frac{x^3\sqrt{\operatorname{arcsinh}(ax)}}{15a^2} \\
&\quad + \frac{3}{100}x^5\sqrt{\operatorname{arcsinh}(ax)} - \frac{4\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^{3/2}}{15a^5} \\
&\quad + \frac{2x^2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^{3/2}}{15a^3} - \frac{x^4\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^{3/2}}{10a} \\
&\quad + \frac{1}{5}x^5\operatorname{arcsinh}(ax)^{5/2} + \frac{3\text{Subst}\left(\int e^{-5x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{3200a^5} - \frac{3\text{Subst}\left(\int e^{5x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{3200a^5}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2x\sqrt{\operatorname{arcsinh}(ax)}}{5a^4} - \frac{x^3\sqrt{\operatorname{arcsinh}(ax)}}{15a^2} \\
&\quad + \frac{3}{100}x^5\sqrt{\operatorname{arcsinh}(ax)} - \frac{4\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^{3/2}}{15a^5} \\
&\quad + \frac{2x^2\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^{3/2}}{15a^3} - \frac{x^4\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^{3/2}}{10a} \\
&\quad + \frac{1}{5}x^5\operatorname{arcsinh}(ax)^{5/2} + \frac{67\sqrt{\pi}\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{640a^5} - \frac{\sqrt{3\pi}\operatorname{erf}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{1280a^5} + \frac{3\sqrt{\frac{\pi}{5}}\operatorname{erf}\left(\sqrt{5}\sqrt{\operatorname{arc}\right.}{6400a^5}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2x\sqrt{\mathrm{arcsinh}(ax)}}{5a^4} - \frac{x^3\sqrt{\mathrm{arcsinh}(ax)}}{15a^2} \\
&\quad + \frac{3}{100}x^5\sqrt{\mathrm{arcsinh}(ax)} - \frac{4\sqrt{1+a^2x^2}\,\mathrm{arcsinh}(ax)^{3/2}}{15a^5} \\
&\quad + \frac{2x^2\sqrt{1+a^2x^2}\,\mathrm{arcsinh}(ax)^{3/2}}{15a^3} - \frac{x^4\sqrt{1+a^2x^2}\,\mathrm{arcsinh}(ax)^{3/2}}{10a} \\
&\quad + \frac{1}{5}x^5\mathrm{arcsinh}(ax)^{5/2} + \frac{15\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{\mathrm{arcsinh}(ax)}\right)}{128a^5} - \frac{\sqrt{\frac{\pi}{3}}\,\mathrm{erf}\left(\sqrt{3}\sqrt{\mathrm{arcsinh}(ax)}\right)}{240a^5} - \frac{\sqrt{3\pi}\,\mathrm{erf}\Big(\sqrt{3}\sqrt{\mathrm{ar}}}{1280a}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.40

$$
\int x^4\mathrm{arcsinh}(ax)^{5/2}\,dx = \frac{\frac{27\sqrt{5}\sqrt{\mathrm{arcsinh}(ax)}\,\Gamma\left(\frac{7}{2},-5\mathrm{arcsinh}(ax)\right)}{\sqrt{-\mathrm{arcsinh}(ax)}} + \frac{625\sqrt{3}\sqrt{-\mathrm{arcsinh}(ax)}\,\Gamma\left(\frac{7}{2},-3\mathrm{arcsinh}(ax)\right)}{\sqrt{\mathrm{arcsinh}(ax)}} + 33750}{}
$$

```
[In] Integrate[x^4*ArcSinh[a*x]^(5/2),x]
```

```
[Out] ((27*Sqrt[5]*Sqrt[ArcSinh[a*x]]*Gamma[7/2, -5*ArcSinh[a*x]])/Sqrt[-ArcSinh[
a*x]] + (625*Sqrt[3]*Sqrt[-ArcSinh[a*x]]*Gamma[7/2, -3*ArcSinh[a*x]])/Sqrt[
ArcSinh[a*x]] + (33750*Sqrt[ArcSinh[a*x]]*Gamma[7/2, -ArcSinh[a*x]])/Sqrt[-
ArcSinh[a*x]] - 33750*Gamma[7/2, ArcSinh[a*x]] + 625*Sqrt[3]*Gamma[7/2, 3*A
rcSinh[a*x]] - 27*Sqrt[5]*Gamma[7/2, 5*ArcSinh[a*x]])/(540000*a^5)
```

Maple [F]

$$
\int x^4\,\mathrm{arcsinh}\,(ax)^{\frac{5}{2}}\,dx
$$

```
[In] int(x^4*arcsinh(a*x)^(5/2),x)
```

```
[Out] int(x^4*arcsinh(a*x)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$
\int x^4\mathrm{arcsinh}(ax)^{5/2}\,dx = \text{Exception raised: TypeError}
$$

```
[In] integrate(x^4*arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int x^4 \text{arcsinh}(ax)^{5/2} \, dx = \text{Timed out}$$

```
[In] integrate(x**4*asinh(a*x)**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int x^4 \text{arcsinh}(ax)^{5/2} \, dx = \int x^4 \operatorname{arsinh}\left(ax\right)^{\frac{5}{2}} \, dx$$

```
[In] integrate(x^4*arcsinh(a*x)^(5/2),x, algorithm="maxima")
[Out] integrate(x^4*arcsinh(a*x)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int x^4 \text{arcsinh}(ax)^{5/2} \, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^4*arcsinh(a*x)^(5/2),x, algorithm="giac")
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \text{arcsinh}(ax)^{5/2} \, dx = \int x^4 \operatorname{asinh}(a\,x)^{5/2} \, dx$$

```
[In] int(x^4*asinh(a*x)^(5/2),x)
[Out] int(x^4*asinh(a*x)^(5/2), x)
```

3.87 $\int x^3 \textbf{arcsinh}(ax)^{5/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 247

$$\int x^3\text{arcsinh}(ax)^{5/2}\,dx=-\frac{225\sqrt{\text{arcsinh}(ax)}}{2048a^4}-\frac{45x^2\sqrt{\text{arcsinh}(ax)}}{256a^2}+\frac{15}{256}x^4\sqrt{\text{arcsinh}(ax)}$$
$$+\frac{15x\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}{64a^3}-\frac{5x^3\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}{32a}-\frac{3\text{arcsinh}(ax)^{5/2}}{32a^4}$$
$$+\frac{1}{4}x^4\text{arcsinh}(ax)^{5/2}-\frac{15\sqrt{\pi}\text{erf}\left(2\sqrt{\text{arcsinh}(ax)}\right)}{16384a^4}+\frac{15\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{512a^4}-\frac{15\sqrt{\pi}\text{erfi}\left(2\sqrt{\text{arcsinh}(a\right.}}{16384a^4}$$

```
[Out] -3/32*arcsinh(a*x)^(5/2)/a^4+1/4*x^4*arcsinh(a*x)^(5/2)+15/1024*erf(2^(1/2)
*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^4+15/1024*erfi(2^(1/2)*arcsinh(a*x)
^(1/2))*2^(1/2)*Pi^(1/2)/a^4-15/16384*erf(2*arcsinh(a*x)^(1/2))*Pi^(1/2)/a^
4-15/16384*erfi(2*arcsinh(a*x)^(1/2))*Pi^(1/2)/a^4+15/64*x*arcsinh(a*x)^(3/
2)*(a^2*x^2+1)^(1/2)/a^3-5/32*x^3*arcsinh(a*x)^(3/2)*(a^2*x^2+1)^(1/2)/a-22
5/2048*arcsinh(a*x)^(1/2)/a^4-45/256*x^2*arcsinh(a*x)^(1/2)/a^2+15/256*x^4*
arcsinh(a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used

$= \{5777, 5812, 5783, 5819, 3393, 3388, 2211, 2235, 2236\}$

$$\begin{aligned}\int x^3 \operatorname{arcsinh}(ax)^{5/2}\,dx = &-\frac{15\sqrt{\pi}\operatorname{erf}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{16384a^4}\\ &+\frac{15\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{512a^4}-\frac{15\sqrt{\pi}\operatorname{erfi}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{16384a^4}\\ &+\frac{15\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{512a^4}-\frac{3\operatorname{arcsinh}(ax)^{5/2}}{32a^4}-\frac{225\sqrt{\operatorname{arcsinh}(ax)}}{2048a^4}\\ &-\frac{45x^2\sqrt{\operatorname{arcsinh}(ax)}}{256a^2}-\frac{5x^3\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^{3/2}}{32a}\\ &+\frac{15x\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^{3/2}}{64a^3}+\frac{1}{4}x^4\operatorname{arcsinh}(ax)^{5/2}+\frac{15}{256}x^4\sqrt{\operatorname{arcsinh}(ax)}\end{aligned}$$

```
[In] Int[x^3*ArcSinh[a*x]^(5/2),x]
```

```
[Out] (-225*Sqrt[ArcSinh[a*x]])/(2048*a^4) - (45*x^2*Sqrt[ArcSinh[a*x]])/(256*a^2
) + (15*x^4*Sqrt[ArcSinh[a*x]])/256 + (15*x*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^
(3/2))/(64*a^3) - (5*x^3*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(3/2))/(32*a) - (3*
ArcSinh[a*x]^(5/2))/(32*a^4) + (x^4*ArcSinh[a*x]^(5/2))/4 - (15*Sqrt[Pi]*Er
f[2*Sqrt[ArcSinh[a*x]]])/(16384*a^4) + (15*Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcS
inh[a*x]]])/(512*a^4) - (15*Sqrt[Pi]*Erfi[2*Sqrt[ArcSinh[a*x]]])/(16384*a^4
) + (15*Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(512*a^4)
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
```

```
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{4}x^4\text{arcsinh}(ax)^{5/2} - \frac{1}{8}(5a)\int \frac{x^4\text{arcsinh}(ax)^{3/2}}{\sqrt{1+a^2x^2}}\,dx$$

$$
\begin{aligned}
&= -\frac{5x^3\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}{32a} \\
&\quad + \frac{1}{4}x^4\text{arcsinh}(ax)^{5/2} + \frac{15}{64}\int x^3\sqrt{\text{arcsinh}(ax)}\,dx + \frac{15\int \frac{x^2\text{arcsinh}(ax)^{3/2}}{\sqrt{1+a^2x^2}}\,dx}{32a}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{15}{256}x^4\sqrt{\text{arcsinh}(ax)} + \frac{15x\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}{64a^3} \\
&\quad - \frac{5x^3\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}{32a} + \frac{1}{4}x^4\text{arcsinh}(ax)^{5/2} - \frac{15\int \frac{\text{arcsinh}(ax)^{3/2}}{\sqrt{1+a^2x^2}}\,dx}{64a^3} \\
&\quad - \frac{45\int x\sqrt{\text{arcsinh}(ax)}\,dx}{128a^2} - \frac{1}{512}(15a)\int \frac{x^4}{\sqrt{1+a^2x^2}\sqrt{\text{arcsinh}(ax)}}\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{45x^2\sqrt{\text{arcsinh}(ax)}}{256a^2} + \frac{15}{256}x^4\sqrt{\text{arcsinh}(ax)} + \frac{15x\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}{64a^3} \\
&\quad - \frac{5x^3\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}{32a} - \frac{3\text{arcsinh}(ax)^{5/2}}{32a^4} \\
&\quad + \frac{1}{4}x^4\text{arcsinh}(ax)^{5/2} - \frac{15\text{Subst}\left(\int \frac{\sinh^4(x)}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{512a^4} + \frac{45\int \frac{x^2}{\sqrt{1+a^2x^2}\sqrt{\text{arcsinh}(ax)}}\,dx}{512a}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{45x^2\sqrt{\text{arcsinh}(ax)}}{256a^2} + \frac{15}{256}x^4\sqrt{\text{arcsinh}(ax)} + \frac{15x\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}{64a^3} \\
&\quad - \frac{5x^3\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}{32a} - \frac{3\text{arcsinh}(ax)^{5/2}}{32a^4} \\
&\quad + \frac{1}{4}x^4\text{arcsinh}(ax)^{5/2} - \frac{15\text{Subst}\left(\int \left(\frac{3}{8\sqrt{x}} - \frac{\cosh(2x)}{2\sqrt{x}} + \frac{\cosh(4x)}{8\sqrt{x}}\right)\,dx, x, \text{arcsinh}(ax)\right)}{512a^4} + \frac{45\text{Subst}\left(\int \frac{\sinh^2}{\sqrt{}}\right.}{}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{45\sqrt{\text{arcsinh}(ax)}}{2048a^4} - \frac{45x^2\sqrt{\text{arcsinh}(ax)}}{256a^2} + \frac{15}{256}x^4\sqrt{\text{arcsinh}(ax)} \\
&\quad + \frac{15x\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}{64a^3} - \frac{5x^3\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}{32a} - \frac{3\text{arcsinh}(ax)^{5/2}}{32a^4} \\
&\quad + \frac{1}{4}x^4\text{arcsinh}(ax)^{5/2} - \frac{15\text{Subst}\left(\int \frac{\cosh(4x)}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{4096a^4} + \frac{15\text{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax\right.}{1024a^4}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{225\sqrt{\text{arcsinh}(ax)}}{2048a^4} - \frac{45x^2\sqrt{\text{arcsinh}(ax)}}{256a^2} + \frac{15}{256}x^4\sqrt{\text{arcsinh}(ax)} \\
&\quad + \frac{15x\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}{64a^3} - \frac{5x^3\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}{32a} - \frac{3\text{arcsinh}(ax)^{5/2}}{32a^4} \\
&\quad + \frac{1}{4}x^4\text{arcsinh}(ax)^{5/2} - \frac{15\text{Subst}\left(\int \frac{e^{-4x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{8192a^4} - \frac{15\text{Subst}\left(\int \frac{e^{4x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{8192a^4} + \frac{1}{}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{225\sqrt{\operatorname{arcsinh}(ax)}}{2048a^4} - \frac{45x^2\sqrt{\operatorname{arcsinh}(ax)}}{256a^2} + \frac{15}{256}x^4\sqrt{\operatorname{arcsinh}(ax)} \\
&\quad + \frac{15x\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^{3/2}}{64a^3} - \frac{5x^3\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^{3/2}}{32a} - \frac{3\operatorname{arcsinh}(ax)^{5/2}}{32a^4} \\
&\quad + \frac{1}{4}x^4\operatorname{arcsinh}(ax)^{5/2} - \frac{15\text{Subst}\left(\int e^{-4x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{4096a^4} - \frac{15\text{Subst}\left(\int e^{4x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(\right.}{4096a^4}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{225\sqrt{\operatorname{arcsinh}(ax)}}{2048a^4} - \frac{45x^2\sqrt{\operatorname{arcsinh}(ax)}}{256a^2} + \frac{15}{256}x^4\sqrt{\operatorname{arcsinh}(ax)} \\
&\quad + \frac{15x\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^{3/2}}{64a^3} - \frac{5x^3\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^{3/2}}{32a} - \frac{3\operatorname{arcsinh}(ax)^{5/2}}{32a^4} \\
&\quad + \frac{1}{4}x^4\operatorname{arcsinh}(ax)^{5/2} - \frac{15\sqrt{\pi}\operatorname{erf}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{16384a^4} + \frac{15\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{2048a^4} - \frac{15\sqrt{\pi}\operatorname{erfi}\left(2\sqrt{\right.}}{163}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{225\sqrt{\operatorname{arcsinh}(ax)}}{2048a^4} - \frac{45x^2\sqrt{\operatorname{arcsinh}(ax)}}{256a^2} + \frac{15}{256}x^4\sqrt{\operatorname{arcsinh}(ax)} \\
&\quad + \frac{15x\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^{3/2}}{64a^3} - \frac{5x^3\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^{3/2}}{32a} - \frac{3\operatorname{arcsinh}(ax)^{5/2}}{32a^4} \\
&\quad + \frac{1}{4}x^4\operatorname{arcsinh}(ax)^{5/2} - \frac{15\sqrt{\pi}\operatorname{erf}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{16384a^4} + \frac{15\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{512a^4} - \frac{15\sqrt{\pi}\operatorname{erfi}\left(2\sqrt{\right.}}{163}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.40

$$
\int x^3\operatorname{arcsinh}(ax)^{5/2}\,dx = \frac{\frac{\sqrt{\operatorname{arcsinh}(ax)}\Gamma\left(\frac{7}{2},-4\operatorname{arcsinh}(ax)\right)}{\sqrt{-\operatorname{arcsinh}(ax)}} + \frac{16\sqrt{2}\sqrt{-\operatorname{arcsinh}(ax)}\Gamma\left(\frac{7}{2},-2\operatorname{arcsinh}(ax)\right)}{\sqrt{\operatorname{arcsinh}(ax)}} - 16\sqrt{2}\Gamma\left(\frac{7}{2},\right.}{2048a^4}
$$

```
[In] Integrate[x^3*ArcSinh[a*x]^(5/2),x]
```

```
[Out] ((Sqrt[ArcSinh[a*x]]*Gamma[7/2, -4*ArcSinh[a*x]])/Sqrt[-ArcSinh[a*x]] + (16
*Sqrt[2]*Sqrt[-ArcSinh[a*x]]*Gamma[7/2, -2*ArcSinh[a*x]])/Sqrt[ArcSinh[a*x]
] - 16*Sqrt[2]*Gamma[7/2, 2*ArcSinh[a*x]] + Gamma[7/2, 4*ArcSinh[a*x]])/(20
48*a^4)
```

Maple [F]

$$\int x^{3} \operatorname{arcsinh}\left(a x\right)^{\frac{5}{2}}d x$$

```
[In] int(x^3*arcsinh(a*x)^(5/2),x)
```

```
[Out] int(x^3*arcsinh(a*x)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^3 \text{arcsinh}(ax)^{5/2} \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3*arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x^3 \text{arcsinh}(ax)^{5/2} \, dx = \int x^{3} \operatorname{asinh}^{\frac{5}{2}}{\left(a x \right)}\, dx$$

```
[In] integrate(x**3*asinh(a*x)**(5/2),x)
```

```
[Out] Integral(x**3*asinh(a*x)**(5/2), x)
```

Maxima [F]

$$\int x^3 \text{arcsinh}(ax)^{5/2} \, dx = \int { x^{3} \operatorname{arsinh}\left(a x\right)^{\frac{5}{2}} \,d x }$$

```
[In] integrate(x^3*arcsinh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3*arcsinh(a*x)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int x^3 \text{arcsinh}(ax)^{5/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3*arcsinh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \text{arcsinh}(ax)^{5/2}\,dx = \int x^3\,\text{asinh}(a\,x)^{5/2}\,dx$$

```
[In] int(x^3*asinh(a*x)^(5/2),x)
```

```
[Out] int(x^3*asinh(a*x)^(5/2), x)
```

3.88 $\int x^2\text{arcsinh}(ax)^{5/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 210

$$\int x^2\text{arcsinh}(ax)^{5/2}\,dx=-\frac{5x\sqrt{\text{arcsinh}(ax)}}{6a^2}+\frac{5}{36}x^3\sqrt{\text{arcsinh}(ax)}$$
$$+\frac{5\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}{9a^3}-\frac{5x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}{18a}$$
$$+\frac{1}{3}x^3\text{arcsinh}(ax)^{5/2}-\frac{15\sqrt{\pi}\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)}{64a^3}+\frac{5\sqrt{\frac{\pi}{3}}\text{erf}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)}{576a^3}+\frac{15\sqrt{\pi}\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)}{64a^3}-$$

```
[Out] 1/3*x^3*arcsinh(a*x)^(5/2)+5/1728*erf(3^(1/2)*arcsinh(a*x)^(1/2))*3^(1/2)*P
i^(1/2)/a^3-5/1728*erfi(3^(1/2)*arcsinh(a*x)^(1/2))*3^(1/2)*Pi^(1/2)/a^3-15
/64*erf(arcsinh(a*x)^(1/2))*Pi^(1/2)/a^3+15/64*erfi(arcsinh(a*x)^(1/2))*Pi^
(1/2)/a^3+5/9*arcsinh(a*x)^(3/2)*(a^2*x^2+1)^(1/2)/a^3-5/18*x^2*arcsinh(a*x
)^(3/2)*(a^2*x^2+1)^(1/2)/a-5/6*x*arcsinh(a*x)^(1/2)/a^2+5/36*x^3*arcsinh(a
*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {5777, 5812, 5798, 5772, 5819, 3389, 2211, 2235, 2236, 3393}

$$\begin{aligned}\int x^2\text{arcsinh}(ax)^{5/2}\,dx = &-\frac{15\sqrt{\pi}\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)}{64a^3}\\ &+\frac{5\sqrt{\frac{\pi}{3}}\text{erf}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)}{576a^3}+\frac{15\sqrt{\pi}\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)}{64a^3}\\ &-\frac{5\sqrt{\frac{\pi}{3}}\text{erfi}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)}{576a^3}-\frac{5x^2\sqrt{a^2x^2+1}\text{arcsinh}(ax)^{3/2}}{18a}-\frac{5x\sqrt{\text{arcsinh}(ax)}}{6a^2}\\ &+\frac{5\sqrt{a^2x^2+1}\text{arcsinh}(ax)^{3/2}}{9a^3}+\frac{1}{3}x^3\text{arcsinh}(ax)^{5/2}+\frac{5}{36}x^3\sqrt{\text{arcsinh}(ax)}\end{aligned}$$

```
[In] Int[x^2*ArcSinh[a*x]^(5/2),x]
```

```
[Out] (-5*x*Sqrt[ArcSinh[a*x]])/(6*a^2) + (5*x^3*Sqrt[ArcSinh[a*x]])/36 + (5*Sqrt
[1 + a^2*x^2]*ArcSinh[a*x]^(3/2))/(9*a^3) - (5*x^2*Sqrt[1 + a^2*x^2]*ArcSin
h[a*x]^(3/2))/(18*a) + (x^3*ArcSinh[a*x]^(5/2))/3 - (15*Sqrt[Pi]*Erf[Sqrt[A
rcSinh[a*x]]])/(64*a^3) + (5*Sqrt[Pi/3]*Erf[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(5
76*a^3) + (15*Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x]]])/(64*a^3) - (5*Sqrt[Pi/3]*E
rfi[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(576*a^3)
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
```

```
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
```

```
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\text{arcsinh}(ax)^{5/2} - \frac{1}{6}(5a)\int \frac{x^3\text{arcsinh}(ax)^{3/2}}{\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{5x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}{18a} \\
&\quad + \frac{1}{3}x^3\text{arcsinh}(ax)^{5/2} + \frac{5}{12}\int x^2\sqrt{\text{arcsinh}(ax)}\,dx + \frac{5\int \frac{x\text{arcsinh}(ax)^{3/2}}{\sqrt{1+a^2x^2}}\,dx}{9a} \\
&= \frac{5}{36}x^3\sqrt{\text{arcsinh}(ax)} + \frac{5\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}{9a^3} - \frac{5x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}{18a} \\
&\quad + \frac{1}{3}x^3\text{arcsinh}(ax)^{5/2} - \frac{5\int \sqrt{\text{arcsinh}(ax)}\,dx}{6a^2} - \frac{1}{72}(5a)\int \frac{x^3}{\sqrt{1+a^2x^2}\sqrt{\text{arcsinh}(ax)}}\,dx \\
&= -\frac{5x\sqrt{\text{arcsinh}(ax)}}{6a^2} + \frac{5}{36}x^3\sqrt{\text{arcsinh}(ax)} + \frac{5\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}{9a^3} \\
&\quad - \frac{5x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}{18a} + \frac{1}{3}x^3\text{arcsinh}(ax)^{5/2} \\
&\quad - \frac{5\text{Subst}\left(\int \frac{\sinh^3(x)}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{72a^3} + \frac{5\int \frac{x}{\sqrt{1+a^2x^2}\sqrt{\text{arcsinh}(ax)}}\,dx}{12a} \\
&= -\frac{5x\sqrt{\text{arcsinh}(ax)}}{6a^2} + \frac{5}{36}x^3\sqrt{\text{arcsinh}(ax)} \\
&\quad + \frac{5\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}{9a^3} - \frac{5x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}{18a} \\
&\quad + \frac{1}{3}x^3\text{arcsinh}(ax)^{5/2} - \frac{(5i)\text{Subst}\left(\int \left(\frac{3i\sinh(x)}{4\sqrt{x}} - \frac{i\sinh(3x)}{4\sqrt{x}}\right)dx, x, \text{arcsinh}(ax)\right)}{72a^3} \\
&\quad + \frac{5\text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{12a^3} \\
&= -\frac{5x\sqrt{\text{arcsinh}(ax)}}{6a^2} + \frac{5}{36}x^3\sqrt{\text{arcsinh}(ax)} + \frac{5\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}{9a^3} \\
&\quad - \frac{5x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}{18a} + \frac{1}{3}x^3\text{arcsinh}(ax)^{5/2} \\
&\quad - \frac{5\text{Subst}\left(\int \frac{\sinh(3x)}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{288a^3} + \frac{5\text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{96a^3} \\
&\quad - \frac{5\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{24a^3} + \frac{5\text{Subst}\left(\int \frac{e^{x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{24a^3}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{5x\sqrt{\text{arcsinh}(ax)}}{6a^2} + \frac{5}{36}x^3\sqrt{\text{arcsinh}(ax)} + \frac{5\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}{9a^3} \\
&\quad - \frac{5x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}{18a} + \frac{1}{3}x^3\text{arcsinh}(ax)^{5/2} \\
&\quad + \frac{5\text{Subst}\left(\int \frac{e^{-3x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{576a^3} - \frac{5\text{Subst}\left(\int \frac{e^{3x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{576a^3} \\
&\quad - \frac{5\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{192a^3} + \frac{5\text{Subst}\left(\int \frac{e^{x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{192a^3} \\
&\quad - \frac{5\text{Subst}\left(\int e^{-x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{12a^3} + \frac{5\text{Subst}\left(\int e^{x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{12a^3}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{5x\sqrt{\text{arcsinh}(ax)}}{6a^2} + \frac{5}{36}x^3\sqrt{\text{arcsinh}(ax)} \\
&\quad + \frac{5\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}{9a^3} - \frac{5x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}{18a} \\
&\quad + \frac{1}{3}x^3\text{arcsinh}(ax)^{5/2} - \frac{5\sqrt{\pi}\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)}{24a^3} + \frac{5\sqrt{\pi}\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)}{24a^3} \\
&\quad + \frac{5\text{Subst}\left(\int e^{-3x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{288a^3} - \frac{5\text{Subst}\left(\int e^{3x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{288a^3} \\
&\quad - \frac{5\text{Subst}\left(\int e^{-x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{96a^3} + \frac{5\text{Subst}\left(\int e^{x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{96a^3}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{5x\sqrt{\text{arcsinh}(ax)}}{6a^2} + \frac{5}{36}x^3\sqrt{\text{arcsinh}(ax)} \\
&\quad + \frac{5\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}{9a^3} - \frac{5x^2\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}{18a} \\
&\quad + \frac{1}{3}x^3\text{arcsinh}(ax)^{5/2} - \frac{15\sqrt{\pi}\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)}{64a^3} + \frac{5\sqrt{\frac{\pi}{3}}\text{erf}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)}{576a^3} \\
&\quad + \frac{15\sqrt{\pi}\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)}{64a^3} - \frac{5\sqrt{\frac{\pi}{3}}\text{erfi}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)}{576a^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.47

$$\int x^2\text{arcsinh}(ax)^{5/2}\,dx = \frac{\frac{\sqrt{3}\sqrt{\text{arcsinh}(ax)}\Gamma\left(\frac{7}{2},-3\text{arcsinh}(ax)\right)}{\sqrt{-\text{arcsinh}(ax)}}+\frac{81\sqrt{-\text{arcsinh}(ax)}\Gamma\left(\frac{7}{2},-\text{arcsinh}(ax)\right)}{\sqrt{\text{arcsinh}(ax)}}+81\Gamma(\frac{7}{2},\text{arcs}}{648a^3}$$

```
[In] Integrate[x^2*ArcSinh[a*x]^(5/2),x]
```

```
[Out] ((Sqrt[3]*Sqrt[ArcSinh[a*x]]*Gamma[7/2, -3*ArcSinh[a*x]])/Sqrt[-ArcSinh[a*x
]] + (81*Sqrt[-ArcSinh[a*x]]*Gamma[7/2, -ArcSinh[a*x]])/Sqrt[ArcSinh[a*x]]
+ 81*Gamma[7/2, ArcSinh[a*x]] - Sqrt[3]*Gamma[7/2, 3*ArcSinh[a*x]])/(648*a^
3)
```

Maple [F]

$$\int x^2 \operatorname{arcsinh}(ax)^{\frac{5}{2}}\,dx$$

```
[In] int(x^2*arcsinh(a*x)^(5/2),x)
```

```
[Out] int(x^2*arcsinh(a*x)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^2\text{arcsinh}(ax)^{5/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2*arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x^2\text{arcsinh}(ax)^{5/2}\,dx = \int x^2 \operatorname{asinh}^{\frac{5}{2}}(ax)\,dx$$

```
[In] integrate(x**2*asinh(a*x)**(5/2),x)
```

```
[Out] Integral(x**2*asinh(a*x)**(5/2), x)
```

Maxima [F]

$$\int x^2 \text{arcsinh}(ax)^{5/2}\,dx = \int x^2\,\text{arsinh}\,(ax)^{\frac{5}{2}}\,dx$$

```
[In] integrate(x^2*arcsinh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*arcsinh(a*x)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int x^2 \text{arcsinh}(ax)^{5/2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^2*arcsinh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \text{arcsinh}(ax)^{5/2}\,dx = \int x^2\,\text{asinh}(a\,x)^{5/2}\,dx$$

```
[In] int(x^2*asinh(a*x)^(5/2),x)
```

```
[Out] int(x^2*asinh(a*x)^(5/2), x)
```

3.89 $\int x\text{arcsinh}(ax)^{5/2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 152

$$\begin{aligned}\int x\text{arcsinh}(ax)^{5/2}\,dx &= \frac{15\sqrt{\text{arcsinh}(ax)}}{64a^2}+\frac{15}{32}x^2\sqrt{\text{arcsinh}(ax)}\\&-\frac{5x\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}{8a}+\frac{\text{arcsinh}(ax)^{5/2}}{4a^2}\\&+\frac{1}{2}x^2\text{arcsinh}(ax)^{5/2}-\frac{15\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{256a^2}-\frac{15\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{256a^2}\end{aligned}$$

```
[Out] 1/4*arcsinh(a*x)^(5/2)/a^2+1/2*x^2*arcsinh(a*x)^(5/2)-15/512*erf(2^(1/2)*ar
csinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^2-15/512*erfi(2^(1/2)*arcsinh(a*x)^(1/
2))*2^(1/2)*Pi^(1/2)/a^2-5/8*x*arcsinh(a*x)^(3/2)*(a^2*x^2+1)^(1/2)/a+15/64
*arcsinh(a*x)^(1/2)/a^2+15/32*x^2*arcsinh(a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.900, Rules used = {5777, 5812, 5783, 5819, 3393, 3388, 2211, 2235, 2236}

$$\begin{aligned}\int x\text{arcsinh}(ax)^{5/2}\,dx &= -\frac{15\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{256a^2}\\&-\frac{15\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{256a^2}-\frac{5x\sqrt{a^2x^2+1}\text{arcsinh}(ax)^{3/2}}{8a}\\&+\frac{\text{arcsinh}(ax)^{5/2}}{4a^2}+\frac{15\sqrt{\text{arcsinh}(ax)}}{64a^2}+\frac{1}{2}x^2\text{arcsinh}(ax)^{5/2}+\frac{15}{32}x^2\sqrt{\text{arcsinh}(ax)}\end{aligned}$$

```
[In] Int[x*ArcSinh[a*x]^(5/2),x]
```

```
[Out] (15*Sqrt[ArcSinh[a*x]])/(64*a^2) + (15*x^2*Sqrt[ArcSinh[a*x]])/32 - (5*x*Sq
rt[1 + a^2*x^2]*ArcSinh[a*x]^(3/2))/(8*a) + ArcSinh[a*x]^(5/2)/(4*a^2) + (x
^2*ArcSinh[a*x]^(5/2))/2 - (15*Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/
(256*a^2) - (15*Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(256*a^2)
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
```

```
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}x^2\text{arcsinh}(ax)^{5/2} - \frac{1}{4}(5a)\int \frac{x^2\text{arcsinh}(ax)^{3/2}}{\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{5x\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}{8a} \\
&\quad + \frac{1}{2}x^2\text{arcsinh}(ax)^{5/2} + \frac{15}{16}\int x\sqrt{\text{arcsinh}(ax)}\,dx + \frac{5\int \frac{\text{arcsinh}(ax)^{3/2}}{\sqrt{1+a^2x^2}}\,dx}{8a} \\
&= \frac{15}{32}x^2\sqrt{\text{arcsinh}(ax)} - \frac{5x\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}{8a} + \frac{\text{arcsinh}(ax)^{5/2}}{4a^2} \\
&\quad + \frac{1}{2}x^2\text{arcsinh}(ax)^{5/2} - \frac{1}{64}(15a)\int \frac{x^2}{\sqrt{1+a^2x^2}\sqrt{\text{arcsinh}(ax)}}\,dx \\
&= \frac{15}{32}x^2\sqrt{\text{arcsinh}(ax)} - \frac{5x\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}{8a} + \frac{\text{arcsinh}(ax)^{5/2}}{4a^2} \\
&\quad + \frac{1}{2}x^2\text{arcsinh}(ax)^{5/2} - \frac{15\text{Subst}\left(\int \frac{\sinh^2(x)}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{64a^2} \\
&= \frac{15}{32}x^2\sqrt{\text{arcsinh}(ax)} - \frac{5x\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}{8a} + \frac{\text{arcsinh}(ax)^{5/2}}{4a^2} \\
&\quad + \frac{1}{2}x^2\text{arcsinh}(ax)^{5/2} + \frac{15\text{Subst}\left(\int \left(\frac{1}{2\sqrt{x}} - \frac{\cosh(2x)}{2\sqrt{x}}\right)\,dx, x, \text{arcsinh}(ax)\right)}{64a^2}
\end{aligned}$$

$$\begin{aligned}
&= \frac{15\sqrt{\text{arcsinh}(ax)}}{64a^2}+\frac{15}{32}x^2\sqrt{\text{arcsinh}(ax)}-\frac{5x\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}{8a} \\
&\quad +\frac{\text{arcsinh}(ax)^{5/2}}{4a^2}+\frac{1}{2}x^2\text{arcsinh}(ax)^{5/2}-\frac{15\text{Subst}\left(\int\frac{\cosh(2x)}{\sqrt{x}}\,dx,x,\text{arcsinh}(ax)\right)}{128a^2} \\
&= \frac{15\sqrt{\text{arcsinh}(ax)}}{64a^2}+\frac{15}{32}x^2\sqrt{\text{arcsinh}(ax)} \\
&\quad -\frac{5x\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}{8a}+\frac{\text{arcsinh}(ax)^{5/2}}{4a^2}+\frac{1}{2}x^2\text{arcsinh}(ax)^{5/2} \\
&\quad -\frac{15\text{Subst}\left(\int\frac{e^{-2x}}{\sqrt{x}}\,dx,x,\text{arcsinh}(ax)\right)}{256a^2}-\frac{15\text{Subst}\left(\int\frac{e^{2x}}{\sqrt{x}}\,dx,x,\text{arcsinh}(ax)\right)}{256a^2} \\
&= \frac{15\sqrt{\text{arcsinh}(ax)}}{64a^2}+\frac{15}{32}x^2\sqrt{\text{arcsinh}(ax)}-\frac{5x\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}{8a} \\
&\quad +\frac{\text{arcsinh}(ax)^{5/2}}{4a^2}+\frac{1}{2}x^2\text{arcsinh}(ax)^{5/2}-\frac{15\text{Subst}\left(\int e^{-2x^2}\,dx,x,\sqrt{\text{arcsinh}(ax)}\right)}{128a^2} \\
&\quad -\frac{15\text{Subst}\left(\int e^{2x^2}\,dx,x,\sqrt{\text{arcsinh}(ax)}\right)}{128a^2} \\
&= \frac{15\sqrt{\text{arcsinh}(ax)}}{64a^2}+\frac{15}{32}x^2\sqrt{\text{arcsinh}(ax)}-\frac{5x\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}{8a}+\frac{\text{arcsinh}(ax)^{5/2}}{4a^2} \\
&\quad +\frac{1}{2}x^2\text{arcsinh}(ax)^{5/2}-\frac{15\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{256a^2}-\frac{15\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{256a^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.34

$$\int x\text{arcsinh}(ax)^{5/2}\,dx=\frac{\frac{\sqrt{\text{arcsinh}(ax)}\Gamma\left(\frac{7}{2},-2\text{arcsinh}(ax)\right)}{\sqrt{-\text{arcsinh}(ax)}}+\Gamma\left(\frac{7}{2},2\text{arcsinh}(ax)\right)}{32\sqrt{2}a^2}$$

```
[In] Integrate[x*ArcSinh[a*x]^(5/2),x]
```

```
[Out] ((Sqrt[ArcSinh[a*x]]*Gamma[7/2, -2*ArcSinh[a*x]])/Sqrt[-ArcSinh[a*x]] + Gam
ma[7/2, 2*ArcSinh[a*x]])/(32*Sqrt[2]*a^2)
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.89

method	result
default	$-\frac{\sqrt{2}\,\Big(-128\operatorname{arcsinh}(ax)^{\frac{5}{2}}\sqrt{2}\,\sqrt{\pi}\,a^{2}x^{2}+160\operatorname{arcsinh}(ax)^{\frac{3}{2}}\sqrt{2}\,\sqrt{\pi}\,\sqrt{a^{2}x^{2}+1}\,ax-120\sqrt{2}\,\sqrt{\operatorname{arcsinh}(ax)}\,\sqrt{\pi}\,a^{2}x^{2}-64\operatorname{arcsinh}(ax)^{\frac{5}{2}}\sqrt{2}}{512\sqrt{\pi}\,a^{2}}$

```
[In] int(x*arcsinh(a*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/512*2^(1/2)*(-128*arcsinh(a*x)^(5/2)*2^(1/2)*Pi^(1/2)*a^2*x^2+160*arcsin
h(a*x)^(3/2)*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*a*x-120*2^(1/2)*arcsinh(a*x
)^(1/2)*Pi^(1/2)*a^2*x^2-64*arcsinh(a*x)^(5/2)*2^(1/2)*Pi^(1/2)-60*2^(1/2)*
arcsinh(a*x)^(1/2)*Pi^(1/2)+15*Pi*erf(2^(1/2)*arcsinh(a*x)^(1/2))+15*Pi*erf
i(2^(1/2)*arcsinh(a*x)^(1/2)))/Pi^(1/2)/a^2
```

Fricas [F(-2)]

Exception generated.

$$\int x\text{arcsinh}(ax)^{5/2}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(x*arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x\text{arcsinh}(ax)^{5/2}\,dx=\int x\operatorname{asinh}^{\frac{5}{2}}(ax)\,dx$$

```
[In] integrate(x*asinh(a*x)**(5/2),x)
```

```
[Out] Integral(x*asinh(a*x)**(5/2), x)
```

Maxima [F]

$$\int x\mathrm{arcsinh}(ax)^{5/2}\,dx = \int x\,\mathrm{arsinh}\left(ax\right)^{\frac{5}{2}}\,dx$$

```
[In] integrate(x*arcsinh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x*arcsinh(a*x)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int x\mathrm{arcsinh}(ax)^{5/2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x*arcsinh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x\mathrm{arcsinh}(ax)^{5/2}\,dx = \int x\,\mathrm{asinh}(a\,x)^{5/2}\,dx$$

```
[In] int(x*asinh(a*x)^(5/2),x)
```

```
[Out] int(x*asinh(a*x)^(5/2), x)
```

3.90 $\int \operatorname{arcsinh}(ax)^{5/2}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 94

$$\int \operatorname{arcsinh}(ax)^{5/2}\,dx=\frac{15}{4}x\sqrt{\operatorname{arcsinh}(ax)}-\frac{5\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^{3/2}}{2a}+x\operatorname{arcsinh}(ax)^{5/2}+\frac{15\sqrt{\pi}\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{16a}-\frac{15\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{16a}$$

```
[Out] x*arcsinh(a*x)^(5/2)+15/16*erf(arcsinh(a*x)^(1/2))*Pi^(1/2)/a-15/16*erfi(ar
csinh(a*x)^(1/2))*Pi^(1/2)/a-5/2*arcsinh(a*x)^(3/2)*(a^2*x^2+1)^(1/2)/a+15/
4*x*arcsinh(a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {5772, 5798, 5819, 3389, 2211, 2235, 2236}

$$\int \operatorname{arcsinh}(ax)^{5/2}\,dx=-\frac{5\sqrt{a^2x^2+1}\operatorname{arcsinh}(ax)^{3/2}}{2a}+\frac{15\sqrt{\pi}\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{16a}-\frac{15\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{16a}+x\operatorname{arcsinh}(ax)^{5/2}+\frac{15}{4}x\sqrt{\operatorname{arcsinh}(ax)}$$

```
[In] Int[ArcSinh[a*x]^(5/2),x]
```

```
[Out] (15*x*Sqrt[ArcSinh[a*x]])/4 - (5*Sqrt[1 + a^2*x^2]*ArcSinh[a*x]^(3/2))/(2*a
) + x*ArcSinh[a*x]^(5/2) + (15*Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]])/(16*a) - (
15*Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x]]])/(16*a)
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= x\text{arcsinh}(ax)^{5/2} - \frac{1}{2}(5a)\int \frac{x\text{arcsinh}(ax)^{3/2}}{\sqrt{1+a^2x^2}}\,dx \\
&= -\frac{5\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}{2a} + x\text{arcsinh}(ax)^{5/2} + \frac{15}{4}\int \sqrt{\text{arcsinh}(ax)}\,dx \\
&= \frac{15}{4}x\sqrt{\text{arcsinh}(ax)} - \frac{5\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}{2a} \\
&\quad + x\text{arcsinh}(ax)^{5/2} - \frac{1}{8}(15a)\int \frac{x}{\sqrt{1+a^2x^2}\sqrt{\text{arcsinh}(ax)}}\,dx \\
&= \frac{15}{4}x\sqrt{\text{arcsinh}(ax)} - \frac{5\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}{2a} \\
&\quad + x\text{arcsinh}(ax)^{5/2} - \frac{15\text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{8a} \\
&= \frac{15}{4}x\sqrt{\text{arcsinh}(ax)} - \frac{5\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}{2a} + x\text{arcsinh}(ax)^{5/2} \\
&\quad + \frac{15\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{16a} - \frac{15\text{Subst}\left(\int \frac{e^{x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{16a} \\
&= \frac{15}{4}x\sqrt{\text{arcsinh}(ax)} - \frac{5\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}{2a} + x\text{arcsinh}(ax)^{5/2} \\
&\quad + \frac{15\text{Subst}\left(\int e^{-x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{8a} - \frac{15\text{Subst}\left(\int e^{x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{8a} \\
&= \frac{15}{4}x\sqrt{\text{arcsinh}(ax)} - \frac{5\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}{2a} \\
&\quad + x\text{arcsinh}(ax)^{5/2} + \frac{15\sqrt{\pi}\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)}{16a} - \frac{15\sqrt{\pi}\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)}{16a}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.48

$$
\int \text{arcsinh}(ax)^{5/2}\,dx = -\frac{\frac{\sqrt{-\text{arcsinh}(ax)}\Gamma\left(\frac{7}{2}, -\text{arcsinh}(ax)\right)}{\sqrt{\text{arcsinh}(ax)}} + \Gamma\left(\frac{7}{2}, \text{arcsinh}(ax)\right)}{2a}
$$

```
[In] Integrate[ArcSinh[a*x]^(5/2),x]
```

```
[Out] -1/2*((Sqrt[-ArcSinh[a*x]]*Gamma[7/2, -ArcSinh[a*x]])/Sqrt[ArcSinh[a*x]] +
Gamma[7/2, ArcSinh[a*x]])/a
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.83

method	result
default	$-\frac{-16 \operatorname{arcsinh}(ax)^{\frac{5}{2}} \sqrt{\pi}\, ax+40 \operatorname{arcsinh}(ax)^{\frac{3}{2}} \sqrt{\pi}\, \sqrt{a^2x^2+1}-60 \sqrt{\operatorname{arcsinh}(ax)}\, \sqrt{\pi}\, ax-15\pi \operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)+15\pi \operatorname{erfi}(\sqrt{\operatorname{arcsinh}(a}}{16\sqrt{\pi}\, a}$

```
[In] int(arcsinh(a*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*(-16*arcsinh(a*x)^(5/2)*Pi^(1/2)*a*x+40*arcsinh(a*x)^(3/2)*Pi^(1/2)*(
a^2*x^2+1)^(1/2)-60*arcsinh(a*x)^(1/2)*Pi^(1/2)*a*x-15*Pi*erf(arcsinh(a*x)^
(1/2))+15*Pi*erfi(arcsinh(a*x)^(1/2)))/Pi^(1/2)/a
```

Fricas [F(-2)]

Exception generated.

$$\int \operatorname{arcsinh}(ax)^{5/2}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \operatorname{arcsinh}(ax)^{5/2}\, dx = \int \operatorname{asinh}^{\frac{5}{2}}(ax)\, dx$$

```
[In] integrate(asinh(a*x)**(5/2),x)
```

```
[Out] Integral(asinh(a*x)**(5/2), x)
```

Maxima [F]

$$\int \operatorname{arcsinh}(ax)^{5/2}\, dx = \int \operatorname{arsinh}(ax)^{\frac{5}{2}}\, dx$$

```
[In] integrate(arcsinh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^(5/2), x)
```

Giac [**F(-2)**]

Exception generated.

$$\int \operatorname{arcsinh}(ax)^{5/2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(arcsinh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [**F(-1)**]

Timed out.

$$\int \operatorname{arcsinh}(ax)^{5/2}\,dx = \int \operatorname{asinh}(a\,x)^{5/2}\,dx$$

```
[In] int(asinh(a*x)^(5/2),x)
```

```
[Out] int(asinh(a*x)^(5/2), x)
```

3.91 $\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{x}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{x}\,dx=\operatorname{Int}\left(\frac{\operatorname{arcsinh}(ax)^{5/2}}{x},x\right)$$

```
[Out] Unintegrable(arcsinh(a*x)^(5/2)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{x}\,dx=\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{x}\,dx$$

```
[In] Int[ArcSinh[a*x]^(5/2)/x,x]
```

```
[Out] Defer[Int][ArcSinh[a*x]^(5/2)/x, x]
```

Rubi steps

$$\text{integral}=\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{x}\,dx = \int \frac{\operatorname{arcsinh}(ax)^{5/2}}{x}\,dx$$

```
[In] Integrate[ArcSinh[a*x]^(5/2)/x,x]
```

```
[Out] Integrate[ArcSinh[a*x]^(5/2)/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\operatorname{arcsinh}\left(ax\right)^{\frac{5}{2}}}{x}dx$$

```
[In] int(arcsinh(a*x)^(5/2)/x,x)
```

```
[Out] int(arcsinh(a*x)^(5/2)/x,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{x}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(arcsinh(a*x)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 19.29 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{x}\,dx = \int \frac{\operatorname{asinh}^{\frac{5}{2}}\left(ax\right)}{x}\,dx$$

```
[In] integrate(asinh(a*x)**(5/2)/x,x)
```

```
[Out] Integral(asinh(a*x)**(5/2)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{x}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^{\frac{5}{2}}}{x}\,dx$$

```
[In] integrate(arcsinh(a*x)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^(5/2)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{x}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^{\frac{5}{2}}}{x}\,dx$$

```
[In] integrate(arcsinh(a*x)^(5/2)/x,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^(5/2)/x, x)
```

Mupad [N/A]

Not integrable

Time = 2.37 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}(ax)^{5/2}}{x}\,dx = \int \frac{\operatorname{asinh}(a\,x)^{5/2}}{x}\,dx$$

```
[In] int(asinh(a*x)^(5/2)/x,x)
```

```
[Out] int(asinh(a*x)^(5/2)/x, x)
```

3.92 $\int \frac{x^4}{\sqrt{\text{arcsinh}(ax)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 163

$$\int \frac{x^4}{\sqrt{\text{arcsinh}(ax)}}\,dx = \frac{\sqrt{\pi}\,\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)}{16a^5} - \frac{\sqrt{3\pi}\,\text{erf}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)}{32a^5} + \frac{\sqrt{\frac{\pi}{5}}\,\text{erf}\left(\sqrt{5}\sqrt{\text{arcsinh}(ax)}\right)}{32a^5} + \frac{\sqrt{\pi}\,\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)}{16a^5} - \frac{\sqrt{3\pi}\,\text{erfi}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)}{32a^5} + \frac{\sqrt{\frac{\pi}{5}}\,\text{erfi}\left(\sqrt{5}\sqrt{\text{arcsinh}(ax)}\right)}{32a^5}$$

```
[Out] 1/160*erf(5^(1/2)*arcsinh(a*x)^(1/2))*5^(1/2)*Pi^(1/2)/a^5+1/160*erfi(5^(1/
2)*arcsinh(a*x)^(1/2))*5^(1/2)*Pi^(1/2)/a^5+1/16*erf(arcsinh(a*x)^(1/2))*Pi
^(1/2)/a^5+1/16*erfi(arcsinh(a*x)^(1/2))*Pi^(1/2)/a^5-1/32*erf(3^(1/2)*arcs
inh(a*x)^(1/2))*3^(1/2)*Pi^(1/2)/a^5-1/32*erfi(3^(1/2)*arcsinh(a*x)^(1/2))*
3^(1/2)*Pi^(1/2)/a^5
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used

$= \{5780, 5556, 3388, 2211, 2235, 2236\}$

$$\int \frac{x^4}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \frac{\sqrt{\pi}\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{16a^5} - \frac{\sqrt{3\pi}\operatorname{erf}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{32a^5} + \frac{\sqrt{\frac{\pi}{5}}\operatorname{erf}\left(\sqrt{5}\sqrt{\operatorname{arcsinh}(ax)}\right)}{32a^5} + \frac{\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{16a^5} - \frac{\sqrt{3\pi}\operatorname{erfi}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{32a^5} + \frac{\sqrt{\frac{\pi}{5}}\operatorname{erfi}\left(\sqrt{5}\sqrt{\operatorname{arcsinh}(ax)}\right)}{32a^5}$$

```
[In] Int[x^4/Sqrt[ArcSinh[a*x]],x]
```

```
[Out] (Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]])/(16*a^5) - (Sqrt[3*Pi]*Erf[Sqrt[3]*Sqrt[
ArcSinh[a*x]]])/(32*a^5) + (Sqrt[Pi/5]*Erf[Sqrt[5]*Sqrt[ArcSinh[a*x]]])/(32
*a^5) + (Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x]]])/(16*a^5) - (Sqrt[3*Pi]*Erfi[Sqr
t[3]*Sqrt[ArcSinh[a*x]]])/(32*a^5) + (Sqrt[Pi/5]*Erfi[Sqrt[5]*Sqrt[ArcSinh[
a*x]]])/(32*a^5)
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
```

```
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\cosh(x)\sinh^4(x)}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{a^5}\\
&= \frac{\text{Subst}\left(\int \left(\frac{\cosh(x)}{8\sqrt{x}} - \frac{3\cosh(3x)}{16\sqrt{x}} + \frac{\cosh(5x)}{16\sqrt{x}}\right) dx, x, \text{arcsinh}(ax)\right)}{a^5}\\
&= \frac{\text{Subst}\left(\int \frac{\cosh(5x)}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{16a^5} + \frac{\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{8a^5}\\
&\quad - \frac{3\text{Subst}\left(\int \frac{\cosh(3x)}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{16a^5}\\
&= \frac{\text{Subst}\left(\int \frac{e^{-5x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{32a^5} + \frac{\text{Subst}\left(\int \frac{e^{5x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{32a^5}\\
&\quad + \frac{\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{16a^5} + \frac{\text{Subst}\left(\int \frac{e^{x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{16a^5}\\
&\quad - \frac{3\text{Subst}\left(\int \frac{e^{-3x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{32a^5} - \frac{3\text{Subst}\left(\int \frac{e^{3x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{32a^5}\\
&= \frac{\text{Subst}\left(\int e^{-5x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{16a^5} + \frac{\text{Subst}\left(\int e^{5x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{16a^5}\\
&\quad + \frac{\text{Subst}\left(\int e^{-x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{8a^5} + \frac{\text{Subst}\left(\int e^{x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{8a^5}\\
&\quad - \frac{3\text{Subst}\left(\int e^{-3x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{16a^5} - \frac{3\text{Subst}\left(\int e^{3x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{16a^5}\\
&= \frac{\sqrt{\pi}\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)}{16a^5} - \frac{\sqrt{3\pi}\text{erf}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)}{32a^5} + \frac{\sqrt{\frac{\pi}{5}}\text{erf}\left(\sqrt{5}\sqrt{\text{arcsinh}(ax)}\right)}{32a^5}\\
&\quad + \frac{\sqrt{\pi}\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)}{16a^5} - \frac{\sqrt{3\pi}\text{erfi}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)}{32a^5} + \frac{\sqrt{\frac{\pi}{5}}\text{erfi}\left(\sqrt{5}\sqrt{\text{arcsinh}(ax)}\right)}{32a^5}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.93

$$\int \frac{x^4}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \frac{\frac{\sqrt{5}\sqrt{-\operatorname{arcsinh}(ax)}\Gamma\left(\frac{1}{2},-5\operatorname{arcsinh}(ax)\right)}{\sqrt{\operatorname{arcsinh}(ax)}}+\frac{5\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\Gamma\left(\frac{1}{2},-3\operatorname{arcsinh}(ax)\right)}{\sqrt{-\operatorname{arcsinh}(ax)}}+\frac{10\sqrt{-\operatorname{arcsinh}(ax)}\Gamma\left(\frac{1}{2},-\operatorname{arcsinh}(ax)\right)}{\sqrt{\operatorname{arcsinh}(ax)}}}{160a^5}$$

```
[In] Integrate[x^4/Sqrt[ArcSinh[a*x]],x]
```

```
[Out] ((Sqrt[5]*Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -5*ArcSinh[a*x]])/Sqrt[ArcSinh[a*x
]] + (5*Sqrt[3]*Sqrt[ArcSinh[a*x]]*Gamma[1/2, -3*ArcSinh[a*x]])/Sqrt[-ArcSi
nh[a*x]] + (10*Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -ArcSinh[a*x]])/Sqrt[ArcSinh[
a*x]] - 10*Gamma[1/2, ArcSinh[a*x]] + 5*Sqrt[3]*Gamma[1/2, 3*ArcSinh[a*x]]
- Sqrt[5]*Gamma[1/2, 5*ArcSinh[a*x]])/(160*a^5)
```

Maple [F]

$$\int \frac{x^4}{\sqrt{\operatorname{arcsinh}\left(ax\right)}}dx$$

```
[In] int(x^4/arcsinh(a*x)^(1/2),x)
```

```
[Out] int(x^4/arcsinh(a*x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^4}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^4/arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^4}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{x^4}{\sqrt{\operatorname{asinh}(ax)}}\,dx$$

```
[In] integrate(x**4/asinh(a*x)**(1/2),x)
[Out] Integral(x**4/sqrt(asinh(a*x)), x)
```

Maxima [F]

$$\int \frac{x^4}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{x^4}{\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

```
[In] integrate(x^4/arcsinh(a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(x^4/sqrt(arcsinh(a*x)), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{x^4}{\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

```
[In] integrate(x^4/arcsinh(a*x)^(1/2),x, algorithm="giac")
[Out] integrate(x^4/sqrt(arcsinh(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{x^4}{\sqrt{\operatorname{asinh}(a\,x)}}\,dx$$

```
[In] int(x^4/asinh(a*x)^(1/2),x)
[Out] int(x^4/asinh(a*x)^(1/2), x)
```

3.93 $\int \frac{x^3}{\sqrt{\textbf{arcsinh}(ax)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 109

$$\int \frac{x^3}{\sqrt{\text{arcsinh}(ax)}}\,dx = -\frac{\sqrt{\pi}\,\text{erf}\left(2\sqrt{\text{arcsinh}(ax)}\right)}{32a^4} + \frac{\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{8a^4} + \frac{\sqrt{\pi}\,\text{erfi}\left(2\sqrt{\text{arcsinh}(ax)}\right)}{32a^4} - \frac{\sqrt{\frac{\pi}{2}}\,\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{8a^4}$$

```
[Out] 1/16*erf(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^4-1/16*erfi(2^(1/2)
*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^4-1/32*erf(2*arcsinh(a*x)^(1/2))*Pi
^(1/2)/a^4+1/32*erfi(2*arcsinh(a*x)^(1/2))*Pi^(1/2)/a^4
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5780, 5556, 3389, 2211, 2235, 2236}

$$\int \frac{x^3}{\sqrt{\text{arcsinh}(ax)}}\,dx = -\frac{\sqrt{\pi}\,\text{erf}\left(2\sqrt{\text{arcsinh}(ax)}\right)}{32a^4} + \frac{\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{8a^4} + \frac{\sqrt{\pi}\,\text{erfi}\left(2\sqrt{\text{arcsinh}(ax)}\right)}{32a^4} - \frac{\sqrt{\frac{\pi}{2}}\,\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{8a^4}$$

```
[In] Int[x^3/Sqrt[ArcSinh[a*x]],x]
```

```
[Out] -1/32*(Sqrt[Pi]*Erf[2*Sqrt[ArcSinh[a*x]]])/a^4 + (Sqrt[Pi/2]*Erf[Sqrt[2]*Sq
rt[ArcSinh[a*x]]])/(8*a^4) + (Sqrt[Pi]*Erfi[2*Sqrt[ArcSinh[a*x]]])/(32*a^4)
 - (Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(8*a^4)
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\cosh(x)\sinh^3(x)}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{a^4}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{\sinh(2x)}{4\sqrt{x}}+\frac{\sinh(4x)}{8\sqrt{x}}\right)\,dx, x, \operatorname{arcsinh}(ax)\right)}{a^4}\\
&= \frac{\text{Subst}\left(\int \frac{\sinh(4x)}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{8a^4}-\frac{\text{Subst}\left(\int \frac{\sinh(2x)}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{4a^4}
\end{aligned}
$$

$$= -\frac{\text{Subst}\left(\int \frac{e^{-4x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{16a^4} + \frac{\text{Subst}\left(\int \frac{e^{4x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{16a^4} + \frac{\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{8a^4} - \frac{\text{Subst}\left(\int \frac{e^{2x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{8a^4}$$

$$= -\frac{\text{Subst}\left(\int e^{-4x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{8a^4} + \frac{\text{Subst}\left(\int e^{4x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{8a^4} + \frac{\text{Subst}\left(\int e^{-2x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{4a^4} - \frac{\text{Subst}\left(\int e^{2x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{4a^4}$$

$$= -\frac{\sqrt{\pi}\,\text{erf}\left(2\sqrt{\text{arcsinh}(ax)}\right)}{32a^4} + \frac{\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{8a^4} + \frac{\sqrt{\pi}\,\text{erfi}\left(2\sqrt{\text{arcsinh}(ax)}\right)}{32a^4} - \frac{\sqrt{\frac{\pi}{2}}\,\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{8a^4}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.91

$$\int \frac{x^3}{\sqrt{\text{arcsinh}(ax)}}\,dx$$

$$= \frac{\frac{\sqrt{-\text{arcsinh}(ax)}\Gamma\left(\frac{1}{2},-4\text{arcsinh}(ax)\right)}{\sqrt{\text{arcsinh}(ax)}} + \frac{2\sqrt{2}\sqrt{\text{arcsinh}(ax)}\Gamma\left(\frac{1}{2},-2\text{arcsinh}(ax)\right)}{\sqrt{-\text{arcsinh}(ax)}} - 2\sqrt{2}\Gamma\left(\frac{1}{2}, 2\text{arcsinh}(ax)\right) + \Gamma(\frac{1}{2}, 4\text{arc}}{32a^4}$$

```
[In] Integrate[x^3/Sqrt[ArcSinh[a*x]],x]
```

```
[Out] ((Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -4*ArcSinh[a*x]])/Sqrt[ArcSinh[a*x]] + (2*
Sqrt[2]*Sqrt[ArcSinh[a*x]]*Gamma[1/2, -2*ArcSinh[a*x]])/Sqrt[-ArcSinh[a*x]]
 - 2*Sqrt[2]*Gamma[1/2, 2*ArcSinh[a*x]] + Gamma[1/2, 4*ArcSinh[a*x]])/(32*a
^4)
```

Maple [F]

$$\int \frac{x^3}{\sqrt{\text{arcsinh}\,(ax)}}dx$$

```
[In] int(x^3/arcsinh(a*x)^(1/2),x)
```

```
[Out] int(x^3/arcsinh(a*x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3/arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{x^3}{\sqrt{\operatorname{asinh}(ax)}}\,dx$$

```
[In] integrate(x**3/asinh(a*x)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(asinh(a*x)), x)
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{x^3}{\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

```
[In] integrate(x^3/arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/sqrt(arcsinh(a*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3/arcsinh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{x^3}{\sqrt{\operatorname{asinh}\left(a\,x\right)}}\,dx$$

```
[In] int(x^3/asinh(a*x)^(1/2),x)
```

```
[Out] int(x^3/asinh(a*x)^(1/2), x)
```

3.94 $\int \frac{x^2}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 105

$$\int \frac{x^2}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = -\frac{\sqrt{\pi}\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{8a^3} + \frac{\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{8a^3} - \frac{\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{8a^3} + \frac{\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{8a^3}$$

```
[Out] 1/24*erf(3^(1/2)*arcsinh(a*x)^(1/2))*3^(1/2)*Pi^(1/2)/a^3+1/24*erfi(3^(1/2)
*arcsinh(a*x)^(1/2))*3^(1/2)*Pi^(1/2)/a^3-1/8*erf(arcsinh(a*x)^(1/2))*Pi^(1
/2)/a^3-1/8*erfi(arcsinh(a*x)^(1/2))*Pi^(1/2)/a^3
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5780, 5556, 3388, 2211, 2235, 2236}

$$\int \frac{x^2}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = -\frac{\sqrt{\pi}\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{8a^3} + \frac{\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{8a^3} - \frac{\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{8a^3} + \frac{\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{8a^3}$$

```
[In] Int[x^2/Sqrt[ArcSinh[a*x]],x]
```

```
[Out] -1/8*(Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]])/a^3 + (Sqrt[Pi/3]*Erf[Sqrt[3]*Sqrt[
ArcSinh[a*x]]])/(8*a^3) - (Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x]]])/(8*a^3) + (Sq
rt[Pi/3]*Erfi[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(8*a^3)
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{a^3}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{\cosh(x)}{4\sqrt{x}}+\frac{\cosh(3x)}{4\sqrt{x}}\right)\,dx, x, \operatorname{arcsinh}(ax)\right)}{a^3}
\end{aligned}
$$

$$= -\frac{\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{4a^3} + \frac{\text{Subst}\left(\int \frac{\cosh(3x)}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{4a^3}$$

$$= \frac{\text{Subst}\left(\int \frac{e^{-3x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{8a^3} - \frac{\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{8a^3} - \frac{\text{Subst}\left(\int \frac{e^{x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{8a^3} + \frac{\text{Subst}\left(\int \frac{e^{3x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{8a^3}$$

$$= \frac{\text{Subst}\left(\int e^{-3x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{4a^3} - \frac{\text{Subst}\left(\int e^{-x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{4a^3} - \frac{\text{Subst}\left(\int e^{x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{4a^3} + \frac{\text{Subst}\left(\int e^{3x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{4a^3}$$

$$= -\frac{\sqrt{\pi}\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)}{8a^3} + \frac{\sqrt{\frac{\pi}{3}}\text{erf}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)}{8a^3} - \frac{\sqrt{\pi}\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)}{8a^3} + \frac{\sqrt{\frac{\pi}{3}}\text{erfi}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)}{8a^3}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{\sqrt{\text{arcsinh}(ax)}}\,dx$$

$$= \frac{\frac{\sqrt{3}\sqrt{-\text{arcsinh}(ax)}\Gamma\big(\frac{1}{2},-3\text{arcsinh}(ax)\big)}{\sqrt{\text{arcsinh}(ax)}} + \frac{3\sqrt{\text{arcsinh}(ax)}\Gamma\big(\frac{1}{2},-\text{arcsinh}(ax)\big)}{\sqrt{-\text{arcsinh}(ax)}} + 3\Gamma\big(\frac{1}{2},\text{arcsinh}(ax)\big) - \sqrt{3}\Gamma\big(\frac{1}{2},3\text{arcs}}{24a^3}$$

```
[In] Integrate[x^2/Sqrt[ArcSinh[a*x]],x]
```

```
[Out] ((Sqrt[3]*Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -3*ArcSinh[a*x]])/Sqrt[ArcSinh[a*x
]] + (3*Sqrt[ArcSinh[a*x]]*Gamma[1/2, -ArcSinh[a*x]])/Sqrt[-ArcSinh[a*x]] +
 3*Gamma[1/2, ArcSinh[a*x]] - Sqrt[3]*Gamma[1/2, 3*ArcSinh[a*x]])/(24*a^3)
```

Maple [F]

$$\int \frac{x^2}{\sqrt{\operatorname{arcsinh}(ax)}}dx$$

```
[In] int(x^2/arcsinh(a*x)^(1/2),x)
```

```
[Out] int(x^2/arcsinh(a*x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2/arcsinh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{x^2}{\sqrt{\operatorname{asinh}(ax)}}\,dx$$

```
[In] integrate(x**2/asinh(a*x)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(asinh(a*x)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{x^2}{\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

```
[In] integrate(x^2/arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(arcsinh(a*x)), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{x^2}{\sqrt{\operatorname{arsinh}\left(ax\right)}}\,dx$$

```
[In] integrate(x^2/arcsinh(a*x)^(1/2),x, algorithm="giac")
[Out] integrate(x^2/sqrt(arcsinh(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{x^2}{\sqrt{\operatorname{asinh}\left(a\,x\right)}}\,dx$$

```
[In] int(x^2/asinh(a*x)^(1/2),x)
[Out] int(x^2/asinh(a*x)^(1/2), x)
```

3.95 $\int \frac{x}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 63

$$\int \frac{x}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = -\frac{\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{4a^2} + \frac{\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{4a^2}$$

```
[Out] -1/8*erf(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^2+1/8*erfi(2^(1/2)*
arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {5780, 5556, 12, 3389, 2211, 2235, 2236}

$$\int \frac{x}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \frac{\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{4a^2} - \frac{\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{4a^2}$$

```
[In] Int[x/Sqrt[ArcSinh[a*x]],x]
```

```
[Out] -1/4*(Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/a^2 + (Sqrt[Pi/2]*Erfi[Sq
rt[2]*Sqrt[ArcSinh[a*x]]])/(4*a^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{a^2} \\
&= \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{2\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{a^2} \\
&= \frac{\text{Subst}\left(\int \frac{\sinh(2x)}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{2a^2}
\end{aligned}
$$

$$= -\frac{\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{4a^2} + \frac{\text{Subst}\left(\int \frac{e^{2x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{4a^2}$$

$$= -\frac{\text{Subst}\left(\int e^{-2x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{2a^2} + \frac{\text{Subst}\left(\int e^{2x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{2a^2}$$

$$= -\frac{\sqrt{\frac{\pi}{2}}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{4a^2} + \frac{\sqrt{\frac{\pi}{2}}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{4a^2}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int \frac{x}{\sqrt{\text{arcsinh}(ax)}}\,dx = \frac{\frac{\sqrt{-\text{arcsinh}(ax)}\Gamma\left(\frac{1}{2},-2\text{arcsinh}(ax)\right)}{\sqrt{\text{arcsinh}(ax)}} + \Gamma\left(\frac{1}{2}, 2\text{arcsinh}(ax)\right)}{4\sqrt{2}a^2}$$

```
[In] Integrate[x/Sqrt[ArcSinh[a*x]],x]
```

```
[Out] ((Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -2*ArcSinh[a*x]])/Sqrt[ArcSinh[a*x]] + Gam
ma[1/2, 2*ArcSinh[a*x]])/(4*Sqrt[2]*a^2)
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.59

method	result	size
default	$-\frac{\sqrt{\pi}\,\sqrt{2}\,\left(\text{erf}\left(\sqrt{2}\,\sqrt{\text{arcsinh}(ax)}\right)-\text{erfi}\left(\sqrt{2}\,\sqrt{\text{arcsinh}(ax)}\right)\right)}{8a^2}$	37

```
[In] int(x/arcsinh(a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*Pi^(1/2)*2^(1/2)*(erf(2^(1/2)*arcsinh(a*x)^(1/2))-erfi(2^(1/2)*arcsinh
(a*x)^(1/2)))/a^2
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x/arcsinh(a*x)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{x}{\sqrt{\operatorname{asinh}(ax)}}\,dx$$

```
[In] integrate(x/asinh(a*x)**(1/2),x)
[Out] Integral(x/sqrt(asinh(a*x)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{x}{\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

```
[In] integrate(x/arcsinh(a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(x/sqrt(arcsinh(a*x)), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{x}{\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

```
[In] integrate(x/arcsinh(a*x)^(1/2),x, algorithm="giac")
[Out] integrate(x/sqrt(arcsinh(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{x}{\sqrt{\operatorname{asinh}\left(a\,x\right)}}\,dx$$

```
[In] int(x/asinh(a*x)^(1/2),x)
[Out] int(x/asinh(a*x)^(1/2), x)
```

3.96 $\int \frac{1}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 43

$$\int \frac{1}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \frac{\sqrt{\pi}\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{2a} + \frac{\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{2a}$$

```
[Out] 1/2*erf(arcsinh(a*x)^(1/2))*Pi^(1/2)/a+1/2*erfi(arcsinh(a*x)^(1/2))*Pi^(1/2
)/a
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5774, 3388, 2211, 2235, 2236}

$$\int \frac{1}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \frac{\sqrt{\pi}\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{2a} + \frac{\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{2a}$$

```
[In] Int[1/Sqrt[ArcSinh[a*x]],x]
```

```
[Out] (Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]])/(2*a) + (Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x]
]])/(2*a)
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{a}\\
&= \frac{\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{2a} + \frac{\text{Subst}\left(\int \frac{e^{x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{2a}\\
&= \frac{\text{Subst}\left(\int e^{-x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{a} + \frac{\text{Subst}\left(\int e^{x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{a}\\
&= \frac{\sqrt{\pi}\,\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)}{2a} + \frac{\sqrt{\pi}\,\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)}{2a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.09

$$\int \frac{1}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \frac{\frac{\sqrt{-\operatorname{arcsinh}(ax)}\Gamma\left(\frac{1}{2},-\operatorname{arcsinh}(ax)\right)}{\sqrt{\operatorname{arcsinh}(ax)}} - \Gamma\left(\frac{1}{2},\operatorname{arcsinh}(ax)\right)}{2a}$$

```
[In] Integrate[1/Sqrt[ArcSinh[a*x]],x]
[Out] ((Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -ArcSinh[a*x]])/Sqrt[ArcSinh[a*x]] - Gamma
[1/2, ArcSinh[a*x]])/(2*a)
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.56

method	result	size
default	$\frac{\sqrt{\pi}\left(\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)+\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)\right)}{2a}$	24

```
[In] int(1/arcsinh(a*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*Pi^(1/2)*(erf(arcsinh(a*x)^(1/2))+erfi(arcsinh(a*x)^(1/2)))/a
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/arcsinh(a*x)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{1}{\sqrt{\operatorname{asinh}{\left(ax \right)}}}\,dx$$

```
[In] integrate(1/asinh(a*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(asinh(a*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{1}{\sqrt{\operatorname{arsinh}\left(ax\right)}}\,dx$$

```
[In] integrate(1/arcsinh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(arcsinh(a*x)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{1}{\sqrt{\operatorname{arsinh}\left(ax\right)}}\,dx$$

```
[In] integrate(1/arcsinh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(arcsinh(a*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{1}{\sqrt{\operatorname{asinh}\left(a\,x\right)}}\,dx$$

```
[In] int(1/asinh(a*x)^(1/2),x)
```

```
[Out] int(1/asinh(a*x)^(1/2), x)
```

3.97 $\int \frac{1}{x\sqrt{\operatorname{arcsinh}(ax)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{1}{x\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \operatorname{Int}\left(\frac{1}{x\sqrt{\operatorname{arcsinh}(ax)}},x\right)$$

```
[Out] Unintegrable(1/x/arcsinh(a*x)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{1}{x\sqrt{\operatorname{arcsinh}(ax)}}\,dx$$

```
[In] Int[1/(x*Sqrt[ArcSinh[a*x]]),x]
```

```
[Out] Defer[Int][1/(x*Sqrt[ArcSinh[a*x]]), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x\sqrt{\operatorname{arcsinh}(ax)}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{1}{x\sqrt{\operatorname{arcsinh}(ax)}}\,dx$$

```
[In] Integrate[1/(x*Sqrt[ArcSinh[a*x]]),x]
[Out] Integrate[1/(x*Sqrt[ArcSinh[a*x]]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{x\sqrt{\operatorname{arcsinh}\left(ax\right)}}dx$$

```
[In] int(1/x/arcsinh(a*x)^(1/2),x)
[Out] int(1/x/arcsinh(a*x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/x/arcsinh(a*x)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{1}{x\sqrt{\operatorname{asinh}\left(ax\right)}}\,dx$$

```
[In] integrate(1/x/asinh(a*x)**(1/2),x)
[Out] Integral(1/(x*sqrt(asinh(a*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{\operatorname{arcsinh}(ax)}}\,dx=\int \frac{1}{x\sqrt{\operatorname{arsinh}\left(ax\right)}}\,dx$$

```
[In] integrate(1/x/arcsinh(a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(x*sqrt(arcsinh(a*x))), x)
```

Giac [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{\operatorname{arcsinh}(ax)}}\,dx=\int \frac{1}{x\sqrt{\operatorname{arsinh}\left(ax\right)}}\,dx$$

```
[In] integrate(1/x/arcsinh(a*x)^(1/2),x, algorithm="giac")
[Out] integrate(1/(x*sqrt(arcsinh(a*x))), x)
```

Mupad [N/A]

Not integrable

Time = 2.35 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{\operatorname{arcsinh}(ax)}}\,dx=\int \frac{1}{x\,\sqrt{\operatorname{asinh}\left(a\,x\right)}}\,dx$$

```
[In] int(1/(x*asinh(a*x)^(1/2)),x)
[Out] int(1/(x*asinh(a*x)^(1/2)), x)
```

3.98 $\int \frac{1}{x^2\sqrt{\textbf{arcsinh}(ax)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{1}{x^2\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \operatorname{Int}\left(\frac{1}{x^2\sqrt{\operatorname{arcsinh}(ax)}},x\right)$$

```
[Out] Unintegrable(1/x^2/arcsinh(a*x)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{1}{x^2\sqrt{\operatorname{arcsinh}(ax)}}\,dx$$

```
[In] Int[1/(x^2*Sqrt[ArcSinh[a*x]]),x]
```

```
[Out] Defer[Int][1/(x^2*Sqrt[ArcSinh[a*x]]), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x^2\sqrt{\operatorname{arcsinh}(ax)}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.34 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x^2\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{1}{x^2\sqrt{\operatorname{arcsinh}(ax)}}\,dx$$

```
[In] Integrate[1/(x^2*Sqrt[ArcSinh[a*x]]),x]
[Out] Integrate[1/(x^2*Sqrt[ArcSinh[a*x]]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^2\sqrt{\operatorname{arcsinh}\left(ax\right)}}dx$$

```
[In] int(1/x^2/arcsinh(a*x)^(1/2),x)
[Out] int(1/x^2/arcsinh(a*x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^2\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/x^2/arcsinh(a*x)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x^2\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{1}{x^2\sqrt{\operatorname{asinh}\left(ax\right)}}\,dx$$

```
[In] integrate(1/x**2/asinh(a*x)**(1/2),x)
[Out] Integral(1/(x**2*sqrt(asinh(a*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{1}{x^2\sqrt{\operatorname{arsinh}\left(ax\right)}}\,dx$$

```
[In] integrate(1/x^2/arcsinh(a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(x^2*sqrt(arcsinh(a*x))), x)
```

Giac [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{1}{x^2\sqrt{\operatorname{arsinh}\left(ax\right)}}\,dx$$

```
[In] integrate(1/x^2/arcsinh(a*x)^(1/2),x, algorithm="giac")
[Out] integrate(1/(x^2*sqrt(arcsinh(a*x))), x)
```

Mupad [N/A]

Not integrable

Time = 2.33 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{1}{x^2\,\sqrt{\operatorname{asinh}\left(a\,x\right)}}\,dx$$

```
[In] int(1/(x^2*asinh(a*x)^(1/2)),x)
[Out] int(1/(x^2*asinh(a*x)^(1/2)), x)
```

3.99 $\int \frac{x^4}{\operatorname{arcsinh}(ax)^{3/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 188

$$\begin{aligned}\int \frac{x^4}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = &-\frac{2x^4\sqrt{1+a^2x^2}}{a\sqrt{\operatorname{arcsinh}(ax)}}-\frac{\sqrt{\pi}\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{8a^5}\\&+\frac{3\sqrt{3\pi}\operatorname{erf}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{16a^5}-\frac{\sqrt{5\pi}\operatorname{erf}\left(\sqrt{5}\sqrt{\operatorname{arcsinh}(ax)}\right)}{16a^5}+\frac{\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{8a^5}\\&-\frac{3\sqrt{3\pi}\operatorname{erfi}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{16a^5}+\frac{\sqrt{5\pi}\operatorname{erfi}\left(\sqrt{5}\sqrt{\operatorname{arcsinh}(ax)}\right)}{16a^5}\end{aligned}$$

```
[Out] -1/8*erf(arcsinh(a*x)^(1/2))*Pi^(1/2)/a^5+1/8*erfi(arcsinh(a*x)^(1/2))*Pi^(
1/2)/a^5+3/16*erf(3^(1/2)*arcsinh(a*x)^(1/2))*3^(1/2)*Pi^(1/2)/a^5-3/16*erf
i(3^(1/2)*arcsinh(a*x)^(1/2))*3^(1/2)*Pi^(1/2)/a^5-1/16*erf(5^(1/2)*arcsinh
(a*x)^(1/2))*5^(1/2)*Pi^(1/2)/a^5+1/16*erfi(5^(1/2)*arcsinh(a*x)^(1/2))*5^(
1/2)*Pi^(1/2)/a^5-2*x^4*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used

$= \{5778, 3389, 2211, 2235, 2236\}$

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = -\frac{\sqrt{\pi}\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{8a^5} + \frac{3\sqrt{3\pi}\operatorname{erf}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{16a^5} - \frac{\sqrt{5\pi}\operatorname{erf}\left(\sqrt{5}\sqrt{\operatorname{arcsinh}(ax)}\right)}{16a^5} + \frac{\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{8a^5} - \frac{3\sqrt{3\pi}\operatorname{erfi}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{16a^5} + \frac{\sqrt{5\pi}\operatorname{erfi}\left(\sqrt{5}\sqrt{\operatorname{arcsinh}(ax)}\right)}{16a^5} - \frac{2x^4\sqrt{a^2x^2+1}}{a\sqrt{\operatorname{arcsinh}(ax)}}$$

```
[In] Int[x^4/ArcSinh[a*x]^(3/2),x]
```

```
[Out] (-2*x^4*Sqrt[1 + a^2*x^2])/(a*Sqrt[ArcSinh[a*x]]) - (Sqrt[Pi]*Erf[Sqrt[ArcS
inh[a*x]]])/(8*a^5) + (3*Sqrt[3*Pi]*Erf[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(16*a^
5) - (Sqrt[5*Pi]*Erf[Sqrt[5]*Sqrt[ArcSinh[a*x]]])/(16*a^5) + (Sqrt[Pi]*Erfi
[Sqrt[ArcSinh[a*x]]])/(8*a^5) - (3*Sqrt[3*Pi]*Erfi[Sqrt[3]*Sqrt[ArcSinh[a*x
]]])/(16*a^5) + (Sqrt[5*Pi]*Erfi[Sqrt[5]*Sqrt[ArcSinh[a*x]]])/(16*a^5)
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
```

```
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2x^4\sqrt{1+a^2x^2}}{a\sqrt{\operatorname{arcsinh}(ax)}}+\frac{2\text{Subst}\left(\int\left(\frac{\sinh(x)}{8\sqrt{x}}-\frac{9\sinh(3x)}{16\sqrt{x}}+\frac{5\sinh(5x)}{16\sqrt{x}}\right)dx,x,\operatorname{arcsinh}(ax)\right)}{a^5}\\
&= -\frac{2x^4\sqrt{1+a^2x^2}}{a\sqrt{\operatorname{arcsinh}(ax)}}+\frac{\text{Subst}\left(\int\frac{\sinh(x)}{\sqrt{x}}\,dx,x,\operatorname{arcsinh}(ax)\right)}{4a^5}\\
&\quad+\frac{5\text{Subst}\left(\int\frac{\sinh(5x)}{\sqrt{x}}\,dx,x,\operatorname{arcsinh}(ax)\right)}{8a^5}-\frac{9\text{Subst}\left(\int\frac{\sinh(3x)}{\sqrt{x}}\,dx,x,\operatorname{arcsinh}(ax)\right)}{8a^5}\\
&= -\frac{2x^4\sqrt{1+a^2x^2}}{a\sqrt{\operatorname{arcsinh}(ax)}}-\frac{\text{Subst}\left(\int\frac{e^{-x}}{\sqrt{x}}\,dx,x,\operatorname{arcsinh}(ax)\right)}{8a^5}\\
&\quad+\frac{\text{Subst}\left(\int\frac{e^{x}}{\sqrt{x}}\,dx,x,\operatorname{arcsinh}(ax)\right)}{8a^5}-\frac{5\text{Subst}\left(\int\frac{e^{-5x}}{\sqrt{x}}\,dx,x,\operatorname{arcsinh}(ax)\right)}{16a^5}\\
&\quad+\frac{5\text{Subst}\left(\int\frac{e^{5x}}{\sqrt{x}}\,dx,x,\operatorname{arcsinh}(ax)\right)}{16a^5}+\frac{9\text{Subst}\left(\int\frac{e^{-3x}}{\sqrt{x}}\,dx,x,\operatorname{arcsinh}(ax)\right)}{16a^5}\\
&\quad-\frac{9\text{Subst}\left(\int\frac{e^{3x}}{\sqrt{x}}\,dx,x,\operatorname{arcsinh}(ax)\right)}{16a^5}\\
&= -\frac{2x^4\sqrt{1+a^2x^2}}{a\sqrt{\operatorname{arcsinh}(ax)}}-\frac{\text{Subst}\left(\int e^{-x^2}\,dx,x,\sqrt{\operatorname{arcsinh}(ax)}\right)}{4a^5}\\
&\quad+\frac{\text{Subst}\left(\int e^{x^2}\,dx,x,\sqrt{\operatorname{arcsinh}(ax)}\right)}{4a^5}-\frac{5\text{Subst}\left(\int e^{-5x^2}\,dx,x,\sqrt{\operatorname{arcsinh}(ax)}\right)}{8a^5}\\
&\quad+\frac{5\text{Subst}\left(\int e^{5x^2}\,dx,x,\sqrt{\operatorname{arcsinh}(ax)}\right)}{8a^5}+\frac{9\text{Subst}\left(\int e^{-3x^2}\,dx,x,\sqrt{\operatorname{arcsinh}(ax)}\right)}{8a^5}\\
&\quad-\frac{9\text{Subst}\left(\int e^{3x^2}\,dx,x,\sqrt{\operatorname{arcsinh}(ax)}\right)}{8a^5}
\end{aligned}$$

$$= -\frac{2x^4\sqrt{1+a^2x^2}}{a\sqrt{\text{arcsinh}(ax)}} - \frac{\sqrt{\pi}\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)}{8a^5} + \frac{3\sqrt{3\pi}\text{erf}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)}{16a^5}$$
$$- \frac{\sqrt{5\pi}\text{erf}\left(\sqrt{5}\sqrt{\text{arcsinh}(ax)}\right)}{16a^5} + \frac{\sqrt{\pi}\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)}{8a^5}$$
$$- \frac{3\sqrt{3\pi}\text{erfi}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)}{16a^5} + \frac{\sqrt{5\pi}\text{erfi}\left(\sqrt{5}\sqrt{\text{arcsinh}(ax)}\right)}{16a^5}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.41

$$\int \frac{x^4}{\text{arcsinh}(ax)^{3/2}}\,dx = \frac{e^{-5\text{arcsinh}(ax)}\left(-1+3e^{2\text{arcsinh}(ax)}-2e^{4\text{arcsinh}(ax)}-2e^{6\text{arcsinh}(ax)}+3e^{8\text{arcsinh}(ax)}-e^{10\text{arcsin}}\right.}{}$$

```
[In] Integrate[x^4/ArcSinh[a*x]^(3/2),x]
```

```
[Out] (-1 + 3*E^(2*ArcSinh[a*x]) - 2*E^(4*ArcSinh[a*x]) - 2*E^(6*ArcSinh[a*x]) +
3*E^(8*ArcSinh[a*x]) - E^(10*ArcSinh[a*x]) + Sqrt[5]*E^(5*ArcSinh[a*x])*Sqr
t[-ArcSinh[a*x]]*Gamma[1/2, -5*ArcSinh[a*x]] - 3*Sqrt[3]*E^(5*ArcSinh[a*x])
*Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -3*ArcSinh[a*x]] + 2*E^(5*ArcSinh[a*x])*Sqr
t[-ArcSinh[a*x]]*Gamma[1/2, -ArcSinh[a*x]] + 2*E^(5*ArcSinh[a*x])*Sqrt[ArcS
inh[a*x]]*Gamma[1/2, ArcSinh[a*x]] - 3*Sqrt[3]*E^(5*ArcSinh[a*x])*Sqrt[ArcS
inh[a*x]]*Gamma[1/2, 3*ArcSinh[a*x]] + Sqrt[5]*E^(5*ArcSinh[a*x])*Sqrt[ArcS
inh[a*x]]*Gamma[1/2, 5*ArcSinh[a*x]])/(16*a^5*E^(5*ArcSinh[a*x])*Sqrt[ArcSi
nh[a*x]])
```

Maple [F]

$$\int \frac{x^4}{\text{arcsinh}\,(ax)^{\frac{3}{2}}}dx$$

```
[In] int(x^4/arcsinh(a*x)^(3/2),x)
```

```
[Out] int(x^4/arcsinh(a*x)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^4/arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{x^4}{\operatorname{asinh}^{\frac{3}{2}}(ax)}\,dx$$

```
[In] integrate(x**4/asinh(a*x)**(3/2),x)
```

```
[Out] Integral(x**4/asinh(a*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{x^4}{\operatorname{arsinh}(ax)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^4/arcsinh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/arcsinh(a*x)^(3/2), x)
```

Giac [F]

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{x^4}{\operatorname{arsinh}(ax)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^4/arcsinh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/arcsinh(a*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{x^4}{\operatorname{asinh}\left(a\,x\right)^{3/2}}\,dx$$

```
[In] int(x^4/asinh(a*x)^(3/2),x)
```

```
[Out] int(x^4/asinh(a*x)^(3/2), x)
```

3.100 $\int \frac{x^3}{\operatorname{arcsinh}(ax)^{3/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 138

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = -\frac{2x^3\sqrt{1+a^2x^2}}{a\sqrt{\operatorname{arcsinh}(ax)}} + \frac{\sqrt{\pi}\operatorname{erf}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{4a^4} - \frac{\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{2a^4} + \frac{\sqrt{\pi}\operatorname{erfi}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{4a^4} - \frac{\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{2a^4}$$

```
[Out] -1/4*erf(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^4-1/4*erfi(2^(1/2)*
arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^4+1/4*erf(2*arcsinh(a*x)^(1/2))*Pi^(
1/2)/a^4+1/4*erfi(2*arcsinh(a*x)^(1/2))*Pi^(1/2)/a^4-2*x^3*(a^2*x^2+1)^(1/2
)/a/arcsinh(a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5778, 3388, 2211, 2235, 2236}

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \frac{\sqrt{\pi}\operatorname{erf}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{4a^4} - \frac{\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{2a^4} + \frac{\sqrt{\pi}\operatorname{erfi}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{4a^4} - \frac{\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{2a^4} - \frac{2x^3\sqrt{a^2x^2+1}}{a\sqrt{\operatorname{arcsinh}(ax)}}$$

```
[In] Int[x^3/ArcSinh[a*x]^(3/2),x]
```

```
[Out] (-2*x^3*Sqrt[1 + a^2*x^2])/(a*Sqrt[ArcSinh[a*x]]) + (Sqrt[Pi]*Erf[2*Sqrt[Ar
cSinh[a*x]]])/(4*a^4) - (Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(2*a^4
```

```
) + (Sqrt[Pi]*Erfi[2*Sqrt[ArcSinh[a*x]]])/(4*a^4) - (Sqrt[Pi/2]*Erfi[Sqrt[2
]*Sqrt[ArcSinh[a*x]]])/(2*a^4)
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rubi steps

$$\text{integral} = -\frac{2x^3\sqrt{1+a^2x^2}}{a\sqrt{\operatorname{arcsinh}(ax)}} + \frac{2\text{Subst}\left(\int \left(-\frac{\cosh(2x)}{2\sqrt{x}} + \frac{\cosh(4x)}{2\sqrt{x}}\right) dx, x, \operatorname{arcsinh}(ax)\right)}{a^4}$$

$$= -\frac{2x^3\sqrt{1+a^2x^2}}{a\sqrt{\operatorname{arcsinh}(ax)}} - \frac{\text{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{x}} dx, x, \operatorname{arcsinh}(ax)\right)}{a^4} + \frac{\text{Subst}\left(\int \frac{\cosh(4x)}{\sqrt{x}} dx, x, \operatorname{arcsinh}(ax)\right)}{a^4}$$

$$
\begin{aligned}
&= -\frac{2x^3\sqrt{1+a^2x^2}}{a\sqrt{\operatorname{arcsinh}(ax)}} + \frac{\operatorname{Subst}\left(\int \frac{e^{-4x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{2a^4} \\
&\quad - \frac{\operatorname{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{2a^4} \\
&\quad - \frac{\operatorname{Subst}\left(\int \frac{e^{2x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{2a^4} + \frac{\operatorname{Subst}\left(\int \frac{e^{4x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{2a^4} \\
&= -\frac{2x^3\sqrt{1+a^2x^2}}{a\sqrt{\operatorname{arcsinh}(ax)}} + \frac{\operatorname{Subst}\left(\int e^{-4x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{a^4} \\
&\quad - \frac{\operatorname{Subst}\left(\int e^{-2x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{a^4} \\
&\quad - \frac{\operatorname{Subst}\left(\int e^{2x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{a^4} + \frac{\operatorname{Subst}\left(\int e^{4x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{a^4} \\
&= -\frac{2x^3\sqrt{1+a^2x^2}}{a\sqrt{\operatorname{arcsinh}(ax)}} + \frac{\sqrt{\pi}\operatorname{erf}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{4a^4} - \frac{\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{2a^4} \\
&\quad + \frac{\sqrt{\pi}\operatorname{erfi}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{4a^4} - \frac{\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{2a^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.91

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \frac{\sqrt{-\operatorname{arcsinh}(ax)}\Gamma\left(\frac{1}{2}, -4\operatorname{arcsinh}(ax)\right) - \sqrt{2}\sqrt{-\operatorname{arcsinh}(ax)}\Gamma\left(\frac{1}{2}, -2\operatorname{arcsinh}(ax)\right) +}{}$$

```
[In] Integrate[x^3/ArcSinh[a*x]^(3/2),x]
```

```
[Out] (Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -4*ArcSinh[a*x]] - Sqrt[2]*Sqrt[-ArcSinh[a*
x]]*Gamma[1/2, -2*ArcSinh[a*x]] + Sqrt[2]*Sqrt[ArcSinh[a*x]]*Gamma[1/2, 2*A
rcSinh[a*x]] - Sqrt[ArcSinh[a*x]]*Gamma[1/2, 4*ArcSinh[a*x]] + 2*Sinh[2*Arc
Sinh[a*x]] - Sinh[4*ArcSinh[a*x]])/(4*a^4*Sqrt[ArcSinh[a*x]])
```

Maple [F]

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^{\frac{3}{2}}}dx$$

```
[In] int(x^3/arcsinh(a*x)^(3/2),x)
```

```
[Out] int(x^3/arcsinh(a*x)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3/arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{x^3}{\operatorname{asinh}^{\frac{3}{2}}(ax)}\,dx$$

```
[In] integrate(x**3/asinh(a*x)**(3/2),x)
```

```
[Out] Integral(x**3/asinh(a*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{x^3}{\operatorname{arsinh}(ax)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^3/arcsinh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/arcsinh(a*x)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3/arcsinh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{x^3}{\operatorname{asinh}\left(a\,x\right)^{3/2}} \,d x$$

```
[In] int(x^3/asinh(a*x)^(3/2),x)
```

```
[Out] int(x^3/asinh(a*x)^(3/2), x)
```

3.101 $\int \frac{x^2}{\textbf{arcsinh}(ax)^{3/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 130

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = -\frac{2x^2\sqrt{1+a^2x^2}}{a\sqrt{\operatorname{arcsinh}(ax)}} + \frac{\sqrt{\pi}\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{4a^3} - \frac{\sqrt{3\pi}\operatorname{erf}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{4a^3} - \frac{\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{4a^3} + \frac{\sqrt{3\pi}\operatorname{erfi}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{4a^3}$$

```
[Out] 1/4*erf(arcsinh(a*x)^(1/2))*Pi^(1/2)/a^3-1/4*erfi(arcsinh(a*x)^(1/2))*Pi^(1
/2)/a^3-1/4*erf(3^(1/2)*arcsinh(a*x)^(1/2))*3^(1/2)*Pi^(1/2)/a^3+1/4*erfi(3
^(1/2)*arcsinh(a*x)^(1/2))*3^(1/2)*Pi^(1/2)/a^3-2*x^2*(a^2*x^2+1)^(1/2)/a/a
rcsinh(a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5778, 3389, 2211, 2235, 2236}

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \frac{\sqrt{\pi}\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{4a^3} - \frac{\sqrt{3\pi}\operatorname{erf}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{4a^3} - \frac{\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{4a^3} + \frac{\sqrt{3\pi}\operatorname{erfi}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{4a^3} - \frac{2x^2\sqrt{a^2x^2+1}}{a\sqrt{\operatorname{arcsinh}(ax)}}$$

```
[In] Int[x^2/ArcSinh[a*x]^(3/2),x]
```

```
[Out] (-2*x^2*Sqrt[1 + a^2*x^2])/(a*Sqrt[ArcSinh[a*x]]) + (Sqrt[Pi]*Erf[Sqrt[ArcS
inh[a*x]]])/(4*a^3) - (Sqrt[3*Pi]*Erf[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(4*a^3)
```

```
- (Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x]]])/(4*a^3) + (Sqrt[3*Pi]*Erfi[Sqrt[3]*Sq
rt[ArcSinh[a*x]]])/(4*a^3)
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2x^2\sqrt{1+a^2x^2}}{a\sqrt{\operatorname{arcsinh}(ax)}}+\frac{2\text{Subst}\left(\int\left(-\frac{\sinh(x)}{4\sqrt{x}}+\frac{3\sinh(3x)}{4\sqrt{x}}\right)dx,x,\operatorname{arcsinh}(ax)\right)}{a^3}\\
&= -\frac{2x^2\sqrt{1+a^2x^2}}{a\sqrt{\operatorname{arcsinh}(ax)}}-\frac{\text{Subst}\left(\int\frac{\sinh(x)}{\sqrt{x}}\,dx,x,\operatorname{arcsinh}(ax)\right)}{2a^3}\\
&\quad+\frac{3\text{Subst}\left(\int\frac{\sinh(3x)}{\sqrt{x}}\,dx,x,\operatorname{arcsinh}(ax)\right)}{2a^3}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{2x^2\sqrt{1+a^2x^2}}{a\sqrt{\operatorname{arcsinh}(ax)}} + \frac{\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{4a^3} \\
&\quad - \frac{\text{Subst}\left(\int \frac{e^{x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{4a^3} - \frac{3\text{Subst}\left(\int \frac{e^{-3x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{4a^3} \\
&\quad + \frac{3\text{Subst}\left(\int \frac{e^{3x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{4a^3} \\
&= -\frac{2x^2\sqrt{1+a^2x^2}}{a\sqrt{\operatorname{arcsinh}(ax)}} + \frac{\text{Subst}\left(\int e^{-x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{2a^3} \\
&\quad - \frac{\text{Subst}\left(\int e^{x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{2a^3} - \frac{3\text{Subst}\left(\int e^{-3x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{2a^3} \\
&\quad + \frac{3\text{Subst}\left(\int e^{3x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{2a^3} \\
&= -\frac{2x^2\sqrt{1+a^2x^2}}{a\sqrt{\operatorname{arcsinh}(ax)}} + \frac{\sqrt{\pi}\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{4a^3} - \frac{\sqrt{3\pi}\operatorname{erf}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{4a^3} \\
&\quad - \frac{\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{4a^3} + \frac{\sqrt{3\pi}\operatorname{erfi}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{4a^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.08

$$
\int \frac{x^2}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \frac{-e^{-3\operatorname{arcsinh}(ax)} + e^{-\operatorname{arcsinh}(ax)} + e^{\operatorname{arcsinh}(ax)} - e^{3\operatorname{arcsinh}(ax)} + \sqrt{3}\sqrt{-\operatorname{arcsinh}(ax)}\Gamma\left(\frac{1}{2}, -3a\right.}{}
$$

```
[In] Integrate[x^2/ArcSinh[a*x]^(3/2),x]
```

```
[Out] (-E^(-3*ArcSinh[a*x]) + E^(-ArcSinh[a*x]) + E^ArcSinh[a*x] - E^(3*ArcSinh[a
*x]) + Sqrt[3]*Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -3*ArcSinh[a*x]] - Sqrt[-ArcS
inh[a*x]]*Gamma[1/2, -ArcSinh[a*x]] - Sqrt[ArcSinh[a*x]]*Gamma[1/2, ArcSinh
[a*x]] + Sqrt[3]*Sqrt[ArcSinh[a*x]]*Gamma[1/2, 3*ArcSinh[a*x]])/(4*a^3*Sqrt
[ArcSinh[a*x]])
```

Maple [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^{\frac{3}{2}}}dx$$

```
[In] int(x^2/arcsinh(a*x)^(3/2),x)
```

```
[Out] int(x^2/arcsinh(a*x)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2/arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{x^2}{\operatorname{asinh}^{\frac{3}{2}}(ax)}\,dx$$

```
[In] integrate(x**2/asinh(a*x)**(3/2),x)
```

```
[Out] Integral(x**2/asinh(a*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{x^2}{\operatorname{arsinh}(ax)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2/arcsinh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/arcsinh(a*x)^(3/2), x)
```

Giac [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{x^2}{\operatorname{arsinh}\left(ax\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2/arcsinh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/arcsinh(a*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{x^2}{\operatorname{asinh}\left(a\,x\right)^{3/2}}\,dx$$

```
[In] int(x^2/asinh(a*x)^(3/2),x)
```

```
[Out] int(x^2/asinh(a*x)^(3/2), x)
```

3.102 $\int \frac{x}{\operatorname{arcsinh}(ax)^{3/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 84

$$\int \frac{x}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = -\frac{2x\sqrt{1+a^2x^2}}{a\sqrt{\operatorname{arcsinh}(ax)}} + \frac{\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{a^2} + \frac{\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{a^2}$$

```
[Out] 1/2*erf(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^2+1/2*erfi(2^(1/2)*a
rcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^2-2*x*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x
)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5778, 3388, 2211, 2235, 2236}

$$\int \frac{x}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \frac{\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{a^2} + \frac{\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{a^2} - \frac{2x\sqrt{a^2x^2+1}}{a\sqrt{\operatorname{arcsinh}(ax)}}$$

```
[In] Int[x/ArcSinh[a*x]^(3/2),x]
```

```
[Out] (-2*x*Sqrt[1 + a^2*x^2])/(a*Sqrt[ArcSinh[a*x]]) + (Sqrt[Pi/2]*Erf[Sqrt[2]*S
qrt[ArcSinh[a*x]]])/a^2 + (Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/a^2
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2x\sqrt{1+a^2x^2}}{a\sqrt{\operatorname{arcsinh}(ax)}}+\frac{2\text{Subst}\left(\int\frac{\cosh(2x)}{\sqrt{x}}\,dx,x,\operatorname{arcsinh}(ax)\right)}{a^2}\\
&= -\frac{2x\sqrt{1+a^2x^2}}{a\sqrt{\operatorname{arcsinh}(ax)}}+\frac{\text{Subst}\left(\int\frac{e^{-2x}}{\sqrt{x}}\,dx,x,\operatorname{arcsinh}(ax)\right)}{a^2}+\frac{\text{Subst}\left(\int\frac{e^{2x}}{\sqrt{x}}\,dx,x,\operatorname{arcsinh}(ax)\right)}{a^2}
\end{aligned}$$

$$= -\frac{2x\sqrt{1+a^2x^2}}{a\sqrt{\operatorname{arcsinh}(ax)}} + \frac{2\text{Subst}\left(\int e^{-2x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{a^2} + \frac{2\text{Subst}\left(\int e^{2x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{a^2}$$

$$= -\frac{2x\sqrt{1+a^2x^2}}{a\sqrt{\operatorname{arcsinh}(ax)}} + \frac{\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{a^2} + \frac{\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{a^2}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.87

$$\int \frac{x}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \frac{\frac{\sqrt{-\operatorname{arcsinh}(ax)}\Gamma\left(\frac{1}{2},-2\operatorname{arcsinh}(ax)\right)}{\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}} - \frac{\Gamma\left(\frac{1}{2},2\operatorname{arcsinh}(ax)\right)}{\sqrt{2}} - \frac{\sinh(2\operatorname{arcsinh}(ax))}{\sqrt{\operatorname{arcsinh}(ax)}}}{a^2}$$

```
[In] Integrate[x/ArcSinh[a*x]^(3/2),x]
```

```
[Out] ((Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -2*ArcSinh[a*x]])/(Sqrt[2]*Sqrt[ArcSinh[a*
x]]) - Gamma[1/2, 2*ArcSinh[a*x]]/Sqrt[2] - Sinh[2*ArcSinh[a*x]]/Sqrt[ArcSi
nh[a*x]])/a^2
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.98

method	result	size
default	$-\frac{\sqrt{2}\left(2\sqrt{2}\,\sqrt{\operatorname{arcsinh}(ax)}\,\sqrt{\pi}\,\sqrt{a^2x^2+1}\,ax-\operatorname{arcsinh}(ax)\pi\,\operatorname{erf}\left(\sqrt{2}\,\sqrt{\operatorname{arcsinh}(ax)}\right)-\operatorname{arcsinh}(ax)\pi\,\operatorname{erfi}\left(\sqrt{2}\,\sqrt{\operatorname{arcsinh}(ax)}\right)\right)}{2\sqrt{\pi}\,a^2\,\operatorname{arcsinh}(ax)}$	82

```
[In] int(x/arcsinh(a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*2^(1/2)*(2*2^(1/2)*arcsinh(a*x)^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*a*x-a
rcsinh(a*x)*Pi*erf(2^(1/2)*arcsinh(a*x)^(1/2))-arcsinh(a*x)*Pi*erfi(2^(1/2)
*arcsinh(a*x)^(1/2)))/Pi^(1/2)/a^2/arcsinh(a*x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x/arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{x}{\operatorname{asinh}^{\frac{3}{2}}(ax)}\,dx$$

```
[In] integrate(x/asinh(a*x)**(3/2),x)
```

```
[Out] Integral(x/asinh(a*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{x}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{x}{\operatorname{arsinh}(ax)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/arcsinh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/arcsinh(a*x)^(3/2), x)
```

Giac [F]

$$\int \frac{x}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{x}{\operatorname{arsinh}(ax)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/arcsinh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/arcsinh(a*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{x}{\operatorname{asinh}\left(a\,x\right)^{3/2}}\,dx$$

```
[In] int(x/asinh(a*x)^(3/2),x)
```

```
[Out] int(x/asinh(a*x)^(3/2), x)
```

3.103 $\int \frac{1}{\mathbf{arcsinh}(ax)^{3/2}} dx$

Optimal result

Integrand size = 8, antiderivative size = 64

$$\int \frac{1}{\operatorname{arcsinh}(ax)^{3/2}} dx = -\frac{2\sqrt{1+a^2x^2}}{a\sqrt{\operatorname{arcsinh}(ax)}} - \frac{\sqrt{\pi}\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{a} + \frac{\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{a}$$

```
[Out] -erf(arcsinh(a*x)^(1/2))*Pi^(1/2)/a+erfi(arcsinh(a*x)^(1/2))*Pi^(1/2)/a-2*(
a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {5773, 5819, 3389, 2211, 2235, 2236}

$$\int \frac{1}{\operatorname{arcsinh}(ax)^{3/2}} dx = -\frac{2\sqrt{a^2x^2+1}}{a\sqrt{\operatorname{arcsinh}(ax)}} - \frac{\sqrt{\pi}\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{a} + \frac{\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{a}$$

```
[In] Int[ArcSinh[a*x]^(-3/2),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2])/(a*Sqrt[ArcSinh[a*x]]) - (Sqrt[Pi]*Erf[Sqrt[ArcSinh[
a*x]]])/a + (Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x]]])/a
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\sqrt{1+a^2x^2}}{a\sqrt{\operatorname{arcsinh}(ax)}}+(2a)\int\frac{x}{\sqrt{1+a^2x^2}\sqrt{\operatorname{arcsinh}(ax)}}\,dx\\
&= -\frac{2\sqrt{1+a^2x^2}}{a\sqrt{\operatorname{arcsinh}(ax)}}+\frac{2\text{Subst}\left(\int\frac{\sinh(x)}{\sqrt{x}}\,dx,x,\operatorname{arcsinh}(ax)\right)}{a}\\
&= -\frac{2\sqrt{1+a^2x^2}}{a\sqrt{\operatorname{arcsinh}(ax)}}-\frac{\text{Subst}\left(\int\frac{e^{-x}}{\sqrt{x}}\,dx,x,\operatorname{arcsinh}(ax)\right)}{a}+\frac{\text{Subst}\left(\int\frac{e^{x}}{\sqrt{x}}\,dx,x,\operatorname{arcsinh}(ax)\right)}{a}
\end{aligned}$$

$$= -\frac{2\sqrt{1+a^2x^2}}{a\sqrt{\operatorname{arcsinh}(ax)}} - \frac{2\mathrm{Subst}\left(\int e^{-x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{a} + \frac{2\mathrm{Subst}\left(\int e^{x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{a}$$

$$= -\frac{2\sqrt{1+a^2x^2}}{a\sqrt{\operatorname{arcsinh}(ax)}} - \frac{\sqrt{\pi}\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{a} + \frac{\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{a}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.08

$$\int \frac{1}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \frac{-e^{-\operatorname{arcsinh}(ax)} - e^{\operatorname{arcsinh}(ax)} + \sqrt{-\operatorname{arcsinh}(ax)}\Gamma\left(\frac{1}{2}, -\operatorname{arcsinh}(ax)\right) + \sqrt{\operatorname{arcsinh}(ax)}\Gamma\left(\frac{1}{2}\right.}{a\sqrt{\operatorname{arcsinh}(ax)}}$$

```
[In] Integrate[ArcSinh[a*x]^(-3/2),x]
```

```
[Out] (-E^(-ArcSinh[a*x]) - E^ArcSinh[a*x] + Sqrt[-ArcSinh[a*x]]*Gamma[1/2, -ArcS
inh[a*x]] + Sqrt[ArcSinh[a*x]]*Gamma[1/2, ArcSinh[a*x]])/(a*Sqrt[ArcSinh[a*
x]])
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

method	result	size
default	$-\frac{\operatorname{arcsinh}(ax)\pi\ \operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)-\operatorname{arcsinh}(ax)\pi\ \operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)+2\sqrt{\operatorname{arcsinh}(ax)}\,\sqrt{\pi}\,\sqrt{a^2x^2+1}}{\sqrt{\pi}\,a\ \operatorname{arcsinh}(ax)}$	65

```
[In] int(1/arcsinh(a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(arcsinh(a*x)*Pi*erf(arcsinh(a*x)^(1/2))-arcsinh(a*x)*Pi*erfi(arcsinh(a*x)
^(1/2))+2*arcsinh(a*x)^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2))/Pi^(1/2)/a/arcsinh
(a*x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/arcsinh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{1}{\operatorname{asinh}^{\frac{3}{2}}(ax)}\,dx$$

```
[In] integrate(1/asinh(a*x)**(3/2),x)
```

```
[Out] Integral(asinh(a*x)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{1}{\operatorname{arsinh}(ax)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/arcsinh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{1}{\operatorname{arsinh}(ax)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/arcsinh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{1}{\operatorname{asinh}\left(a\,x\right)^{3/2}}\,dx$$

```
[In] int(1/asinh(a*x)^(3/2),x)
[Out] int(1/asinh(a*x)^(3/2), x)
```

3.104 $\int \frac{1}{x\operatorname{arcsinh}(ax)^{3/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{1}{x\operatorname{arcsinh}(ax)^{3/2}}\,dx = \operatorname{Int}\left(\frac{1}{x\operatorname{arcsinh}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable(1/x/arcsinh(a*x)^(3/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{1}{x\operatorname{arcsinh}(ax)^{3/2}}\,dx$$

```
[In] Int[1/(x*ArcSinh[a*x]^(3/2)),x]
```

```
[Out] Defer[Int][1/(x*ArcSinh[a*x]^(3/2)), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x\operatorname{arcsinh}(ax)^{3/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x\text{arcsinh}(ax)^{3/2}}\,dx=\int \frac{1}{x\text{arcsinh}(ax)^{3/2}}\,dx$$

```
[In] Integrate[1/(x*ArcSinh[a*x]^(3/2)),x]
[Out] Integrate[1/(x*ArcSinh[a*x]^(3/2)), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{x\operatorname{arcsinh}\left(ax\right)^{\frac{3}{2}}}dx$$

```
[In] int(1/x/arcsinh(a*x)^(3/2),x)
[Out] int(1/x/arcsinh(a*x)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x\text{arcsinh}(ax)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/x/arcsinh(a*x)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.96 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\text{arcsinh}(ax)^{3/2}}\,dx=\int \frac{1}{x\operatorname{asinh}^{\frac{3}{2}}\left(ax\right)}\,dx$$

```
[In] integrate(1/x/asinh(a*x)**(3/2),x)
[Out] Integral(1/(x*asinh(a*x)**(3/2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\text{arcsinh}(ax)^{3/2}}\,dx = \int \frac{1}{x\,\text{arsinh}\,(ax)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x/arcsinh(a*x)^(3/2),x, algorithm="maxima")
[Out] integrate(1/(x*arcsinh(a*x)^(3/2)), x)
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\text{arcsinh}(ax)^{3/2}}\,dx = \int \frac{1}{x\,\text{arsinh}\,(ax)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x/arcsinh(a*x)^(3/2),x, algorithm="giac")
[Out] integrate(1/(x*arcsinh(a*x)^(3/2)), x)
```

Mupad [N/A]

Not integrable

Time = 2.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\text{arcsinh}(ax)^{3/2}}\,dx = \int \frac{1}{x\,\text{asinh}\,(a\,x)^{3/2}}\,dx$$

```
[In] int(1/(x*asinh(a*x)^(3/2)),x)
[Out] int(1/(x*asinh(a*x)^(3/2)), x)
```

3.105 $\int \frac{x^4}{\mathbf{arcsinh}(ax)^{5/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 223

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = -\frac{2x^4\sqrt{1+a^2x^2}}{3a\operatorname{arcsinh}(ax)^{3/2}} - \frac{16x^3}{3a^2\sqrt{\operatorname{arcsinh}(ax)}} - \frac{20x^5}{3\sqrt{\operatorname{arcsinh}(ax)}} + \frac{\sqrt{\pi}\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{12a^5} - \frac{3\sqrt{3\pi}\operatorname{erf}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{8a^5} + \frac{5\sqrt{5\pi}\operatorname{erf}\left(\sqrt{5}\sqrt{\operatorname{arcsinh}(ax)}\right)}{24a^5} + \frac{\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{12a^5} - \frac{3\sqrt{3\pi}\operatorname{erfi}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{8a^5} + \frac{5\sqrt{5\pi}\operatorname{erfi}\left(\sqrt{5}\sqrt{\operatorname{arcsinh}(ax)}\right)}{24a^5}$$

```
[Out] 1/12*erf(arcsinh(a*x)^(1/2))*Pi^(1/2)/a^5+1/12*erfi(arcsinh(a*x)^(1/2))*Pi^
(1/2)/a^5-3/8*erf(3^(1/2)*arcsinh(a*x)^(1/2))*3^(1/2)*Pi^(1/2)/a^5-3/8*erfi
(3^(1/2)*arcsinh(a*x)^(1/2))*3^(1/2)*Pi^(1/2)/a^5+5/24*erf(5^(1/2)*arcsinh(
a*x)^(1/2))*5^(1/2)*Pi^(1/2)/a^5+5/24*erfi(5^(1/2)*arcsinh(a*x)^(1/2))*5^(1
/2)*Pi^(1/2)/a^5-2/3*x^4*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^(3/2)-16/3*x^3/a^
2/arcsinh(a*x)^(1/2)-20/3*x^5/arcsinh(a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 34, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {5779, 5818, 5780, 5556, 3388, 2211, 2235, 2236}

$$\begin{aligned}\int \frac{x^4}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = &\frac{\sqrt{\pi}\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{12a^5} - \frac{3\sqrt{3\pi}\operatorname{erf}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{8a^5}\\ &+ \frac{5\sqrt{5\pi}\operatorname{erf}\left(\sqrt{5}\sqrt{\operatorname{arcsinh}(ax)}\right)}{24a^5} + \frac{\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{12a^5}\\ &- \frac{3\sqrt{3\pi}\operatorname{erfi}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{8a^5} + \frac{5\sqrt{5\pi}\operatorname{erfi}\left(\sqrt{5}\sqrt{\operatorname{arcsinh}(ax)}\right)}{24a^5}\\ &- \frac{16x^3}{3a^2\sqrt{\operatorname{arcsinh}(ax)}} - \frac{2x^4\sqrt{a^2x^2+1}}{3a\operatorname{arcsinh}(ax)^{3/2}} - \frac{20x^5}{3\sqrt{\operatorname{arcsinh}(ax)}}\end{aligned}$$

```
[In] Int[x^4/ArcSinh[a*x]^(5/2),x]
```

```
[Out] (-2*x^4*Sqrt[1 + a^2*x^2])/(3*a*ArcSinh[a*x]^(3/2)) - (16*x^3)/(3*a^2*Sqrt[
ArcSinh[a*x]]) - (20*x^5)/(3*Sqrt[ArcSinh[a*x]]) + (Sqrt[Pi]*Erf[Sqrt[ArcSi
nh[a*x]]])/(12*a^5) - (3*Sqrt[3*Pi]*Erf[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(8*a^5
) + (5*Sqrt[5*Pi]*Erf[Sqrt[5]*Sqrt[ArcSinh[a*x]]])/(24*a^5) + (Sqrt[Pi]*Erf
i[Sqrt[ArcSinh[a*x]]])/(12*a^5) - (3*Sqrt[3*Pi]*Erfi[Sqrt[3]*Sqrt[ArcSinh[a
*x]]])/(8*a^5) + (5*Sqrt[5*Pi]*Erfi[Sqrt[5]*Sqrt[ArcSinh[a*x]]])/(24*a^5)
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
```

```
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/S
qrt[1 + c^2*x^2]), x], x] - Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Sinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2x^4\sqrt{1+a^2x^2}}{3a\text{arcsinh}(ax)^{3/2}} + \frac{8\int \frac{x^3}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}\,dx}{3a} \\
&\quad + \frac{1}{3}(10a)\int \frac{x^5}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}\,dx \\
&= -\frac{2x^4\sqrt{1+a^2x^2}}{3a\text{arcsinh}(ax)^{3/2}} - \frac{16x^3}{3a^2\sqrt{\text{arcsinh}(ax)}} - \frac{20x^5}{3\sqrt{\text{arcsinh}(ax)}} \\
&\quad + \frac{100}{3}\int \frac{x^4}{\sqrt{\text{arcsinh}(ax)}}\,dx + \frac{16\int \frac{x^2}{\sqrt{\text{arcsinh}(ax)}}\,dx}{a^2}
\end{aligned}$$

$$= -\frac{2x^4\sqrt{1+a^2x^2}}{3a\text{arcsinh}(ax)^{3/2}} - \frac{16x^3}{3a^2\sqrt{\text{arcsinh}(ax)}} - \frac{20x^5}{3\sqrt{\text{arcsinh}(ax)}} + \frac{16\text{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{a^5} + \frac{100\text{Subst}\left(\int \frac{\cosh(x)\sinh^4(x)}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{3a^5}$$

$$= -\frac{2x^4\sqrt{1+a^2x^2}}{3a\text{arcsinh}(ax)^{3/2}} - \frac{16x^3}{3a^2\sqrt{\text{arcsinh}(ax)}} - \frac{20x^5}{3\sqrt{\text{arcsinh}(ax)}} + \frac{16\text{Subst}\left(\int \left(-\frac{\cosh(x)}{4\sqrt{x}} + \frac{\cosh(3x)}{4\sqrt{x}}\right)\,dx, x, \text{arcsinh}(ax)\right)}{a^5} + \frac{100\text{Subst}\left(\int \left(\frac{\cosh(x)}{8\sqrt{x}} - \frac{3\cosh(3x)}{16\sqrt{x}} + \frac{\cosh(5x)}{16\sqrt{x}}\right)\,dx, x, \text{arcsinh}(ax)\right)}{3a^5}$$

$$= -\frac{2x^4\sqrt{1+a^2x^2}}{3a\text{arcsinh}(ax)^{3/2}} - \frac{16x^3}{3a^2\sqrt{\text{arcsinh}(ax)}} - \frac{20x^5}{3\sqrt{\text{arcsinh}(ax)}} + \frac{25\text{Subst}\left(\int \frac{\cosh(5x)}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{12a^5} - \frac{4\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{a^5} + \frac{4\text{Subst}\left(\int \frac{\cosh(3x)}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{a^5} + \frac{25\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{6a^5} - \frac{25\text{Subst}\left(\int \frac{\cosh(3x)}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{4a^5}$$

$$= -\frac{2x^4\sqrt{1+a^2x^2}}{3a\text{arcsinh}(ax)^{3/2}} - \frac{16x^3}{3a^2\sqrt{\text{arcsinh}(ax)}} - \frac{20x^5}{3\sqrt{\text{arcsinh}(ax)}} + \frac{25\text{Subst}\left(\int \frac{e^{-5x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{24a^5} + \frac{25\text{Subst}\left(\int \frac{e^{5x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{24a^5} + \frac{2\text{Subst}\left(\int \frac{e^{-3x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{a^5} - \frac{2\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{a^5} - \frac{2\text{Subst}\left(\int \frac{e^{x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{a^5} + \frac{2\text{Subst}\left(\int \frac{e^{3x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{a^5} + \frac{25\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{12a^5} + \frac{25\text{Subst}\left(\int \frac{e^{x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{12a^5} - \frac{25\text{Subst}\left(\int \frac{e^{-3x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{8a^5} - \frac{25\text{Subst}\left(\int \frac{e^{3x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{8a^5}$$

$$
\begin{aligned}
&= -\frac{2x^4\sqrt{1+a^2x^2}}{3a\text{arcsinh}(ax)^{3/2}} - \frac{16x^3}{3a^2\sqrt{\text{arcsinh}(ax)}} - \frac{20x^5}{3\sqrt{\text{arcsinh}(ax)}} \\
&+ \frac{25\text{Subst}\left(\int e^{-5x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{12a^5} + \frac{25\text{Subst}\left(\int e^{5x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{12a^5} \\
&+ \frac{4\text{Subst}\left(\int e^{-3x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{a^5} - \frac{4\text{Subst}\left(\int e^{-x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{a^5} \\
&- \frac{4\text{Subst}\left(\int e^{x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{a^5} + \frac{4\text{Subst}\left(\int e^{3x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{a^5} \\
&+ \frac{25\text{Subst}\left(\int e^{-x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{6a^5} + \frac{25\text{Subst}\left(\int e^{x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{6a^5} \\
&- \frac{25\text{Subst}\left(\int e^{-3x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{4a^5} - \frac{25\text{Subst}\left(\int e^{3x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{4a^5}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2x^4\sqrt{1+a^2x^2}}{3a\text{arcsinh}(ax)^{3/2}} - \frac{16x^3}{3a^2\sqrt{\text{arcsinh}(ax)}} - \frac{20x^5}{3\sqrt{\text{arcsinh}(ax)}} \\
&+ \frac{\sqrt{\pi}\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)}{12a^5} - \frac{3\sqrt{3\pi}\text{erf}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)}{8a^5} \\
&+ \frac{5\sqrt{5\pi}\text{erf}\left(\sqrt{5}\sqrt{\text{arcsinh}(ax)}\right)}{24a^5} + \frac{\sqrt{\pi}\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)}{12a^5} \\
&- \frac{3\sqrt{3\pi}\text{erfi}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)}{8a^5} + \frac{5\sqrt{5\pi}\text{erfi}\left(\sqrt{5}\sqrt{\text{arcsinh}(ax)}\right)}{24a^5}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.52

$$\int \frac{x^4}{\text{arcsinh}(ax)^{5/2}}\,dx = \frac{-\frac{e^{5\text{arcsinh}(ax)}(1+10\text{arcsinh}(ax))+10\sqrt{5}(-\text{arcsinh}(ax))^{3/2}\Gamma\left(\frac{1}{2},-5\text{arcsinh}(ax)\right)}{48\text{arcsinh}(ax)^{3/2}} + \frac{e^{3\text{arcsinh}(ax)}(1+6\text{ar}}{}}{}$$

```
[In] Integrate[x^4/ArcSinh[a*x]^(5/2),x]
```

```
[Out] (-1/48*(E^(5*ArcSinh[a*x])*(1 + 10*ArcSinh[a*x]) + 10*Sqrt[5]*(-ArcSinh[a*x
])^(3/2)*Gamma[1/2, -5*ArcSinh[a*x]])/ArcSinh[a*x]^(3/2) + (E^(3*ArcSinh[a*
x])*(1 + 6*ArcSinh[a*x]) + 6*Sqrt[3]*(-ArcSinh[a*x])^(3/2)*Gamma[1/2, -3*Ar
cSinh[a*x]])/(16*ArcSinh[a*x]^(3/2)) - (E^ArcSinh[a*x]*(1 + 2*ArcSinh[a*x])
 + 2*(-ArcSinh[a*x])^(3/2)*Gamma[1/2, -ArcSinh[a*x]])/(24*ArcSinh[a*x]^(3/2
)) - (1 - 2*ArcSinh[a*x] + 2*E^ArcSinh[a*x]*ArcSinh[a*x]^(3/2)*Gamma[1/2, A
rcSinh[a*x]])/(24*E^ArcSinh[a*x]*ArcSinh[a*x]^(3/2)) + (1/(E^(3*ArcSinh[a*x
])*ArcSinh[a*x]^(3/2)) - 6/(E^(3*ArcSinh[a*x])*Sqrt[ArcSinh[a*x]]) + 6*Sqrt
```

```
[3]*Gamma[1/2, 3*ArcSinh[a*x]])/16 - (1 - 10*ArcSinh[a*x] + 10*Sqrt[5]*E^(5
*ArcSinh[a*x])*ArcSinh[a*x]^(3/2)*Gamma[1/2, 5*ArcSinh[a*x]])/(48*E^(5*ArcS
inh[a*x])*ArcSinh[a*x]^(3/2)))/a^5
```

Maple [F]

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^{\frac{5}{2}}}dx$$

```
[In] int(x^4/arcsinh(a*x)^(5/2),x)
[Out] int(x^4/arcsinh(a*x)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^4/arcsinh(a*x)^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \int \frac{x^4}{\operatorname{asinh}^{\frac{5}{2}}(ax)}\,dx$$

```
[In] integrate(x**4/asinh(a*x)**(5/2),x)
[Out] Integral(x**4/asinh(a*x)**(5/2), x)
```

Maxima [F]

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \int \frac{x^4}{\operatorname{arsinh}(ax)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^4/arcsinh(a*x)^(5/2),x, algorithm="maxima")
[Out] integrate(x^4/arcsinh(a*x)^(5/2), x)
```

Giac [F]

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \int \frac{x^4}{\operatorname{arsinh}\left(ax\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^4/arcsinh(a*x)^(5/2),x, algorithm="giac")
[Out] integrate(x^4/arcsinh(a*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \int \frac{x^4}{{\mathrm{asinh}\left(a\,x\right)}^{5/2}}\,dx$$

```
[In] int(x^4/asinh(a*x)^(5/2),x)
[Out] int(x^4/asinh(a*x)^(5/2), x)
```

3.106 $\int \frac{x^3}{\operatorname{arcsinh}(ax)^{5/2}} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 167

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^{5/2}} \, dx = -\frac{2x^3\sqrt{1+a^2x^2}}{3a\operatorname{arcsinh}(ax)^{3/2}} - \frac{4x^2}{a^2\sqrt{\operatorname{arcsinh}(ax)}} - \frac{16x^4}{3\sqrt{\operatorname{arcsinh}(ax)}} - \frac{2\sqrt{\pi}\operatorname{erf}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{3a^4} + \frac{\sqrt{2\pi}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{3a^4} + \frac{2\sqrt{\pi}\operatorname{erfi}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{3a^4} - \frac{\sqrt{2\pi}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{3a^4}$$

```
[Out] -2/3*erf(2*arcsinh(a*x)^(1/2))*Pi^(1/2)/a^4+2/3*erfi(2*arcsinh(a*x)^(1/2))*
Pi^(1/2)/a^4+1/3*erf(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^4-1/3*e
rfi(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^4-2/3*x^3*(a^2*x^2+1)^(1
/2)/a/arcsinh(a*x)^(3/2)-4*x^2/a^2/arcsinh(a*x)^(1/2)-16/3*x^4/arcsinh(a*x)
^(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used

$= \{5779, 5818, 5780, 5556, 3389, 2211, 2235, 2236, 12\}$

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = -\frac{2\sqrt{\pi}\operatorname{erf}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{3a^4} + \frac{\sqrt{2\pi}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{3a^4} + \frac{2\sqrt{\pi}\operatorname{erfi}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{3a^4} - \frac{\sqrt{2\pi}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{3a^4} - \frac{4x^2}{a^2\sqrt{\operatorname{arcsinh}(ax)}} - \frac{2x^3\sqrt{a^2x^2+1}}{3a\operatorname{arcsinh}(ax)^{3/2}} - \frac{16x^4}{3\sqrt{\operatorname{arcsinh}(ax)}}$$

```
[In] Int[x^3/ArcSinh[a*x]^(5/2),x]
```

```
[Out] (-2*x^3*Sqrt[1 + a^2*x^2])/(3*a*ArcSinh[a*x]^(3/2)) - (4*x^2)/(a^2*Sqrt[Arc
Sinh[a*x]]) - (16*x^4)/(3*Sqrt[ArcSinh[a*x]]) - (2*Sqrt[Pi]*Erf[2*Sqrt[ArcS
inh[a*x]]])/(3*a^4) + (Sqrt[2*Pi]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(3*a^4)
+ (2*Sqrt[Pi]*Erfi[2*Sqrt[ArcSinh[a*x]]])/(3*a^4) - (Sqrt[2*Pi]*Erfi[Sqrt[2
]*Sqrt[ArcSinh[a*x]]])/(3*a^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/S
qrt[1 + c^2*x^2]), x], x] - Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Sinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2x^3\sqrt{1+a^2x^2}}{3a\operatorname{arcsinh}(ax)^{3/2}}+\frac{2\int\frac{x^2}{\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^{3/2}}\,dx}{a}\\&\quad+\frac{1}{3}(8a)\int\frac{x^4}{\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^{3/2}}\,dx\\&=-\frac{2x^3\sqrt{1+a^2x^2}}{3a\operatorname{arcsinh}(ax)^{3/2}}-\frac{4x^2}{a^2\sqrt{\operatorname{arcsinh}(ax)}}-\frac{16x^4}{3\sqrt{\operatorname{arcsinh}(ax)}}\\&\quad+\frac{64}{3}\int\frac{x^3}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx+\frac{8\int\frac{x}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx}{a^2}\end{aligned}$$

$$
\begin{aligned}
&= -\frac{2x^3\sqrt{1+a^2x^2}}{3a\text{arcsinh}(ax)^{3/2}} - \frac{4x^2}{a^2\sqrt{\text{arcsinh}(ax)}} - \frac{16x^4}{3\sqrt{\text{arcsinh}(ax)}} \\
&\quad + \frac{8\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{a^4} \\
&\quad + \frac{64\text{Subst}\left(\int \frac{\cosh(x)\sinh^3(x)}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{3a^4} \\
&= -\frac{2x^3\sqrt{1+a^2x^2}}{3a\text{arcsinh}(ax)^{3/2}} - \frac{4x^2}{a^2\sqrt{\text{arcsinh}(ax)}} - \frac{16x^4}{3\sqrt{\text{arcsinh}(ax)}} \\
&\quad + \frac{8\text{Subst}\left(\int \frac{\sinh(2x)}{2\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{a^4} \\
&\quad + \frac{64\text{Subst}\left(\int \left(-\frac{\sinh(2x)}{4\sqrt{x}} + \frac{\sinh(4x)}{8\sqrt{x}}\right)\,dx, x, \text{arcsinh}(ax)\right)}{3a^4} \\
&= -\frac{2x^3\sqrt{1+a^2x^2}}{3a\text{arcsinh}(ax)^{3/2}} - \frac{4x^2}{a^2\sqrt{\text{arcsinh}(ax)}} - \frac{16x^4}{3\sqrt{\text{arcsinh}(ax)}} \\
&\quad + \frac{8\text{Subst}\left(\int \frac{\sinh(4x)}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{3a^4} + \frac{4\text{Subst}\left(\int \frac{\sinh(2x)}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{a^4} \\
&\quad - \frac{16\text{Subst}\left(\int \frac{\sinh(2x)}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{3a^4} \\
&= -\frac{2x^3\sqrt{1+a^2x^2}}{3a\text{arcsinh}(ax)^{3/2}} - \frac{4x^2}{a^2\sqrt{\text{arcsinh}(ax)}} - \frac{16x^4}{3\sqrt{\text{arcsinh}(ax)}} \\
&\quad - \frac{4\text{Subst}\left(\int \frac{e^{-4x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{3a^4} + \frac{4\text{Subst}\left(\int \frac{e^{4x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{3a^4} \\
&\quad - \frac{2\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{a^4} + \frac{2\text{Subst}\left(\int \frac{e^{2x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{a^4} \\
&\quad + \frac{8\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{3a^4} - \frac{8\text{Subst}\left(\int \frac{e^{2x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{3a^4} \\
&= -\frac{2x^3\sqrt{1+a^2x^2}}{3a\text{arcsinh}(ax)^{3/2}} - \frac{4x^2}{a^2\sqrt{\text{arcsinh}(ax)}} - \frac{16x^4}{3\sqrt{\text{arcsinh}(ax)}} \\
&\quad - \frac{8\text{Subst}\left(\int e^{-4x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{3a^4} + \frac{8\text{Subst}\left(\int e^{4x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{3a^4} \\
&\quad - \frac{4\text{Subst}\left(\int e^{-2x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{a^4} + \frac{4\text{Subst}\left(\int e^{2x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{a^4} \\
&\quad + \frac{16\text{Subst}\left(\int e^{-2x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{3a^4} - \frac{16\text{Subst}\left(\int e^{2x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{3a^4}
\end{aligned}
$$

$$= -\frac{2x^3\sqrt{1+a^2x^2}}{3a\operatorname{arcsinh}(ax)^{3/2}} - \frac{4x^2}{a^2\sqrt{\operatorname{arcsinh}(ax)}} - \frac{16x^4}{3\sqrt{\operatorname{arcsinh}(ax)}} - \frac{2\sqrt{\pi}\operatorname{erf}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{3a^4} + \frac{\sqrt{2\pi}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{3a^4} + \frac{2\sqrt{\pi}\operatorname{erfi}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{3a^4} - \frac{\sqrt{2\pi}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{3a^4}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.10

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \frac{-4e^{-4\operatorname{arcsinh}(ax)}\operatorname{arcsinh}(ax) + 4e^{-2\operatorname{arcsinh}(ax)}\operatorname{arcsinh}(ax) + 4e^{2\operatorname{arcsinh}(ax)}\operatorname{arcsinh}(ax) -}{}$$

```
[In] Integrate[x^3/ArcSinh[a*x]^(5/2),x]
```

```
[Out] ((-4*ArcSinh[a*x])/E^(4*ArcSinh[a*x]) + (4*ArcSinh[a*x])/E^(2*ArcSinh[a*x])
 + 4*E^(2*ArcSinh[a*x])*ArcSinh[a*x] - 4*E^(4*ArcSinh[a*x])*ArcSinh[a*x] -
8*(-ArcSinh[a*x])^(3/2)*Gamma[1/2, -4*ArcSinh[a*x]] + 4*Sqrt[2]*(-ArcSinh[a
*x])^(3/2)*Gamma[1/2, -2*ArcSinh[a*x]] - 4*Sqrt[2]*ArcSinh[a*x]^(3/2)*Gamma
[1/2, 2*ArcSinh[a*x]] + 8*ArcSinh[a*x]^(3/2)*Gamma[1/2, 4*ArcSinh[a*x]] + 2
*Sinh[2*ArcSinh[a*x]] - Sinh[4*ArcSinh[a*x]])/(12*a^4*ArcSinh[a*x]^(3/2))
```

Maple [F]

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^{\frac{5}{2}}}dx$$

```
[In] int(x^3/arcsinh(a*x)^(5/2),x)
```

```
[Out] int(x^3/arcsinh(a*x)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3/arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \int \frac{x^3}{\operatorname{asinh}^{\frac{5}{2}}(ax)}\,dx$$

```
[In] integrate(x**3/asinh(a*x)**(5/2),x)
[Out] Integral(x**3/asinh(a*x)**(5/2), x)
```

Maxima [F]

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \int \frac{x^3}{\operatorname{arsinh}(ax)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^3/arcsinh(a*x)^(5/2),x, algorithm="maxima")
[Out] integrate(x^3/arcsinh(a*x)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3/arcsinh(a*x)^(5/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \int \frac{x^3}{\operatorname{asinh}(a\,x)^{5/2}}\,dx$$

```
[In] int(x^3/asinh(a*x)^(5/2),x)
[Out] int(x^3/asinh(a*x)^(5/2), x)
```

3.107 $\int \frac{x^2}{\operatorname{arcsinh}(ax)^{5/2}} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 161

$$\begin{aligned}\int \frac{x^2}{\operatorname{arcsinh}(ax)^{5/2}} \, dx = & -\frac{2x^2\sqrt{1+a^2x^2}}{3a\operatorname{arcsinh}(ax)^{3/2}} - \frac{8x}{3a^2\sqrt{\operatorname{arcsinh}(ax)}} \\ & - \frac{4x^3}{\sqrt{\operatorname{arcsinh}(ax)}} - \frac{\sqrt{\pi}\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{6a^3} + \frac{\sqrt{3\pi}\operatorname{erf}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{2a^3} \\ & - \frac{\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{6a^3} + \frac{\sqrt{3\pi}\operatorname{erfi}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{2a^3}\end{aligned}$$

```
[Out] -1/6*erf(arcsinh(a*x)^(1/2))*Pi^(1/2)/a^3-1/6*erfi(arcsinh(a*x)^(1/2))*Pi^(
1/2)/a^3+1/2*erf(3^(1/2)*arcsinh(a*x)^(1/2))*3^(1/2)*Pi^(1/2)/a^3+1/2*erfi(
3^(1/2)*arcsinh(a*x)^(1/2))*3^(1/2)*Pi^(1/2)/a^3-2/3*x^2*(a^2*x^2+1)^(1/2)/
a/arcsinh(a*x)^(3/2)-8/3*x/a^2/arcsinh(a*x)^(1/2)-4*x^3/arcsinh(a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5779, 5818, 5780, 5556, 3388, 2211, 2235, 2236, 5774}

$$\begin{aligned}\int \frac{x^2}{\operatorname{arcsinh}(ax)^{5/2}} \, dx = & -\frac{\sqrt{\pi}\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{6a^3} + \frac{\sqrt{3\pi}\operatorname{erf}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{2a^3} \\ & - \frac{\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{6a^3} + \frac{\sqrt{3\pi}\operatorname{erfi}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{2a^3} \\ & - \frac{2x^2\sqrt{a^2x^2+1}}{3a\operatorname{arcsinh}(ax)^{3/2}} - \frac{8x}{3a^2\sqrt{\operatorname{arcsinh}(ax)}} - \frac{4x^3}{\sqrt{\operatorname{arcsinh}(ax)}}\end{aligned}$$

```
[In] Int[x^2/ArcSinh[a*x]^(5/2),x]
```

```
[Out] (-2*x^2*Sqrt[1 + a^2*x^2])/(3*a*ArcSinh[a*x]^(3/2)) - (8*x)/(3*a^2*Sqrt[Arc
Sinh[a*x]]) - (4*x^3)/Sqrt[ArcSinh[a*x]] - (Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]
])/(6*a^3) + (Sqrt[3*Pi]*Erf[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(2*a^3) - (Sqrt[P
i]*Erfi[Sqrt[ArcSinh[a*x]]])/(6*a^3) + (Sqrt[3*Pi]*Erfi[Sqrt[3]*Sqrt[ArcSin
h[a*x]]])/(2*a^3)
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/S
qrt[1 + c^2*x^2]), x], x] - Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Sinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2x^2\sqrt{1+a^2x^2}}{3a\operatorname{arcsinh}(ax)^{3/2}} + \frac{4\int \frac{x}{\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^{3/2}}\,dx}{3a} \\
&\quad + (2a)\int \frac{x^3}{\sqrt{1+a^2x^2}\operatorname{arcsinh}(ax)^{3/2}}\,dx \\
&= -\frac{2x^2\sqrt{1+a^2x^2}}{3a\operatorname{arcsinh}(ax)^{3/2}} - \frac{8x}{3a^2\sqrt{\operatorname{arcsinh}(ax)}} - \frac{4x^3}{\sqrt{\operatorname{arcsinh}(ax)}} \\
&\quad + 12\int \frac{x^2}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx + \frac{8\int \frac{1}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx}{3a^2} \\
&= -\frac{2x^2\sqrt{1+a^2x^2}}{3a\operatorname{arcsinh}(ax)^{3/2}} - \frac{8x}{3a^2\sqrt{\operatorname{arcsinh}(ax)}} \\
&\quad - \frac{4x^3}{\sqrt{\operatorname{arcsinh}(ax)}} + \frac{8\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{3a^3} \\
&\quad + \frac{12\text{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{a^3}
\end{aligned}
$$

$$= -\frac{2x^2\sqrt{1+a^2x^2}}{3a\text{arcsinh}(ax)^{3/2}} - \frac{8x}{3a^2\sqrt{\text{arcsinh}(ax)}} - \frac{4x^3}{\sqrt{\text{arcsinh}(ax)}} + \frac{4\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{3a^3} + \frac{4\text{Subst}\left(\int \frac{e^{x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{3a^3} + \frac{12\text{Subst}\left(\int \left(-\frac{\cosh(x)}{4\sqrt{x}} + \frac{\cosh(3x)}{4\sqrt{x}}\right)\,dx, x, \text{arcsinh}(ax)\right)}{a^3}$$

$$= -\frac{2x^2\sqrt{1+a^2x^2}}{3a\text{arcsinh}(ax)^{3/2}} - \frac{8x}{3a^2\sqrt{\text{arcsinh}(ax)}} - \frac{4x^3}{\sqrt{\text{arcsinh}(ax)}} + \frac{8\text{Subst}\left(\int e^{-x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{3a^3} + \frac{8\text{Subst}\left(\int e^{x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{3a^3} - \frac{3\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{a^3} + \frac{3\text{Subst}\left(\int \frac{\cosh(3x)}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{a^3}$$

$$= -\frac{2x^2\sqrt{1+a^2x^2}}{3a\text{arcsinh}(ax)^{3/2}} - \frac{8x}{3a^2\sqrt{\text{arcsinh}(ax)}} - \frac{4x^3}{\sqrt{\text{arcsinh}(ax)}} + \frac{4\sqrt{\pi}\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)}{3a^3} + \frac{4\sqrt{\pi}\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)}{3a^3} + \frac{3\text{Subst}\left(\int \frac{e^{-3x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{2a^3} - \frac{3\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{2a^3} - \frac{3\text{Subst}\left(\int \frac{e^{x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{2a^3} + \frac{3\text{Subst}\left(\int \frac{e^{3x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{2a^3}$$

$$= -\frac{2x^2\sqrt{1+a^2x^2}}{3a\text{arcsinh}(ax)^{3/2}} - \frac{8x}{3a^2\sqrt{\text{arcsinh}(ax)}} - \frac{4x^3}{\sqrt{\text{arcsinh}(ax)}} + \frac{4\sqrt{\pi}\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)}{3a^3} + \frac{4\sqrt{\pi}\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)}{3a^3} + \frac{3\text{Subst}\left(\int e^{-3x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{a^3} - \frac{3\text{Subst}\left(\int e^{-x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{a^3} - \frac{3\text{Subst}\left(\int e^{x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{a^3} + \frac{3\text{Subst}\left(\int e^{3x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{a^3}$$

$$= -\frac{2x^2\sqrt{1+a^2x^2}}{3a\text{arcsinh}(ax)^{3/2}} - \frac{8x}{3a^2\sqrt{\text{arcsinh}(ax)}} - \frac{4x^3}{\sqrt{\text{arcsinh}(ax)}} - \frac{\sqrt{\pi}\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)}{6a^3} + \frac{\sqrt{3\pi}\text{erf}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)}{2a^3} - \frac{\sqrt{\pi}\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)}{6a^3} + \frac{\sqrt{3\pi}\text{erfi}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)}{2a^3}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.38

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \frac{-\frac{e^{3\operatorname{arcsinh}(ax)}(1+6\operatorname{arcsinh}(ax))+6\sqrt{3}(-\operatorname{arcsinh}(ax))^{3/2}\Gamma\left(\frac{1}{2},-3\operatorname{arcsinh}(ax)\right)}{12\operatorname{arcsinh}(ax)^{3/2}} + \frac{e^{\operatorname{arcsinh}(ax)}(1+2\operatorname{arcs}}{}}{}$$

```
[In] Integrate[x^2/ArcSinh[a*x]^(5/2),x]
```

```
[Out] (-1/12*(E^(3*ArcSinh[a*x])*(1 + 6*ArcSinh[a*x]) + 6*Sqrt[3]*(-ArcSinh[a*x])
^(3/2)*Gamma[1/2, -3*ArcSinh[a*x]])/ArcSinh[a*x]^(3/2) + (E^ArcSinh[a*x]*(1
 + 2*ArcSinh[a*x]) + 2*(-ArcSinh[a*x])^(3/2)*Gamma[1/2, -ArcSinh[a*x]])/(12
*ArcSinh[a*x]^(3/2)) + (1 - 2*ArcSinh[a*x] + 2*E^ArcSinh[a*x]*ArcSinh[a*x]^
(3/2)*Gamma[1/2, ArcSinh[a*x]])/(12*E^ArcSinh[a*x]*ArcSinh[a*x]^(3/2)) + (-
(1/(E^(3*ArcSinh[a*x])*ArcSinh[a*x]^(3/2))) + 6/(E^(3*ArcSinh[a*x])*Sqrt[Ar
cSinh[a*x]]) - 6*Sqrt[3]*Gamma[1/2, 3*ArcSinh[a*x]])/12)/a^3
```

Maple [F]

$$\int \frac{x^2}{\operatorname{arcsinh}\left(ax\right)^{\frac{5}{2}}}dx$$

```
[In] int(x^2/arcsinh(a*x)^(5/2),x)
```

```
[Out] int(x^2/arcsinh(a*x)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2/arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \int \frac{x^2}{\operatorname{asinh}^{\frac{5}{2}}(ax)}\,dx$$

```
[In] integrate(x**2/asinh(a*x)**(5/2),x)
[Out] Integral(x**2/asinh(a*x)**(5/2), x)
```

Maxima [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \int \frac{x^2}{\operatorname{arsinh}(ax)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^2/arcsinh(a*x)^(5/2),x, algorithm="maxima")
[Out] integrate(x^2/arcsinh(a*x)^(5/2), x)
```

Giac [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \int \frac{x^2}{\operatorname{arsinh}(ax)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^2/arcsinh(a*x)^(5/2),x, algorithm="giac")
[Out] integrate(x^2/arcsinh(a*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \int \frac{x^2}{\operatorname{asinh}(a\,x)^{5/2}}\,dx$$

```
[In] int(x^2/asinh(a*x)^(5/2),x)
[Out] int(x^2/asinh(a*x)^(5/2), x)
```

3.108 $\int \frac{x}{\mathrm{arcsinh}(ax)^{5/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 118

$$\int \frac{x}{\mathrm{arcsinh}(ax)^{5/2}}\,dx = -\frac{2x\sqrt{1+a^2x^2}}{3a\,\mathrm{arcsinh}(ax)^{3/2}} - \frac{4}{3a^2\sqrt{\mathrm{arcsinh}(ax)}} - \frac{8x^2}{3\sqrt{\mathrm{arcsinh}(ax)}} - \frac{2\sqrt{2\pi}\,\mathrm{erf}\left(\sqrt{2}\sqrt{\mathrm{arcsinh}(ax)}\right)}{3a^2} + \frac{2\sqrt{2\pi}\,\mathrm{erfi}\left(\sqrt{2}\sqrt{\mathrm{arcsinh}(ax)}\right)}{3a^2}$$

```
[Out] -2/3*erf(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^2+2/3*erfi(2^(1/2)*
arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^2-2/3*x*(a^2*x^2+1)^(1/2)/a/arcsinh(
a*x)^(3/2)-4/3/a^2/arcsinh(a*x)^(1/2)-8/3*x^2/arcsinh(a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5779, 5818, 5780, 5556, 12, 3389, 2211, 2235, 2236, 5783}

$$\int \frac{x}{\mathrm{arcsinh}(ax)^{5/2}}\,dx = -\frac{2\sqrt{2\pi}\,\mathrm{erf}\left(\sqrt{2}\sqrt{\mathrm{arcsinh}(ax)}\right)}{3a^2} + \frac{2\sqrt{2\pi}\,\mathrm{erfi}\left(\sqrt{2}\sqrt{\mathrm{arcsinh}(ax)}\right)}{3a^2} - \frac{2x\sqrt{a^2x^2+1}}{3a\,\mathrm{arcsinh}(ax)^{3/2}} - \frac{4}{3a^2\sqrt{\mathrm{arcsinh}(ax)}} - \frac{8x^2}{3\sqrt{\mathrm{arcsinh}(ax)}}$$

```
[In] Int[x/ArcSinh[a*x]^(5/2),x]
```

```
[Out] (-2*x*Sqrt[1 + a^2*x^2])/(3*a*ArcSinh[a*x]^(3/2)) - 4/(3*a^2*Sqrt[ArcSinh[a
*x]]) - (8*x^2)/(3*Sqrt[ArcSinh[a*x]]) - (2*Sqrt[2*Pi]*Erf[Sqrt[2]*Sqrt[Arc
Sinh[a*x]]])/(3*a^2) + (2*Sqrt[2*Pi]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(3*a
^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/S
qrt[1 + c^2*x^2]), x], x] - Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Sinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2x\sqrt{1+a^2x^2}}{3a\text{arcsinh}(ax)^{3/2}}+\frac{2\int\frac{1}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}\,dx}{3a}\\
&\quad+\frac{1}{3}(4a)\int\frac{x^2}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}\,dx\\
&=-\frac{2x\sqrt{1+a^2x^2}}{3a\text{arcsinh}(ax)^{3/2}}-\frac{4}{3a^2\sqrt{\text{arcsinh}(ax)}}-\frac{8x^2}{3\sqrt{\text{arcsinh}(ax)}}+\frac{16}{3}\int\frac{x}{\sqrt{\text{arcsinh}(ax)}}\,dx\\
&=-\frac{2x\sqrt{1+a^2x^2}}{3a\text{arcsinh}(ax)^{3/2}}-\frac{4}{3a^2\sqrt{\text{arcsinh}(ax)}}-\frac{8x^2}{3\sqrt{\text{arcsinh}(ax)}}\\
&\quad+\frac{16\text{Subst}\left(\int\frac{\cosh(x)\sinh(x)}{\sqrt{x}}\,dx,x,\text{arcsinh}(ax)\right)}{3a^2}\\
&=-\frac{2x\sqrt{1+a^2x^2}}{3a\text{arcsinh}(ax)^{3/2}}-\frac{4}{3a^2\sqrt{\text{arcsinh}(ax)}}-\frac{8x^2}{3\sqrt{\text{arcsinh}(ax)}}\\
&\quad+\frac{16\text{Subst}\left(\int\frac{\sinh(2x)}{2\sqrt{x}}\,dx,x,\text{arcsinh}(ax)\right)}{3a^2}\\
&=-\frac{2x\sqrt{1+a^2x^2}}{3a\text{arcsinh}(ax)^{3/2}}-\frac{4}{3a^2\sqrt{\text{arcsinh}(ax)}}-\frac{8x^2}{3\sqrt{\text{arcsinh}(ax)}}\\
&\quad+\frac{8\text{Subst}\left(\int\frac{\sinh(2x)}{\sqrt{x}}\,dx,x,\text{arcsinh}(ax)\right)}{3a^2}
\end{aligned}
$$

$$= -\frac{2x\sqrt{1+a^2x^2}}{3a\text{arcsinh}(ax)^{3/2}} - \frac{4}{3a^2\sqrt{\text{arcsinh}(ax)}} - \frac{8x^2}{3\sqrt{\text{arcsinh}(ax)}} - \frac{4\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{3a^2} + \frac{4\text{Subst}\left(\int \frac{e^{2x}}{\sqrt{x}}\,dx, x, \text{arcsinh}(ax)\right)}{3a^2}$$

$$= -\frac{2x\sqrt{1+a^2x^2}}{3a\text{arcsinh}(ax)^{3/2}} - \frac{4}{3a^2\sqrt{\text{arcsinh}(ax)}} - \frac{8x^2}{3\sqrt{\text{arcsinh}(ax)}} - \frac{8\text{Subst}\left(\int e^{-2x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{3a^2} + \frac{8\text{Subst}\left(\int e^{2x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{3a^2}$$

$$= -\frac{2x\sqrt{1+a^2x^2}}{3a\text{arcsinh}(ax)^{3/2}} - \frac{4}{3a^2\sqrt{\text{arcsinh}(ax)}} - \frac{8x^2}{3\sqrt{\text{arcsinh}(ax)}} - \frac{2\sqrt{2\pi}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{3a^2} + \frac{2\sqrt{2\pi}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{3a^2}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.83

$$\int \frac{x}{\text{arcsinh}(ax)^{5/2}}\,dx =$$

$$-\frac{2\text{arcsinh}(ax)\left(e^{-2\text{arcsinh}(ax)} + e^{2\text{arcsinh}(ax)} - \sqrt{2}\sqrt{-\text{arcsinh}(ax)}\Gamma\left(\frac{1}{2}, -2\text{arcsinh}(ax)\right) - \sqrt{2}\sqrt{\text{arcsinh}(ax)}\Gamma(}{3a^2\text{arcsinh}(ax)^{3/2}}$$

```
[In] Integrate[x/ArcSinh[a*x]^(5/2),x]
```

```
[Out] -1/3*(2*ArcSinh[a*x]*(E^(-2*ArcSinh[a*x]) + E^(2*ArcSinh[a*x]) - Sqrt[2]*Sq
rt[-ArcSinh[a*x]]*Gamma[1/2, -2*ArcSinh[a*x]] - Sqrt[2]*Sqrt[ArcSinh[a*x]]*
Gamma[1/2, 2*ArcSinh[a*x]]) + Sinh[2*ArcSinh[a*x]])/(a^2*ArcSinh[a*x]^(3/2)
)
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.01

method	result
default	$-\frac{\sqrt{2}\,\left(4\,\text{arcsinh}(ax)^{\frac{3}{2}}\sqrt{2}\,\sqrt{\pi}\,a^2x^2+\sqrt{2}\,\sqrt{\text{arcsinh}(ax)}\,\sqrt{\pi}\,\sqrt{a^2x^2+1}\,ax+2\,\text{arcsinh}(ax)^2\pi\ \text{erf}\left(\sqrt{2}\,\sqrt{\text{arcsinh}(ax)}\right)-2\,\text{arcsinh}(ax)^2\pi\ \mathrm{er}}{3\sqrt{\pi}\,a^2\,\text{arcsinh}(ax)^2}$

```
[In] int(x/arcsinh(a*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*2^(1/2)*(4*arcsinh(a*x)^(3/2)*2^(1/2)*Pi^(1/2)*a^2*x^2+2^(1/2)*arcsinh
(a*x)^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*a*x+2*arcsinh(a*x)^2*Pi*erf(2^(1/2)*
arcsinh(a*x)^(1/2))-2*arcsinh(a*x)^2*Pi*erfi(2^(1/2)*arcsinh(a*x)^(1/2))+2*
arcsinh(a*x)^(3/2)*2^(1/2)*Pi^(1/2))/Pi^(1/2)/a^2/arcsinh(a*x)^2
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x/arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \int \frac{x}{\operatorname{asinh}^{\frac{5}{2}}(ax)}\,dx$$

```
[In] integrate(x/asinh(a*x)**(5/2),x)
```

```
[Out] Integral(x/asinh(a*x)**(5/2), x)
```

Maxima [F]

$$\int \frac{x}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \int \frac{x}{\operatorname{arsinh}(ax)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x/arcsinh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x/arcsinh(a*x)^(5/2), x)
```

Giac [F]

$$\int \frac{x}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \int \frac{x}{\operatorname{arsinh}\left(ax\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x/arcsinh(a*x)^(5/2),x, algorithm="giac")
[Out] integrate(x/arcsinh(a*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\operatorname{arcsinh}(ax)^{5/2}}\,dx = \int \frac{x}{\operatorname{asinh}\left(a\,x\right)^{5/2}}\,dx$$

```
[In] int(x/asinh(a*x)^(5/2),x)
[Out] int(x/asinh(a*x)^(5/2), x)
```

3.109 $\int \frac{1}{\operatorname{arcsinh}(ax)^{5/2}}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 84

$$\int \frac{1}{\operatorname{arcsinh}(ax)^{5/2}}\,dx=-\frac{2\sqrt{1+a^2x^2}}{3a\operatorname{arcsinh}(ax)^{3/2}}-\frac{4x}{3\sqrt{\operatorname{arcsinh}(ax)}}+\frac{2\sqrt{\pi}\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{3a}+\frac{2\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{3a}$$

```
[Out] 2/3*erf(arcsinh(a*x)^(1/2))*Pi^(1/2)/a+2/3*erfi(arcsinh(a*x)^(1/2))*Pi^(1/2
)/a-2/3*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^(3/2)-4/3*x/arcsinh(a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {5773, 5818, 5774, 3388, 2211, 2235, 2236}

$$\int \frac{1}{\operatorname{arcsinh}(ax)^{5/2}}\,dx=-\frac{2\sqrt{a^2x^2+1}}{3a\operatorname{arcsinh}(ax)^{3/2}}+\frac{2\sqrt{\pi}\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{3a}+\frac{2\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{3a}-\frac{4x}{3\sqrt{\operatorname{arcsinh}(ax)}}$$

```
[In] Int[ArcSinh[a*x]^(-5/2),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2])/(3*a*ArcSinh[a*x]^(3/2)) - (4*x)/(3*Sqrt[ArcSinh[a*x
]]) + (2*Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]])/(3*a) + (2*Sqrt[Pi]*Erfi[Sqrt[Ar
cSinh[a*x]]])/(3*a)
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2\sqrt{1+a^2x^2}}{3a\text{arcsinh}(ax)^{3/2}}+\frac{1}{3}(2a)\int\frac{x}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{3/2}}\,dx\\
&= -\frac{2\sqrt{1+a^2x^2}}{3a\text{arcsinh}(ax)^{3/2}}-\frac{4x}{3\sqrt{\text{arcsinh}(ax)}}+\frac{4}{3}\int\frac{1}{\sqrt{\text{arcsinh}(ax)}}\,dx\\
&= -\frac{2\sqrt{1+a^2x^2}}{3a\text{arcsinh}(ax)^{3/2}}-\frac{4x}{3\sqrt{\text{arcsinh}(ax)}}+\frac{4\text{Subst}\left(\int\frac{\cosh(x)}{\sqrt{x}}\,dx,x,\text{arcsinh}(ax)\right)}{3a}\\
&= -\frac{2\sqrt{1+a^2x^2}}{3a\text{arcsinh}(ax)^{3/2}}-\frac{4x}{3\sqrt{\text{arcsinh}(ax)}}\\
&\quad+\frac{2\text{Subst}\left(\int\frac{e^{-x}}{\sqrt{x}}\,dx,x,\text{arcsinh}(ax)\right)}{3a}+\frac{2\text{Subst}\left(\int\frac{e^{x}}{\sqrt{x}}\,dx,x,\text{arcsinh}(ax)\right)}{3a}\\
&= -\frac{2\sqrt{1+a^2x^2}}{3a\text{arcsinh}(ax)^{3/2}}-\frac{4x}{3\sqrt{\text{arcsinh}(ax)}}+\frac{4\text{Subst}\left(\int e^{-x^2}\,dx,x,\sqrt{\text{arcsinh}(ax)}\right)}{3a}\\
&\quad+\frac{4\text{Subst}\left(\int e^{x^2}\,dx,x,\sqrt{\text{arcsinh}(ax)}\right)}{3a}\\
&= -\frac{2\sqrt{1+a^2x^2}}{3a\text{arcsinh}(ax)^{3/2}}-\frac{4x}{3\sqrt{\text{arcsinh}(ax)}}+\frac{2\sqrt{\pi}\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)}{3a}+\frac{2\sqrt{\pi}\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)}{3a}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.25

$$
\int\frac{1}{\text{arcsinh}(ax)^{5/2}}\,dx =
$$

$$
-\frac{e^{-\text{arcsinh}(ax)}\big(1+e^{2\text{arcsinh}(ax)}-2\text{arcsinh}(ax)+2e^{2\text{arcsinh}(ax)}\text{arcsinh}(ax)+2e^{\text{arcsinh}(ax)}(-\text{arcsinh}(ax))^{3/2}\Gamma(\frac{1}{2}}{3a\text{arcsinh}(ax)^{3/2}}
$$

```
[In] Integrate[ArcSinh[a*x]^(-5/2),x]
```

```
[Out] -1/3*(1 + E^(2*ArcSinh[a*x]) - 2*ArcSinh[a*x] + 2*E^(2*ArcSinh[a*x])*ArcSin
h[a*x] + 2*E^ArcSinh[a*x]*(-ArcSinh[a*x])^(3/2)*Gamma[1/2, -ArcSinh[a*x]] +
 2*E^ArcSinh[a*x]*ArcSinh[a*x]^(3/2)*Gamma[1/2, ArcSinh[a*x]])/(a*E^ArcSinh
[a*x]*ArcSinh[a*x]^(3/2))
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{-\frac{4 \operatorname{arcsinh}(ax)^{\frac{3}{2}} \sqrt{\pi}\, a x}{3}+\frac{2 \operatorname{arcsinh}(ax)^{2} \pi \ \operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{3}+\frac{2 \operatorname{arcsinh}(ax)^{2} \pi \ \operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{3}-\frac{2 \sqrt{\operatorname{arcsinh}(ax)}\, \sqrt{\pi}\, \sqrt{a^{2} x^{2}+1}}{3}}{\sqrt{\pi}\, a \operatorname{arcsinh}(ax)^{2}}$	81

```
[In] int(1/arcsinh(a*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(-2*arcsinh(a*x)^(3/2)*Pi^(1/2)*a*x+arcsinh(a*x)^2*Pi*erf(arcsinh(a*x)^
(1/2))+arcsinh(a*x)^2*Pi*erfi(arcsinh(a*x)^(1/2))-arcsinh(a*x)^(1/2)*Pi^(1/
2)*(a^2*x^2+1)^(1/2))/Pi^(1/2)/a/arcsinh(a*x)^2
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\operatorname{arcsinh}(ax)^{5/2}}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/arcsinh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\operatorname{arcsinh}(ax)^{5/2}}\, dx = \int \frac{1}{\operatorname{asinh}^{\frac{5}{2}}(ax)}\, dx$$

```
[In] integrate(1/asinh(a*x)**(5/2),x)
```

```
[Out] Integral(asinh(a*x)**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{\operatorname{arcsinh}(ax)^{5/2}}\, dx = \int \frac{1}{\operatorname{arsinh}(ax)^{\frac{5}{2}}}\, dx$$

```
[In] integrate(1/arcsinh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{\operatorname{arcsinh}(ax)^{5/2}}\,dx=\int \frac{1}{\operatorname{arsinh}\left(ax\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/arcsinh(a*x)^(5/2),x, algorithm="giac")
[Out] integrate(arcsinh(a*x)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\operatorname{arcsinh}(ax)^{5/2}}\,dx=\int \frac{1}{{\mathrm{asinh}\left(a\,x\right)}^{5/2}}\,dx$$

```
[In] int(1/asinh(a*x)^(5/2),x)
[Out] int(1/asinh(a*x)^(5/2), x)
```

3.110 $\int \frac{1}{x\textbf{arcsinh}(ax)^{5/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{1}{x\text{arcsinh}(ax)^{5/2}}\,dx = \text{Int}\left(\frac{1}{x\text{arcsinh}(ax)^{5/2}},x\right)$$

```
[Out] Unintegrable(1/x/arcsinh(a*x)^(5/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\text{arcsinh}(ax)^{5/2}}\,dx = \int \frac{1}{x\text{arcsinh}(ax)^{5/2}}\,dx$$

```
[In] Int[1/(x*ArcSinh[a*x]^(5/2)),x]
```

```
[Out] Defer[Int][1/(x*ArcSinh[a*x]^(5/2)), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x\text{arcsinh}(ax)^{5/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x\text{arcsinh}(ax)^{5/2}}\,dx = \int \frac{1}{x\text{arcsinh}(ax)^{5/2}}\,dx$$

```
[In] Integrate[1/(x*ArcSinh[a*x]^(5/2)),x]
[Out] Integrate[1/(x*ArcSinh[a*x]^(5/2)), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{x \operatorname{arcsinh}\left(a x\right)^{\frac{5}{2}}}d x$$

```
[In] int(1/x/arcsinh(a*x)^(5/2),x)
[Out] int(1/x/arcsinh(a*x)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x\text{arcsinh}(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/x/arcsinh(a*x)^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 7.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\text{arcsinh}(ax)^{5/2}}\,dx = \int \frac{1}{x \operatorname{asinh}^{\frac{5}{2}}{\left(a x \right)}}\,dx$$

```
[In] integrate(1/x/asinh(a*x)**(5/2),x)
[Out] Integral(1/(x*asinh(a*x)**(5/2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\text{arcsinh}(ax)^{5/2}}\,dx = \int \frac{1}{x\,\text{arsinh}\left(ax\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/x/arcsinh(a*x)^(5/2),x, algorithm="maxima")
[Out] integrate(1/(x*arcsinh(a*x)^(5/2)), x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\text{arcsinh}(ax)^{5/2}}\,dx = \int \frac{1}{x\,\text{arsinh}\left(ax\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/x/arcsinh(a*x)^(5/2),x, algorithm="giac")
[Out] integrate(1/(x*arcsinh(a*x)^(5/2)), x)
```

Mupad [N/A]

Not integrable

Time = 2.35 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\text{arcsinh}(ax)^{5/2}}\,dx = \int \frac{1}{x\,\text{asinh}\left(a\,x\right)^{5/2}}\,dx$$

```
[In] int(1/(x*asinh(a*x)^(5/2)),x)
[Out] int(1/(x*asinh(a*x)^(5/2)), x)
```

3.111 $\int \frac{x^4}{\operatorname{arcsinh}(ax)^{7/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 285

$$
\begin{aligned}
\int \frac{x^4}{\operatorname{arcsinh}(ax)^{7/2}}\,dx = & -\frac{2x^4\sqrt{1+a^2x^2}}{5a\operatorname{arcsinh}(ax)^{5/2}} - \frac{16x^3}{15a^2\operatorname{arcsinh}(ax)^{3/2}} \\
& - \frac{4x^5}{3\operatorname{arcsinh}(ax)^{3/2}} - \frac{32x^2\sqrt{1+a^2x^2}}{5a^3\sqrt{\operatorname{arcsinh}(ax)}} - \frac{40x^4\sqrt{1+a^2x^2}}{3a\sqrt{\operatorname{arcsinh}(ax)}} \\
& - \frac{\sqrt{\pi}\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{30a^5} + \frac{9\sqrt{3\pi}\operatorname{erf}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{20a^5} \\
& - \frac{5\sqrt{5\pi}\operatorname{erf}\left(\sqrt{5}\sqrt{\operatorname{arcsinh}(ax)}\right)}{12a^5} + \frac{\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{30a^5} \\
& - \frac{9\sqrt{3\pi}\operatorname{erfi}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{20a^5} + \frac{5\sqrt{5\pi}\operatorname{erfi}\left(\sqrt{5}\sqrt{\operatorname{arcsinh}(ax)}\right)}{12a^5}
\end{aligned}
$$

```
[Out] -16/15*x^3/a^2/arcsinh(a*x)^(3/2)-4/3*x^5/arcsinh(a*x)^(3/2)-1/30*erf(arcsi
nh(a*x)^(1/2))*Pi^(1/2)/a^5+1/30*erfi(arcsinh(a*x)^(1/2))*Pi^(1/2)/a^5+9/20
*erf(3^(1/2)*arcsinh(a*x)^(1/2))*3^(1/2)*Pi^(1/2)/a^5-9/20*erfi(3^(1/2)*arc
sinh(a*x)^(1/2))*3^(1/2)*Pi^(1/2)/a^5-5/12*erf(5^(1/2)*arcsinh(a*x)^(1/2))*
5^(1/2)*Pi^(1/2)/a^5+5/12*erfi(5^(1/2)*arcsinh(a*x)^(1/2))*5^(1/2)*Pi^(1/2)
/a^5-2/5*x^4*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^(5/2)-32/5*x^2*(a^2*x^2+1)^(1
/2)/a^3/arcsinh(a*x)^(1/2)-40/3*x^4*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {5779, 5818, 5778, 3389, 2211, 2235, 2236}

$$\begin{aligned}\int \frac{x^4}{\operatorname{arcsinh}(ax)^{7/2}}\,dx = & -\frac{\sqrt{\pi}\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{30a^5} \\ & +\frac{9\sqrt{3\pi}\operatorname{erf}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{20a^5}-\frac{5\sqrt{5\pi}\operatorname{erf}\left(\sqrt{5}\sqrt{\operatorname{arcsinh}(ax)}\right)}{12a^5} \\ & +\frac{\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{30a^5}-\frac{9\sqrt{3\pi}\operatorname{erfi}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{20a^5} \\ & +\frac{5\sqrt{5\pi}\operatorname{erfi}\left(\sqrt{5}\sqrt{\operatorname{arcsinh}(ax)}\right)}{12a^5}-\frac{16x^3}{15a^2\operatorname{arcsinh}(ax)^{3/2}}-\frac{40x^4\sqrt{a^2x^2+1}}{3a\sqrt{\operatorname{arcsinh}(ax)}} \\ & -\frac{2x^4\sqrt{a^2x^2+1}}{5a\operatorname{arcsinh}(ax)^{5/2}}-\frac{32x^2\sqrt{a^2x^2+1}}{5a^3\sqrt{\operatorname{arcsinh}(ax)}}-\frac{4x^5}{3\operatorname{arcsinh}(ax)^{3/2}}\end{aligned}$$

```
[In] Int[x^4/ArcSinh[a*x]^(7/2),x]
```

```
[Out] (-2*x^4*Sqrt[1 + a^2*x^2])/(5*a*ArcSinh[a*x]^(5/2)) - (16*x^3)/(15*a^2*ArcS
inh[a*x]^(3/2)) - (4*x^5)/(3*ArcSinh[a*x]^(3/2)) - (32*x^2*Sqrt[1 + a^2*x^2
])/(5*a^3*Sqrt[ArcSinh[a*x]]) - (40*x^4*Sqrt[1 + a^2*x^2])/(3*a*Sqrt[ArcSin
h[a*x]]) - (Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]])/(30*a^5) + (9*Sqrt[3*Pi]*Erf[
Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(20*a^5) - (5*Sqrt[5*Pi]*Erf[Sqrt[5]*Sqrt[ArcS
inh[a*x]]])/(12*a^5) + (Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x]]])/(30*a^5) - (9*Sq
rt[3*Pi]*Erfi[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(20*a^5) + (5*Sqrt[5*Pi]*Erfi[Sq
rt[5]*Sqrt[ArcSinh[a*x]]])/(12*a^5)
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/S
qrt[1 + c^2*x^2]), x], x] - Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Sinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2x^4\sqrt{1+a^2x^2}}{5a\text{arcsinh}(ax)^{5/2}} + \frac{8\int \frac{x^3}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{5/2}}\,dx}{5a} \\
&\quad + (2a)\int \frac{x^5}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{5/2}}\,dx \\
&= -\frac{2x^4\sqrt{1+a^2x^2}}{5a\text{arcsinh}(ax)^{5/2}} - \frac{16x^3}{15a^2\text{arcsinh}(ax)^{3/2}} - \frac{4x^5}{3\text{arcsinh}(ax)^{3/2}} \\
&\quad + \frac{20}{3}\int \frac{x^4}{\text{arcsinh}(ax)^{3/2}}\,dx + \frac{16\int \frac{x^2}{\text{arcsinh}(ax)^{3/2}}\,dx}{5a^2}
\end{aligned}
$$

$$= -\frac{2x^4\sqrt{1+a^2x^2}}{5a\text{arcsinh}(ax)^{5/2}} - \frac{16x^3}{15a^2\text{arcsinh}(ax)^{3/2}} - \frac{4x^5}{3\text{arcsinh}(ax)^{3/2}} - \frac{32x^2\sqrt{1+a^2x^2}}{5a^3\sqrt{\text{arcsinh}(ax)}} - \frac{40x^4\sqrt{1+a^2x^2}}{3a\sqrt{\text{arcsinh}(ax)}} + \frac{32\text{Subst}\left(\int\left(-\frac{\sinh(x)}{4\sqrt{x}}+\frac{3\sinh(3x)}{4\sqrt{x}}\right)dx,x,\text{arcsinh}(ax)\right)}{5a^5} + \frac{40\text{Subst}\left(\int\left(\frac{\sinh(x)}{8\sqrt{x}}-\frac{9\sinh(3x)}{16\sqrt{x}}+\frac{5\sinh(5x)}{16\sqrt{x}}\right)dx,x,\text{arcsinh}(ax)\right)}{3a^5}$$

$$= -\frac{2x^4\sqrt{1+a^2x^2}}{5a\text{arcsinh}(ax)^{5/2}} - \frac{16x^3}{15a^2\text{arcsinh}(ax)^{3/2}} - \frac{4x^5}{3\text{arcsinh}(ax)^{3/2}} - \frac{32x^2\sqrt{1+a^2x^2}}{5a^3\sqrt{\text{arcsinh}(ax)}} - \frac{40x^4\sqrt{1+a^2x^2}}{3a\sqrt{\text{arcsinh}(ax)}} - \frac{8\text{Subst}\left(\int\frac{\sinh(x)}{\sqrt{x}}dx,x,\text{arcsinh}(ax)\right)}{5a^5} + \frac{5\text{Subst}\left(\int\frac{\sinh(x)}{\sqrt{x}}dx,x,\text{arcsinh}(ax)\right)}{3a^5} + \frac{25\text{Subst}\left(\int\frac{\sinh(5x)}{\sqrt{x}}dx,x,\text{arcsinh}(ax)\right)}{6a^5} + \frac{24\text{Subst}\left(\int\frac{\sinh(3x)}{\sqrt{x}}dx,x,\text{arcsinh}(ax)\right)}{5a^5} - \frac{15\text{Subst}\left(\int\frac{\sinh(3x)}{\sqrt{x}}dx,x,\text{arcsinh}(ax)\right)}{2a^5}$$

$$= -\frac{2x^4\sqrt{1+a^2x^2}}{5a\text{arcsinh}(ax)^{5/2}} - \frac{16x^3}{15a^2\text{arcsinh}(ax)^{3/2}} - \frac{4x^5}{3\text{arcsinh}(ax)^{3/2}} - \frac{32x^2\sqrt{1+a^2x^2}}{5a^3\sqrt{\text{arcsinh}(ax)}} - \frac{40x^4\sqrt{1+a^2x^2}}{3a\sqrt{\text{arcsinh}(ax)}} + \frac{4\text{Subst}\left(\int\frac{e^{-x}}{\sqrt{x}}dx,x,\text{arcsinh}(ax)\right)}{5a^5} - \frac{4\text{Subst}\left(\int\frac{e^{x}}{\sqrt{x}}dx,x,\text{arcsinh}(ax)\right)}{5a^5} - \frac{5\text{Subst}\left(\int\frac{e^{-x}}{\sqrt{x}}dx,x,\text{arcsinh}(ax)\right)}{6a^5} + \frac{5\text{Subst}\left(\int\frac{e^{x}}{\sqrt{x}}dx,x,\text{arcsinh}(ax)\right)}{6a^5} - \frac{25\text{Subst}\left(\int\frac{e^{-5x}}{\sqrt{x}}dx,x,\text{arcsinh}(ax)\right)}{12a^5} + \frac{25\text{Subst}\left(\int\frac{e^{5x}}{\sqrt{x}}dx,x,\text{arcsinh}(ax)\right)}{12a^5} - \frac{12\text{Subst}\left(\int\frac{e^{-3x}}{\sqrt{x}}dx,x,\text{arcsinh}(ax)\right)}{5a^5} + \frac{12\text{Subst}\left(\int\frac{e^{3x}}{\sqrt{x}}dx,x,\text{arcsinh}(ax)\right)}{5a^5} + \frac{15\text{Subst}\left(\int\frac{e^{-3x}}{\sqrt{x}}dx,x,\text{arcsinh}(ax)\right)}{4a^5} - \frac{15\text{Subst}\left(\int\frac{e^{3x}}{\sqrt{x}}dx,x,\text{arcsinh}(ax)\right)}{4a^5}$$

$$
\begin{aligned}
&= -\frac{2x^4\sqrt{1+a^2x^2}}{5a\text{arcsinh}(ax)^{5/2}} - \frac{16x^3}{15a^2\text{arcsinh}(ax)^{3/2}} \\
&\quad - \frac{4x^5}{3\text{arcsinh}(ax)^{3/2}} - \frac{32x^2\sqrt{1+a^2x^2}}{5a^3\sqrt{\text{arcsinh}(ax)}} - \frac{40x^4\sqrt{1+a^2x^2}}{3a\sqrt{\text{arcsinh}(ax)}} \\
&\quad + \frac{8\text{Subst}\left(\int e^{-x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{5a^5} - \frac{8\text{Subst}\left(\int e^{x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{5a^5} \\
&\quad - \frac{5\text{Subst}\left(\int e^{-x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{3a^5} + \frac{5\text{Subst}\left(\int e^{x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{3a^5} \\
&\quad - \frac{25\text{Subst}\left(\int e^{-5x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{6a^5} + \frac{25\text{Subst}\left(\int e^{5x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{6a^5} \\
&\quad - \frac{24\text{Subst}\left(\int e^{-3x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{5a^5} + \frac{24\text{Subst}\left(\int e^{3x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{5a^5} \\
&\quad + \frac{15\text{Subst}\left(\int e^{-3x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{2a^5} - \frac{15\text{Subst}\left(\int e^{3x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{2a^5} \\
&= -\frac{2x^4\sqrt{1+a^2x^2}}{5a\text{arcsinh}(ax)^{5/2}} - \frac{16x^3}{15a^2\text{arcsinh}(ax)^{3/2}} - \frac{4x^5}{3\text{arcsinh}(ax)^{3/2}} - \frac{32x^2\sqrt{1+a^2x^2}}{5a^3\sqrt{\text{arcsinh}(ax)}} \\
&\quad - \frac{40x^4\sqrt{1+a^2x^2}}{3a\sqrt{\text{arcsinh}(ax)}} - \frac{\sqrt{\pi}\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)}{30a^5} + \frac{9\sqrt{3\pi}\text{erf}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)}{20a^5} \\
&\quad - \frac{5\sqrt{5\pi}\text{erf}\left(\sqrt{5}\sqrt{\text{arcsinh}(ax)}\right)}{12a^5} + \frac{\sqrt{\pi}\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)}{30a^5} \\
&\quad - \frac{9\sqrt{3\pi}\text{erfi}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)}{20a^5} + \frac{5\sqrt{5\pi}\text{erfi}\left(\sqrt{5}\sqrt{\text{arcsinh}(ax)}\right)}{12a^5}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.17

$$\int \frac{x^4}{\text{arcsinh}(ax)^{7/2}}\,dx = \frac{-2e^{\text{arcsinh}(ax)}(3+2\text{arcsinh}(ax)+4\text{arcsinh}(ax)^2)+9e^{3\text{arcsinh}(ax)}(1+2\text{arcsinh}(ax)+}{}$$

```
[In] Integrate[x^4/ArcSinh[a*x]^(7/2),x]
```

```
[Out] (-2*E^ArcSinh[a*x]*(3 + 2*ArcSinh[a*x] + 4*ArcSinh[a*x]^2) + 9*E^(3*ArcSinh
[a*x])*(1 + 2*ArcSinh[a*x] + 12*ArcSinh[a*x]^2) - E^(5*ArcSinh[a*x])*(3 + 1
0*ArcSinh[a*x] + 100*ArcSinh[a*x]^2) + 100*Sqrt[5]*(-ArcSinh[a*x])^(5/2)*Ga
mma[1/2, -5*ArcSinh[a*x]] - 108*Sqrt[3]*(-ArcSinh[a*x])^(5/2)*Gamma[1/2, -3
*ArcSinh[a*x]] + 8*(-ArcSinh[a*x])^(5/2)*Gamma[1/2, -ArcSinh[a*x]] + (-6 +
4*ArcSinh[a*x] - 8*ArcSinh[a*x]^2 + 8*E^ArcSinh[a*x]*ArcSinh[a*x]^(5/2)*Gam
```

```
ma[1/2, ArcSinh[a*x]])/E^ArcSinh[a*x] + (9*(1 - 2*ArcSinh[a*x] + 12*ArcSinh
[a*x]^2 - 12*Sqrt[3]*E^(3*ArcSinh[a*x])*ArcSinh[a*x]^(5/2)*Gamma[1/2, 3*Arc
Sinh[a*x]]))/E^(3*ArcSinh[a*x]) + (-3 + 10*ArcSinh[a*x] - 100*ArcSinh[a*x]^
2 + 100*Sqrt[5]*E^(5*ArcSinh[a*x])*ArcSinh[a*x]^(5/2)*Gamma[1/2, 5*ArcSinh[
a*x]])/E^(5*ArcSinh[a*x]))/(240*a^5*ArcSinh[a*x]^(5/2))
```

Maple [F]

$$\int \frac{x^{4}}{\operatorname{arcsinh}\left(a x\right)^{\frac{7}{2}}}dx$$

```
[In] int(x^4/arcsinh(a*x)^(7/2),x)
```

```
[Out] int(x^4/arcsinh(a*x)^(7/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^{7/2}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(x^4/arcsinh(a*x)^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^{7/2}}\,dx=\int \frac{x^{4}}{\operatorname{asinh}^{\frac{7}{2}}{\left(a x \right)}}\, dx$$

```
[In] integrate(x**4/asinh(a*x)**(7/2),x)
```

```
[Out] Integral(x**4/asinh(a*x)**(7/2), x)
```

Maxima [F]

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^{7/2}}\,dx = \int \frac{x^4}{\operatorname{arsinh}\left(ax\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(x^4/arcsinh(a*x)^(7/2),x, algorithm="maxima")
[Out] integrate(x^4/arcsinh(a*x)^(7/2), x)
```

Giac [F]

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^{7/2}}\,dx = \int \frac{x^4}{\operatorname{arsinh}\left(ax\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(x^4/arcsinh(a*x)^(7/2),x, algorithm="giac")
[Out] integrate(x^4/arcsinh(a*x)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\operatorname{arcsinh}(ax)^{7/2}}\,dx = \int \frac{x^4}{\operatorname{asinh}\left(a\,x\right)^{7/2}}\,dx$$

```
[In] int(x^4/asinh(a*x)^(7/2),x)
[Out] int(x^4/asinh(a*x)^(7/2), x)
```

3.112 $\int \frac{x^3}{\text{arcsinh}(ax)^{7/2}} dx$

Optimal result

Integrand size = 12, antiderivative size = 229

$$\begin{aligned}\int \frac{x^3}{\text{arcsinh}(ax)^{7/2}}\,dx &= -\frac{2x^3\sqrt{1+a^2x^2}}{5a\text{arcsinh}(ax)^{5/2}}-\frac{4x^2}{5a^2\text{arcsinh}(ax)^{3/2}}\\ &-\frac{16x^4}{15\text{arcsinh}(ax)^{3/2}}-\frac{16x\sqrt{1+a^2x^2}}{5a^3\sqrt{\text{arcsinh}(ax)}}-\frac{128x^3\sqrt{1+a^2x^2}}{15a\sqrt{\text{arcsinh}(ax)}}\\ &+\frac{16\sqrt{\pi}\text{erf}\left(2\sqrt{\text{arcsinh}(ax)}\right)}{15a^4}-\frac{4\sqrt{2\pi}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{15a^4}\\ &+\frac{16\sqrt{\pi}\text{erfi}\left(2\sqrt{\text{arcsinh}(ax)}\right)}{15a^4}-\frac{4\sqrt{2\pi}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{15a^4}\end{aligned}$$

```
[Out] -4/5*x^2/a^2/arcsinh(a*x)^(3/2)-16/15*x^4/arcsinh(a*x)^(3/2)+16/15*erf(2*ar
csinh(a*x)^(1/2))*Pi^(1/2)/a^4+16/15*erfi(2*arcsinh(a*x)^(1/2))*Pi^(1/2)/a^
4-4/15*erf(2^(1/2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^4-4/15*erfi(2^(1/
2)*arcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^4-2/5*x^3*(a^2*x^2+1)^(1/2)/a/arc
sinh(a*x)^(5/2)-16/5*x*(a^2*x^2+1)^(1/2)/a^3/arcsinh(a*x)^(1/2)-128/15*x^3*
(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5779, 5818, 5778, 3388, 2211, 2235, 2236}

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^{7/2}}\,dx = \frac{16\sqrt{\pi}\operatorname{erf}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{15a^4} - \frac{4\sqrt{2\pi}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{15a^4} + \frac{16\sqrt{\pi}\operatorname{erfi}\left(2\sqrt{\operatorname{arcsinh}(ax)}\right)}{15a^4} - \frac{4\sqrt{2\pi}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{15a^4} - \frac{4x^2}{5a^2\operatorname{arcsinh}(ax)^{3/2}} - \frac{128x^3\sqrt{a^2x^2+1}}{15a\sqrt{\operatorname{arcsinh}(ax)}} - \frac{2x^3\sqrt{a^2x^2+1}}{5a\operatorname{arcsinh}(ax)^{5/2}} - \frac{16x\sqrt{a^2x^2+1}}{5a^3\sqrt{\operatorname{arcsinh}(ax)}} - \frac{16x^4}{15\operatorname{arcsinh}(ax)^{3/2}}$$

```
[In] Int[x^3/ArcSinh[a*x]^(7/2),x]
```

```
[Out] (-2*x^3*Sqrt[1 + a^2*x^2])/(5*a*ArcSinh[a*x]^(5/2)) - (4*x^2)/(5*a^2*ArcSin
h[a*x]^(3/2)) - (16*x^4)/(15*ArcSinh[a*x]^(3/2)) - (16*x*Sqrt[1 + a^2*x^2])
/(5*a^3*Sqrt[ArcSinh[a*x]]) - (128*x^3*Sqrt[1 + a^2*x^2])/(15*a*Sqrt[ArcSin
h[a*x]]) + (16*Sqrt[Pi]*Erf[2*Sqrt[ArcSinh[a*x]]])/(15*a^4) - (4*Sqrt[2*Pi]
*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(15*a^4) + (16*Sqrt[Pi]*Erfi[2*Sqrt[ArcSi
nh[a*x]]])/(15*a^4) - (4*Sqrt[2*Pi]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(15*a
^4)
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
```

```
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/S
qrt[1 + c^2*x^2]), x], x] - Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Sinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2x^3\sqrt{1+a^2x^2}}{5a\text{arcsinh}(ax)^{5/2}}+\frac{6\int\frac{x^2}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{5/2}}\,dx}{5a} \\
&\quad+\frac{1}{5}(8a)\int\frac{x^4}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{5/2}}\,dx \\
&= -\frac{2x^3\sqrt{1+a^2x^2}}{5a\text{arcsinh}(ax)^{5/2}}-\frac{4x^2}{5a^2\text{arcsinh}(ax)^{3/2}}-\frac{16x^4}{15\text{arcsinh}(ax)^{3/2}} \\
&\quad+\frac{64}{15}\int\frac{x^3}{\text{arcsinh}(ax)^{3/2}}\,dx+\frac{8\int\frac{x}{\text{arcsinh}(ax)^{3/2}}\,dx}{5a^2}
\end{aligned}$$

$$= -\frac{2x^3\sqrt{1+a^2x^2}}{5a\operatorname{arcsinh}(ax)^{5/2}} - \frac{4x^2}{5a^2\operatorname{arcsinh}(ax)^{3/2}} - \frac{16x^4}{15\operatorname{arcsinh}(ax)^{3/2}} - \frac{16x\sqrt{1+a^2x^2}}{5a^3\sqrt{\operatorname{arcsinh}(ax)}}$$

$$- \frac{128x^3\sqrt{1+a^2x^2}}{15a\sqrt{\operatorname{arcsinh}(ax)}} + \frac{16\text{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{5a^4}$$

$$+ \frac{128\text{Subst}\left(\int \left(-\frac{\cosh(2x)}{2\sqrt{x}} + \frac{\cosh(4x)}{2\sqrt{x}}\right)\,dx, x, \operatorname{arcsinh}(ax)\right)}{15a^4}$$

$$= -\frac{2x^3\sqrt{1+a^2x^2}}{5a\operatorname{arcsinh}(ax)^{5/2}} - \frac{4x^2}{5a^2\operatorname{arcsinh}(ax)^{3/2}} - \frac{16x^4}{15\operatorname{arcsinh}(ax)^{3/2}}$$

$$- \frac{16x\sqrt{1+a^2x^2}}{5a^3\sqrt{\operatorname{arcsinh}(ax)}} - \frac{128x^3\sqrt{1+a^2x^2}}{15a\sqrt{\operatorname{arcsinh}(ax)}}$$

$$+ \frac{8\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{5a^4} + \frac{8\text{Subst}\left(\int \frac{e^{2x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{5a^4}$$

$$- \frac{64\text{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{15a^4} + \frac{64\text{Subst}\left(\int \frac{\cosh(4x)}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{15a^4}$$

$$= -\frac{2x^3\sqrt{1+a^2x^2}}{5a\operatorname{arcsinh}(ax)^{5/2}} - \frac{4x^2}{5a^2\operatorname{arcsinh}(ax)^{3/2}} - \frac{16x^4}{15\operatorname{arcsinh}(ax)^{3/2}}$$

$$- \frac{16x\sqrt{1+a^2x^2}}{5a^3\sqrt{\operatorname{arcsinh}(ax)}} - \frac{128x^3\sqrt{1+a^2x^2}}{15a\sqrt{\operatorname{arcsinh}(ax)}}$$

$$+ \frac{32\text{Subst}\left(\int \frac{e^{-4x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{15a^4} - \frac{32\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{15a^4}$$

$$- \frac{32\text{Subst}\left(\int \frac{e^{2x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{15a^4} + \frac{32\text{Subst}\left(\int \frac{e^{4x}}{\sqrt{x}}\,dx, x, \operatorname{arcsinh}(ax)\right)}{15a^4}$$

$$+ \frac{16\text{Subst}\left(\int e^{-2x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{5a^4} + \frac{16\text{Subst}\left(\int e^{2x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{5a^4}$$

$$= -\frac{2x^3\sqrt{1+a^2x^2}}{5a\operatorname{arcsinh}(ax)^{5/2}} - \frac{4x^2}{5a^2\operatorname{arcsinh}(ax)^{3/2}} - \frac{16x^4}{15\operatorname{arcsinh}(ax)^{3/2}} - \frac{16x\sqrt{1+a^2x^2}}{5a^3\sqrt{\operatorname{arcsinh}(ax)}}$$

$$- \frac{128x^3\sqrt{1+a^2x^2}}{15a\sqrt{\operatorname{arcsinh}(ax)}} + \frac{4\sqrt{2\pi}\operatorname{erf}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{5a^4} + \frac{4\sqrt{2\pi}\operatorname{erfi}\left(\sqrt{2}\sqrt{\operatorname{arcsinh}(ax)}\right)}{5a^4}$$

$$+ \frac{64\text{Subst}\left(\int e^{-4x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{15a^4} - \frac{64\text{Subst}\left(\int e^{-2x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{15a^4}$$

$$- \frac{64\text{Subst}\left(\int e^{2x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{15a^4} + \frac{64\text{Subst}\left(\int e^{4x^2}\,dx, x, \sqrt{\operatorname{arcsinh}(ax)}\right)}{15a^4}$$

$$= -\frac{2x^3\sqrt{1+a^2x^2}}{5a\text{arcsinh}(ax)^{5/2}} - \frac{4x^2}{5a^2\text{arcsinh}(ax)^{3/2}} - \frac{16x^4}{15\text{arcsinh}(ax)^{3/2}} - \frac{16x\sqrt{1+a^2x^2}}{5a^3\sqrt{\text{arcsinh}(ax)}}$$
$$- \frac{128x^3\sqrt{1+a^2x^2}}{15a\sqrt{\text{arcsinh}(ax)}} + \frac{16\sqrt{\pi}\text{erf}\left(2\sqrt{\text{arcsinh}(ax)}\right)}{15a^4} - \frac{4\sqrt{2\pi}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{15a^4}$$
$$+ \frac{16\sqrt{\pi}\text{erfi}\left(2\sqrt{\text{arcsinh}(ax)}\right)}{15a^4} - \frac{4\sqrt{2\pi}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{15a^4}$$

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.92

$$\int \frac{x^3}{\text{arcsinh}(ax)^{7/2}}\,dx = \frac{4\text{arcsinh}(ax)\left(e^{-2\text{arcsinh}(ax)}(1-4\text{arcsinh}(ax)) + e^{2\text{arcsinh}(ax)}(1+4\text{arcsinh}(ax)) + 4\sqrt{2}\right.}{}$$

```
[In] Integrate[x^3/ArcSinh[a*x]^(7/2),x]
```

```
[Out] (4*ArcSinh[a*x]*((1 - 4*ArcSinh[a*x])/E^(2*ArcSinh[a*x]) + E^(2*ArcSinh[a*x
])*(1 + 4*ArcSinh[a*x]) + 4*Sqrt[2]*(-ArcSinh[a*x])^(3/2)*Gamma[1/2, -2*Arc
Sinh[a*x]] + 4*Sqrt[2]*ArcSinh[a*x]^(3/2)*Gamma[1/2, 2*ArcSinh[a*x]]) - 4*A
rcSinh[a*x]*((1 - 8*ArcSinh[a*x])/E^(4*ArcSinh[a*x]) + E^(4*ArcSinh[a*x])*(
1 + 8*ArcSinh[a*x]) + 16*(-ArcSinh[a*x])^(3/2)*Gamma[1/2, -4*ArcSinh[a*x]]
+ 16*ArcSinh[a*x]^(3/2)*Gamma[1/2, 4*ArcSinh[a*x]]) + 6*Sinh[2*ArcSinh[a*x]
] - 3*Sinh[4*ArcSinh[a*x]])/(60*a^4*ArcSinh[a*x]^(5/2))
```

Maple [F]

$$\int \frac{x^3}{\text{arcsinh}\,(ax)^{\frac{7}{2}}}dx$$

```
[In] int(x^3/arcsinh(a*x)^(7/2),x)
```

```
[Out] int(x^3/arcsinh(a*x)^(7/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3}{\text{arcsinh}(ax)^{7/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3/arcsinh(a*x)^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^{7/2}}\,dx = \int \frac{x^3}{\operatorname{asinh}^{\frac{7}{2}}(ax)}\,dx$$

```
[In] integrate(x**3/asinh(a*x)**(7/2),x)
```

```
[Out] Integral(x**3/asinh(a*x)**(7/2), x)
```

Maxima [F]

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^{7/2}}\,dx = \int \frac{x^3}{\operatorname{arsinh}(ax)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(x^3/arcsinh(a*x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/arcsinh(a*x)^(7/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^{7/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3/arcsinh(a*x)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\operatorname{arcsinh}(ax)^{7/2}}\,dx = \int \frac{x^3}{{\operatorname{asinh}\left(a\,x\right)}^{7/2}}\,dx$$

```
[In] int(x^3/asinh(a*x)^(7/2),x)
```

```
[Out] int(x^3/asinh(a*x)^(7/2), x)
```

3.113 $\int \frac{x^2}{\operatorname{arcsinh}(ax)^{7/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 222

$$\begin{aligned}\int \frac{x^2}{\operatorname{arcsinh}(ax)^{7/2}}\,dx &= -\frac{2x^2\sqrt{1+a^2x^2}}{5a\operatorname{arcsinh}(ax)^{5/2}}-\frac{8x}{15a^2\operatorname{arcsinh}(ax)^{3/2}}\\ &\quad-\frac{4x^3}{5\operatorname{arcsinh}(ax)^{3/2}}-\frac{16\sqrt{1+a^2x^2}}{15a^3\sqrt{\operatorname{arcsinh}(ax)}}-\frac{24x^2\sqrt{1+a^2x^2}}{5a\sqrt{\operatorname{arcsinh}(ax)}}\\ &\quad+\frac{\sqrt{\pi}\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{15a^3}-\frac{3\sqrt{3\pi}\operatorname{erf}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{5a^3}\\ &\quad-\frac{\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{15a^3}+\frac{3\sqrt{3\pi}\operatorname{erfi}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{5a^3}\end{aligned}$$

```
[Out] -8/15*x/a^2/arcsinh(a*x)^(3/2)-4/5*x^3/arcsinh(a*x)^(3/2)+1/15*erf(arcsinh(
a*x)^(1/2))*Pi^(1/2)/a^3-1/15*erfi(arcsinh(a*x)^(1/2))*Pi^(1/2)/a^3-3/5*erf
(3^(1/2)*arcsinh(a*x)^(1/2))*3^(1/2)*Pi^(1/2)/a^3+3/5*erfi(3^(1/2)*arcsinh(
a*x)^(1/2))*3^(1/2)*Pi^(1/2)/a^3-2/5*x^2*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^(
5/2)-16/15*(a^2*x^2+1)^(1/2)/a^3/arcsinh(a*x)^(1/2)-24/5*x^2*(a^2*x^2+1)^(1
/2)/a/arcsinh(a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5779, 5818, 5778, 3389, 2211, 2235, 2236, 5773, 5819}

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^{7/2}}\,dx = \frac{\sqrt{\pi}\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{15a^3} - \frac{3\sqrt{3\pi}\operatorname{erf}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{5a^3} - \frac{\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{15a^3} + \frac{3\sqrt{3\pi}\operatorname{erfi}\left(\sqrt{3}\sqrt{\operatorname{arcsinh}(ax)}\right)}{5a^3} - \frac{24x^2\sqrt{a^2x^2+1}}{5a\sqrt{\operatorname{arcsinh}(ax)}} - \frac{2x^2\sqrt{a^2x^2+1}}{5a\operatorname{arcsinh}(ax)^{5/2}} - \frac{8x}{15a^2\operatorname{arcsinh}(ax)^{3/2}} - \frac{16\sqrt{a^2x^2+1}}{15a^3\sqrt{\operatorname{arcsinh}(ax)}} - \frac{4x^3}{5\operatorname{arcsinh}(ax)^{3/2}}$$

```
[In] Int[x^2/ArcSinh[a*x]^(7/2),x]
```

```
[Out] (-2*x^2*Sqrt[1 + a^2*x^2])/(5*a*ArcSinh[a*x]^(5/2)) - (8*x)/(15*a^2*ArcSinh
[a*x]^(3/2)) - (4*x^3)/(5*ArcSinh[a*x]^(3/2)) - (16*Sqrt[1 + a^2*x^2])/(15*
a^3*Sqrt[ArcSinh[a*x]]) - (24*x^2*Sqrt[1 + a^2*x^2])/(5*a*Sqrt[ArcSinh[a*x]
]) + (Sqrt[Pi]*Erf[Sqrt[ArcSinh[a*x]]])/(15*a^3) - (3*Sqrt[3*Pi]*Erf[Sqrt[3
]*Sqrt[ArcSinh[a*x]]])/(5*a^3) - (Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x]]])/(15*a^
3) + (3*Sqrt[3*Pi]*Erfi[Sqrt[3]*Sqrt[ArcSinh[a*x]]])/(5*a^3)
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/S
qrt[1 + c^2*x^2]), x], x] - Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Sinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2x^2\sqrt{1+a^2x^2}}{5a\text{arcsinh}(ax)^{5/2}}+\frac{4\int\frac{x}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{5/2}}\,dx}{5a} \\
&\quad+\frac{1}{5}(6a)\int\frac{x^3}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{5/2}}\,dx \\
&= -\frac{2x^2\sqrt{1+a^2x^2}}{5a\text{arcsinh}(ax)^{5/2}}-\frac{8x}{15a^2\text{arcsinh}(ax)^{3/2}}-\frac{4x^3}{5\text{arcsinh}(ax)^{3/2}} \\
&\quad+\frac{12}{5}\int\frac{x^2}{\text{arcsinh}(ax)^{3/2}}\,dx+\frac{8\int\frac{1}{\text{arcsinh}(ax)^{3/2}}\,dx}{15a^2} \\
&= -\frac{2x^2\sqrt{1+a^2x^2}}{5a\text{arcsinh}(ax)^{5/2}}-\frac{8x}{15a^2\text{arcsinh}(ax)^{3/2}}-\frac{4x^3}{5\text{arcsinh}(ax)^{3/2}}-\frac{16\sqrt{1+a^2x^2}}{15a^3\sqrt{\text{arcsinh}(ax)}} \\
&\quad-\frac{24x^2\sqrt{1+a^2x^2}}{5a\sqrt{\text{arcsinh}(ax)}}+\frac{24\text{Subst}\left(\int\left(-\frac{\sinh(x)}{4\sqrt{x}}+\frac{3\sinh(3x)}{4\sqrt{x}}\right)dx,x,\text{arcsinh}(ax)\right)}{5a^3} \\
&\quad+\frac{16\int\frac{x}{\sqrt{1+a^2x^2}\sqrt{\text{arcsinh}(ax)}}\,dx}{15a} \\
&= -\frac{2x^2\sqrt{1+a^2x^2}}{5a\text{arcsinh}(ax)^{5/2}}-\frac{8x}{15a^2\text{arcsinh}(ax)^{3/2}}-\frac{4x^3}{5\text{arcsinh}(ax)^{3/2}} \\
&\quad-\frac{16\sqrt{1+a^2x^2}}{15a^3\sqrt{\text{arcsinh}(ax)}}-\frac{24x^2\sqrt{1+a^2x^2}}{5a\sqrt{\text{arcsinh}(ax)}}+\frac{16\text{Subst}\left(\int\frac{\sinh(x)}{\sqrt{x}}\,dx,x,\text{arcsinh}(ax)\right)}{15a^3} \\
&\quad-\frac{6\text{Subst}\left(\int\frac{\sinh(x)}{\sqrt{x}}\,dx,x,\text{arcsinh}(ax)\right)}{5a^3}+\frac{18\text{Subst}\left(\int\frac{\sinh(3x)}{\sqrt{x}}\,dx,x,\text{arcsinh}(ax)\right)}{5a^3} \\
&= -\frac{2x^2\sqrt{1+a^2x^2}}{5a\text{arcsinh}(ax)^{5/2}}-\frac{8x}{15a^2\text{arcsinh}(ax)^{3/2}}-\frac{4x^3}{5\text{arcsinh}(ax)^{3/2}} \\
&\quad-\frac{16\sqrt{1+a^2x^2}}{15a^3\sqrt{\text{arcsinh}(ax)}}-\frac{24x^2\sqrt{1+a^2x^2}}{5a\sqrt{\text{arcsinh}(ax)}}-\frac{8\text{Subst}\left(\int\frac{e^{-x}}{\sqrt{x}}\,dx,x,\text{arcsinh}(ax)\right)}{15a^3} \\
&\quad+\frac{8\text{Subst}\left(\int\frac{e^{x}}{\sqrt{x}}\,dx,x,\text{arcsinh}(ax)\right)}{15a^3}+\frac{3\text{Subst}\left(\int\frac{e^{-x}}{\sqrt{x}}\,dx,x,\text{arcsinh}(ax)\right)}{5a^3} \\
&\quad-\frac{3\text{Subst}\left(\int\frac{e^{x}}{\sqrt{x}}\,dx,x,\text{arcsinh}(ax)\right)}{5a^3}-\frac{9\text{Subst}\left(\int\frac{e^{-3x}}{\sqrt{x}}\,dx,x,\text{arcsinh}(ax)\right)}{5a^3} \\
&\quad+\frac{9\text{Subst}\left(\int\frac{e^{3x}}{\sqrt{x}}\,dx,x,\text{arcsinh}(ax)\right)}{5a^3}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{2x^2\sqrt{1+a^2x^2}}{5a\text{arcsinh}(ax)^{5/2}} - \frac{8x}{15a^2\text{arcsinh}(ax)^{3/2}} - \frac{4x^3}{5\text{arcsinh}(ax)^{3/2}} \\
&\quad - \frac{16\sqrt{1+a^2x^2}}{15a^3\sqrt{\text{arcsinh}(ax)}} - \frac{24x^2\sqrt{1+a^2x^2}}{5a\sqrt{\text{arcsinh}(ax)}} \\
&\quad - \frac{16\text{Subst}\left(\int e^{-x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{15a^3} + \frac{16\text{Subst}\left(\int e^{x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{15a^3} \\
&\quad + \frac{6\text{Subst}\left(\int e^{-x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{5a^3} - \frac{6\text{Subst}\left(\int e^{x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{5a^3} \\
&\quad - \frac{18\text{Subst}\left(\int e^{-3x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{5a^3} + \frac{18\text{Subst}\left(\int e^{3x^2}\,dx, x, \sqrt{\text{arcsinh}(ax)}\right)}{5a^3} \\
&= -\frac{2x^2\sqrt{1+a^2x^2}}{5a\text{arcsinh}(ax)^{5/2}} - \frac{8x}{15a^2\text{arcsinh}(ax)^{3/2}} - \frac{4x^3}{5\text{arcsinh}(ax)^{3/2}} - \frac{16\sqrt{1+a^2x^2}}{15a^3\sqrt{\text{arcsinh}(ax)}} \\
&\quad - \frac{24x^2\sqrt{1+a^2x^2}}{5a\sqrt{\text{arcsinh}(ax)}} + \frac{\sqrt{\pi}\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)}{15a^3} - \frac{3\sqrt{3\pi}\text{erf}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)}{5a^3} \\
&\quad - \frac{\sqrt{\pi}\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)}{15a^3} + \frac{3\sqrt{3\pi}\text{erfi}\left(\sqrt{3}\sqrt{\text{arcsinh}(ax)}\right)}{5a^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{\text{arcsinh}(ax)^{7/2}}\,dx = \frac{e^{\text{arcsinh}(ax)}(3+2\text{arcsinh}(ax)+4\text{arcsinh}(ax)^2)-3e^{3\text{arcsinh}(ax)}(1+2\text{arcsinh}(ax)+12\text{ar}}{}$$

```
[In] Integrate[x^2/ArcSinh[a*x]^(7/2),x]
```

```
[Out] (E^ArcSinh[a*x]*(3 + 2*ArcSinh[a*x] + 4*ArcSinh[a*x]^2) - 3*E^(3*ArcSinh[a*
x])*(1 + 2*ArcSinh[a*x] + 12*ArcSinh[a*x]^2) + 36*Sqrt[3]*(-ArcSinh[a*x])^(
5/2)*Gamma[1/2, -3*ArcSinh[a*x]] - 4*(-ArcSinh[a*x])^(5/2)*Gamma[1/2, -ArcS
inh[a*x]] + (3 - 2*ArcSinh[a*x] + 4*ArcSinh[a*x]^2 - 4*E^ArcSinh[a*x]*ArcSi
nh[a*x]^(5/2)*Gamma[1/2, ArcSinh[a*x]])/E^ArcSinh[a*x] + (-3 + 6*ArcSinh[a*
x] - 36*ArcSinh[a*x]^2 + 36*Sqrt[3]*E^(3*ArcSinh[a*x])*ArcSinh[a*x]^(5/2)*G
amma[1/2, 3*ArcSinh[a*x]])/E^(3*ArcSinh[a*x]))/(60*a^3*ArcSinh[a*x]^(5/2))
```

Maple [F]

$$\int \frac{x^2}{\operatorname{arcsinh}\left(ax\right)^{\frac{7}{2}}}dx$$

```
[In] int(x^2/arcsinh(a*x)^(7/2),x)
```

```
[Out] int(x^2/arcsinh(a*x)^(7/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^{7/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2/arcsinh(a*x)^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^{7/2}}\,dx = \int \frac{x^2}{\operatorname{asinh}^{\frac{7}{2}}\left(ax\right)}\,dx$$

```
[In] integrate(x**2/asinh(a*x)**(7/2),x)
```

```
[Out] Integral(x**2/asinh(a*x)**(7/2), x)
```

Maxima [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^{7/2}}\,dx = \int \frac{x^2}{\operatorname{arsinh}\left(ax\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(x^2/arcsinh(a*x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/arcsinh(a*x)^(7/2), x)
```

Giac [F]

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^{7/2}}\,dx = \int \frac{x^2}{\operatorname{arsinh}\left(ax\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(x^2/arcsinh(a*x)^(7/2),x, algorithm="giac")
[Out] integrate(x^2/arcsinh(a*x)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\operatorname{arcsinh}(ax)^{7/2}}\,dx = \int \frac{x^2}{{\mathrm{asinh}\left(a\,x\right)}^{7/2}}\,dx$$

```
[In] int(x^2/asinh(a*x)^(7/2),x)
[Out] int(x^2/asinh(a*x)^(7/2), x)
```

3.114 $\int \frac{x}{\text{arcsinh}(ax)^{7/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 147

$$\int \frac{x}{\text{arcsinh}(ax)^{7/2}}\,dx = -\frac{2x\sqrt{1+a^2x^2}}{5a\text{arcsinh}(ax)^{5/2}} - \frac{4}{15a^2\text{arcsinh}(ax)^{3/2}} - \frac{8x^2}{15\text{arcsinh}(ax)^{3/2}} - \frac{32x\sqrt{1+a^2x^2}}{15a\sqrt{\text{arcsinh}(ax)}} + \frac{8\sqrt{2\pi}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{15a^2} + \frac{8\sqrt{2\pi}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{15a^2}$$

```
[Out] -4/15/a^2/arcsinh(a*x)^(3/2)-8/15*x^2/arcsinh(a*x)^(3/2)+8/15*erf(2^(1/2)*a
rcsinh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^2+8/15*erfi(2^(1/2)*arcsinh(a*x)^(1/2
))*2^(1/2)*Pi^(1/2)/a^2-2/5*x*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^(5/2)-32/15*
x*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {5779, 5818, 5778, 3388, 2211, 2235, 2236, 5783}

$$\int \frac{x}{\text{arcsinh}(ax)^{7/2}}\,dx = \frac{8\sqrt{2\pi}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{15a^2} + \frac{8\sqrt{2\pi}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{15a^2} - \frac{32x\sqrt{a^2x^2+1}}{15a\sqrt{\text{arcsinh}(ax)}} - \frac{2x\sqrt{a^2x^2+1}}{5a\text{arcsinh}(ax)^{5/2}} - \frac{4}{15a^2\text{arcsinh}(ax)^{3/2}} - \frac{8x^2}{15\text{arcsinh}(ax)^{3/2}}$$

```
[In] Int[x/ArcSinh[a*x]^(7/2),x]
```

```
[Out] (-2*x*Sqrt[1 + a^2*x^2])/(5*a*ArcSinh[a*x]^(5/2)) - 4/(15*a^2*ArcSinh[a*x]^
(3/2)) - (8*x^2)/(15*ArcSinh[a*x]^(3/2)) - (32*x*Sqrt[1 + a^2*x^2])/(15*a*S
```

```
qrt[ArcSinh[a*x]]) + (8*Sqrt[2*Pi]*Erf[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(15*a^2
) + (8*Sqrt[2*Pi]*Erfi[Sqrt[2]*Sqrt[ArcSinh[a*x]]])/(15*a^2)
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/S
qrt[1 + c^2*x^2]), x], x] - Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Sinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2x\sqrt{1+a^2x^2}}{5a\text{arcsinh}(ax)^{5/2}}+\frac{2\int \frac{1}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{5/2}}\,dx}{5a} \\
&\quad +\frac{1}{5}(4a)\int \frac{x^2}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{5/2}}\,dx \\
&= -\frac{2x\sqrt{1+a^2x^2}}{5a\text{arcsinh}(ax)^{5/2}}-\frac{4}{15a^2\text{arcsinh}(ax)^{3/2}}-\frac{8x^2}{15\text{arcsinh}(ax)^{3/2}}+\frac{16}{15}\int \frac{x}{\text{arcsinh}(ax)^{3/2}}\,dx \\
&= -\frac{2x\sqrt{1+a^2x^2}}{5a\text{arcsinh}(ax)^{5/2}}-\frac{4}{15a^2\text{arcsinh}(ax)^{3/2}}-\frac{8x^2}{15\text{arcsinh}(ax)^{3/2}} \\
&\quad -\frac{32x\sqrt{1+a^2x^2}}{15a\sqrt{\text{arcsinh}(ax)}}+\frac{32\text{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{x}}\,dx,x,\text{arcsinh}(ax)\right)}{15a^2} \\
&= -\frac{2x\sqrt{1+a^2x^2}}{5a\text{arcsinh}(ax)^{5/2}}-\frac{4}{15a^2\text{arcsinh}(ax)^{3/2}}-\frac{8x^2}{15\text{arcsinh}(ax)^{3/2}}-\frac{32x\sqrt{1+a^2x^2}}{15a\sqrt{\text{arcsinh}(ax)}} \\
&\quad +\frac{16\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx,x,\text{arcsinh}(ax)\right)}{15a^2}+\frac{16\text{Subst}\left(\int \frac{e^{2x}}{\sqrt{x}}\,dx,x,\text{arcsinh}(ax)\right)}{15a^2} \\
&= -\frac{2x\sqrt{1+a^2x^2}}{5a\text{arcsinh}(ax)^{5/2}}-\frac{4}{15a^2\text{arcsinh}(ax)^{3/2}}-\frac{8x^2}{15\text{arcsinh}(ax)^{3/2}}-\frac{32x\sqrt{1+a^2x^2}}{15a\sqrt{\text{arcsinh}(ax)}} \\
&\quad +\frac{32\text{Subst}\left(\int e^{-2x^2}\,dx,x,\sqrt{\text{arcsinh}(ax)}\right)}{15a^2}+\frac{32\text{Subst}\left(\int e^{2x^2}\,dx,x,\sqrt{\text{arcsinh}(ax)}\right)}{15a^2} \\
&= -\frac{2x\sqrt{1+a^2x^2}}{5a\text{arcsinh}(ax)^{5/2}}-\frac{4}{15a^2\text{arcsinh}(ax)^{3/2}}-\frac{8x^2}{15\text{arcsinh}(ax)^{3/2}}-\frac{32x\sqrt{1+a^2x^2}}{15a\sqrt{\text{arcsinh}(ax)}} \\
&\quad +\frac{8\sqrt{2\pi}\text{erf}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{15a^2}+\frac{8\sqrt{2\pi}\text{erfi}\left(\sqrt{2}\sqrt{\text{arcsinh}(ax)}\right)}{15a^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.80

$$\int \frac{x}{\operatorname{arcsinh}(ax)^{7/2}}\,dx = -\frac{2\operatorname{arcsinh}(ax)\left(e^{-2\operatorname{arcsinh}(ax)}(1-4\operatorname{arcsinh}(ax))+e^{2\operatorname{arcsinh}(ax)}(1+4\operatorname{arcsinh}(ax))+4\sqrt{2}(-\operatorname{arcsinh}(ax))^{3/2}\Gamma\left(\frac{1}{2}\right.\right.}{15a^2\operatorname{arcsinh}(ax)^{5/2}}$$

```
[In] Integrate[x/ArcSinh[a*x]^(7/2),x]
```

```
[Out] -1/15*(2*ArcSinh[a*x]*((1 - 4*ArcSinh[a*x])/E^(2*ArcSinh[a*x]) + E^(2*ArcSi
nh[a*x])*(1 + 4*ArcSinh[a*x]) + 4*Sqrt[2]*(-ArcSinh[a*x])^(3/2)*Gamma[1/2,
-2*ArcSinh[a*x]] + 4*Sqrt[2]*ArcSinh[a*x]^(3/2)*Gamma[1/2, 2*ArcSinh[a*x]])
 + 3*Sinh[2*ArcSinh[a*x]])/(a^2*ArcSinh[a*x]^(5/2))
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00

method	result
default	$-\frac{\sqrt{2}\left(16\operatorname{arcsinh}(ax)^{\frac{5}{2}}\sqrt{2}\,\sqrt{\pi}\,\sqrt{a^2x^2+1}\,ax+4\operatorname{arcsinh}(ax)^{\frac{3}{2}}\sqrt{2}\,\sqrt{\pi}\,a^2x^2+3\sqrt{2}\,\sqrt{\operatorname{arcsinh}(ax)}\,\sqrt{\pi}\,\sqrt{a^2x^2+1}\,ax-8\operatorname{arcsinh}(ax)^3\pi\ \operatorname{erf}\left(\right.\right.}{15\sqrt{\pi}\,a^2\operatorname{arcsinh}(ax)^3}$

```
[In] int(x/arcsinh(a*x)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*2^(1/2)*(16*arcsinh(a*x)^(5/2)*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*a*x
+4*arcsinh(a*x)^(3/2)*2^(1/2)*Pi^(1/2)*a^2*x^2+3*2^(1/2)*arcsinh(a*x)^(1/2)
*Pi^(1/2)*(a^2*x^2+1)^(1/2)*a*x-8*arcsinh(a*x)^3*Pi*erf(2^(1/2)*arcsinh(a*x
)^(1/2))-8*arcsinh(a*x)^3*Pi*erfi(2^(1/2)*arcsinh(a*x)^(1/2))+2*arcsinh(a*x
)^(3/2)*2^(1/2)*Pi^(1/2))/Pi^(1/2)/a^2/arcsinh(a*x)^3
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{\operatorname{arcsinh}(ax)^{7/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x/arcsinh(a*x)^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x}{\operatorname{arcsinh}(ax)^{7/2}}\,dx = \int \frac{x}{\operatorname{asinh}^{\frac{7}{2}}(ax)}\,dx$$

```
[In] integrate(x/asinh(a*x)**(7/2),x)
[Out] Integral(x/asinh(a*x)**(7/2), x)
```

Maxima [F]

$$\int \frac{x}{\operatorname{arcsinh}(ax)^{7/2}}\,dx = \int \frac{x}{\operatorname{arsinh}(ax)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(x/arcsinh(a*x)^(7/2),x, algorithm="maxima")
[Out] integrate(x/arcsinh(a*x)^(7/2), x)
```

Giac [F]

$$\int \frac{x}{\operatorname{arcsinh}(ax)^{7/2}}\,dx = \int \frac{x}{\operatorname{arsinh}(ax)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(x/arcsinh(a*x)^(7/2),x, algorithm="giac")
[Out] integrate(x/arcsinh(a*x)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\operatorname{arcsinh}(ax)^{7/2}}\,dx = \int \frac{x}{\operatorname{asinh}(a\,x)^{7/2}}\,dx$$

```
[In] int(x/asinh(a*x)^(7/2),x)
[Out] int(x/asinh(a*x)^(7/2), x)
```

3.115 $\int \frac{1}{\operatorname{arcsinh}(ax)^{7/2}}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 112

$$\int \frac{1}{\operatorname{arcsinh}(ax)^{7/2}}\,dx = -\frac{2\sqrt{1+a^2x^2}}{5a\operatorname{arcsinh}(ax)^{5/2}} - \frac{4x}{15\operatorname{arcsinh}(ax)^{3/2}} - \frac{8\sqrt{1+a^2x^2}}{15a\sqrt{\operatorname{arcsinh}(ax)}} - \frac{4\sqrt{\pi}\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{15a} + \frac{4\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{15a}$$

```
[Out] -4/15*x/arcsinh(a*x)^(3/2)-4/15*erf(arcsinh(a*x)^(1/2))*Pi^(1/2)/a+4/15*erf
i(arcsinh(a*x)^(1/2))*Pi^(1/2)/a-2/5*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^(5/2)
-8/15*(a^2*x^2+1)^(1/2)/a/arcsinh(a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {5773, 5818, 5819, 3389, 2211, 2235, 2236}

$$\int \frac{1}{\operatorname{arcsinh}(ax)^{7/2}}\,dx = -\frac{8\sqrt{a^2x^2+1}}{15a\sqrt{\operatorname{arcsinh}(ax)}} - \frac{2\sqrt{a^2x^2+1}}{5a\operatorname{arcsinh}(ax)^{5/2}} - \frac{4\sqrt{\pi}\operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{15a} + \frac{4\sqrt{\pi}\operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)}{15a} - \frac{4x}{15\operatorname{arcsinh}(ax)^{3/2}}$$

```
[In] Int[ArcSinh[a*x]^(-7/2),x]
```

```
[Out] (-2*Sqrt[1 + a^2*x^2])/(5*a*ArcSinh[a*x]^(5/2)) - (4*x)/(15*ArcSinh[a*x]^(3
/2)) - (8*Sqrt[1 + a^2*x^2])/(15*a*Sqrt[ArcSinh[a*x]]) - (4*Sqrt[Pi]*Erf[Sq
rt[ArcSinh[a*x]]])/(15*a) + (4*Sqrt[Pi]*Erfi[Sqrt[ArcSinh[a*x]]])/(15*a)
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
```

```
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2\sqrt{1+a^2x^2}}{5a\text{arcsinh}(ax)^{5/2}}+\frac{1}{5}(2a)\int \frac{x}{\sqrt{1+a^2x^2}\text{arcsinh}(ax)^{5/2}}\,dx\\
&= -\frac{2\sqrt{1+a^2x^2}}{5a\text{arcsinh}(ax)^{5/2}}-\frac{4x}{15\text{arcsinh}(ax)^{3/2}}+\frac{4}{15}\int \frac{1}{\text{arcsinh}(ax)^{3/2}}\,dx\\
&= -\frac{2\sqrt{1+a^2x^2}}{5a\text{arcsinh}(ax)^{5/2}}-\frac{4x}{15\text{arcsinh}(ax)^{3/2}}-\frac{8\sqrt{1+a^2x^2}}{15a\sqrt{\text{arcsinh}(ax)}}\\
&\quad+\frac{1}{15}(8a)\int \frac{x}{\sqrt{1+a^2x^2}\sqrt{\text{arcsinh}(ax)}}\,dx\\
&= -\frac{2\sqrt{1+a^2x^2}}{5a\text{arcsinh}(ax)^{5/2}}-\frac{4x}{15\text{arcsinh}(ax)^{3/2}}-\frac{8\sqrt{1+a^2x^2}}{15a\sqrt{\text{arcsinh}(ax)}}\\
&\quad+\frac{8\text{Subst}\left(\int \frac{\sinh(x)}{\sqrt{x}}\,dx,x,\text{arcsinh}(ax)\right)}{15a}\\
&= -\frac{2\sqrt{1+a^2x^2}}{5a\text{arcsinh}(ax)^{5/2}}-\frac{4x}{15\text{arcsinh}(ax)^{3/2}}-\frac{8\sqrt{1+a^2x^2}}{15a\sqrt{\text{arcsinh}(ax)}}\\
&\quad-\frac{4\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}}\,dx,x,\text{arcsinh}(ax)\right)}{15a}+\frac{4\text{Subst}\left(\int \frac{e^{x}}{\sqrt{x}}\,dx,x,\text{arcsinh}(ax)\right)}{15a}\\
&= -\frac{2\sqrt{1+a^2x^2}}{5a\text{arcsinh}(ax)^{5/2}}-\frac{4x}{15\text{arcsinh}(ax)^{3/2}}-\frac{8\sqrt{1+a^2x^2}}{15a\sqrt{\text{arcsinh}(ax)}}\\
&\quad-\frac{8\text{Subst}\left(\int e^{-x^2}\,dx,x,\sqrt{\text{arcsinh}(ax)}\right)}{15a}+\frac{8\text{Subst}\left(\int e^{x^2}\,dx,x,\sqrt{\text{arcsinh}(ax)}\right)}{15a}\\
&= -\frac{2\sqrt{1+a^2x^2}}{5a\text{arcsinh}(ax)^{5/2}}-\frac{4x}{15\text{arcsinh}(ax)^{3/2}}-\frac{8\sqrt{1+a^2x^2}}{15a\sqrt{\text{arcsinh}(ax)}}\\
&\quad-\frac{4\sqrt{\pi}\text{erf}\left(\sqrt{\text{arcsinh}(ax)}\right)}{15a}+\frac{4\sqrt{\pi}\text{erfi}\left(\sqrt{\text{arcsinh}(ax)}\right)}{15a}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.99

$$\int \frac{1}{\operatorname{arcsinh}(ax)^{7/2}}\,dx = \frac{-2e^{\operatorname{arcsinh}(ax)}(3+2\operatorname{arcsinh}(ax)+4\operatorname{arcsinh}(ax)^2)+8(-\operatorname{arcsinh}(ax))^{5/2}\Gamma\left(\frac{1}{2},-\operatorname{arcsin}\right.}{}$$

```
[In] Integrate[ArcSinh[a*x]^(-7/2),x]
```

```
[Out] (-2*E^ArcSinh[a*x]*(3 + 2*ArcSinh[a*x] + 4*ArcSinh[a*x]^2) + 8*(-ArcSinh[a*
x])^(5/2)*Gamma[1/2, -ArcSinh[a*x]] + (-6 + 4*ArcSinh[a*x] - 8*ArcSinh[a*x]
^2 + 8*E^ArcSinh[a*x]*ArcSinh[a*x]^(5/2)*Gamma[1/2, ArcSinh[a*x]])/E^ArcSin
h[a*x])/(30*a*ArcSinh[a*x]^(5/2))
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.94

method	result
default	$-\frac{2\left(2\operatorname{arcsinh}(ax)^{3}\pi\ \operatorname{erf}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)-2\operatorname{arcsinh}(ax)^{3}\pi\ \operatorname{erfi}\left(\sqrt{\operatorname{arcsinh}(ax)}\right)+4\sqrt{a^{2}x^{2}+1}\ \operatorname{arcsinh}(ax)^{\frac{5}{2}}\sqrt{\pi}+2\operatorname{arcsinh}(ax)^{\frac{3}{2}}\sqrt{\pi}\right.}{15\sqrt{\pi}\,a\operatorname{arcsinh}(ax)^{3}}$

```
[In] int(1/arcsinh(a*x)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/15*(2*arcsinh(a*x)^3*Pi*erf(arcsinh(a*x)^(1/2))-2*arcsinh(a*x)^3*Pi*erfi
(arcsinh(a*x)^(1/2))+4*(a^2*x^2+1)^(1/2)*arcsinh(a*x)^(5/2)*Pi^(1/2)+2*arcs
inh(a*x)^(3/2)*Pi^(1/2)*a*x+3*arcsinh(a*x)^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)
)/Pi^(1/2)/a/arcsinh(a*x)^3
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\operatorname{arcsinh}(ax)^{7/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/arcsinh(a*x)^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\operatorname{arcsinh}(ax)^{7/2}}\,dx = \int \frac{1}{\operatorname{asinh}^{\frac{7}{2}}(ax)}\,dx$$

```
[In] integrate(1/asinh(a*x)**(7/2),x)
[Out] Integral(asinh(a*x)**(-7/2), x)
```

Maxima [F]

$$\int \frac{1}{\operatorname{arcsinh}(ax)^{7/2}}\,dx = \int \frac{1}{\operatorname{arsinh}(ax)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/arcsinh(a*x)^(7/2),x, algorithm="maxima")
[Out] integrate(arcsinh(a*x)^(-7/2), x)
```

Giac [F]

$$\int \frac{1}{\operatorname{arcsinh}(ax)^{7/2}}\,dx = \int \frac{1}{\operatorname{arsinh}(ax)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/arcsinh(a*x)^(7/2),x, algorithm="giac")
[Out] integrate(arcsinh(a*x)^(-7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\operatorname{arcsinh}(ax)^{7/2}}\,dx = \int \frac{1}{\operatorname{asinh}(a\,x)^{7/2}}\,dx$$

```
[In] int(1/asinh(a*x)^(7/2),x)
[Out] int(1/asinh(a*x)^(7/2), x)
```

3.116 $\int \frac{1}{x\textbf{arcsinh}(ax)^{7/2}} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{1}{x\text{arcsinh}(ax)^{7/2}} \, dx = \text{Int}\left(\frac{1}{x\text{arcsinh}(ax)^{7/2}}, x\right)$$

```
[Out] Unintegrable(1/x/arcsinh(a*x)^(7/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\text{arcsinh}(ax)^{7/2}} \, dx = \int \frac{1}{x\text{arcsinh}(ax)^{7/2}} \, dx$$

```
[In] Int[1/(x*ArcSinh[a*x]^(7/2)),x]
```

```
[Out] Defer[Int][1/(x*ArcSinh[a*x]^(7/2)), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x\text{arcsinh}(ax)^{7/2}} \, dx$$

Mathematica [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x\text{arcsinh}(ax)^{7/2}}\,dx = \int \frac{1}{x\text{arcsinh}(ax)^{7/2}}\,dx$$

```
[In] Integrate[1/(x*ArcSinh[a*x]^(7/2)),x]
[Out] Integrate[1/(x*ArcSinh[a*x]^(7/2)), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{x\,\text{arcsinh}\left(ax\right)^{\frac{7}{2}}}dx$$

```
[In] int(1/x/arcsinh(a*x)^(7/2),x)
[Out] int(1/x/arcsinh(a*x)^(7/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x\text{arcsinh}(ax)^{7/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/x/arcsinh(a*x)^(7/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 67.44 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\text{arcsinh}(ax)^{7/2}}\,dx = \int \frac{1}{x\,\text{asinh}^{\frac{7}{2}}\left(ax\right)}\,dx$$

```
[In] integrate(1/x/asinh(a*x)**(7/2),x)
[Out] Integral(1/(x*asinh(a*x)**(7/2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\text{arcsinh}(ax)^{7/2}}\,dx=\int \frac{1}{x\,\text{arsinh}\left(ax\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/x/arcsinh(a*x)^(7/2),x, algorithm="maxima")
[Out] integrate(1/(x*arcsinh(a*x)^(7/2)), x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\text{arcsinh}(ax)^{7/2}}\,dx=\int \frac{1}{x\,\text{arsinh}\left(ax\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/x/arcsinh(a*x)^(7/2),x, algorithm="giac")
[Out] integrate(1/(x*arcsinh(a*x)^(7/2)), x)
```

Mupad [N/A]

Not integrable

Time = 2.32 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\text{arcsinh}(ax)^{7/2}}\,dx=\int \frac{1}{x\,\text{asinh}\left(a\,x\right)^{7/2}}\,dx$$

```
[In] int(1/(x*asinh(a*x)^(7/2)),x)
[Out] int(1/(x*asinh(a*x)^(7/2)), x)
```

3.117 $\int x^m \mathbf{arcsinh}(ax)^4 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int x^m \operatorname{arcsinh}(ax)^4 \, dx = \frac{x^{1+m} \operatorname{arcsinh}(ax)^4}{1+m} - \frac{4a\mathrm{Int}\left(\frac{x^{1+m} \operatorname{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}}, x\right)}{1+m}$$

```
[Out] x^(1+m)*arcsinh(a*x)^4/(1+m)-4*a*Unintegrable(x^(1+m)*arcsinh(a*x)^3/(a^2*x
^2+1)^(1/2),x)/(1+m)
```

Rubi [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^m \operatorname{arcsinh}(ax)^4 \, dx = \int x^m \operatorname{arcsinh}(ax)^4 \, dx$$

```
[In] Int[x^m*ArcSinh[a*x]^4,x]
```

```
[Out] (x^(1 + m)*ArcSinh[a*x]^4)/(1 + m) - (4*a*Defer[Int][(x^(1 + m)*ArcSinh[a*x
]^3)/Sqrt[1 + a^2*x^2], x])/(1 + m)
```

Rubi steps

$$\text{integral} = \frac{x^{1+m} \operatorname{arcsinh}(ax)^4}{1+m} - \frac{(4a) \int \frac{x^{1+m} \operatorname{arcsinh}(ax)^3}{\sqrt{1+a^2x^2}} \, dx}{1+m}$$

Mathematica [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \text{arcsinh}(ax)^4 \, dx = \int x^m \text{arcsinh}(ax)^4 \, dx$$

```
[In] Integrate[x^m*ArcSinh[a*x]^4,x]
[Out] Integrate[x^m*ArcSinh[a*x]^4, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.48 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int x^m \operatorname{arcsinh}\left(a x\right)^4 dx$$

```
[In] int(x^m*arcsinh(a*x)^4,x)
[Out] int(x^m*arcsinh(a*x)^4,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \text{arcsinh}(ax)^4 \, dx = \int x^m \operatorname{arsinh}\left(a x\right)^4 \, dx$$

```
[In] integrate(x^m*arcsinh(a*x)^4,x, algorithm="fricas")
[Out] integral(x^m*arcsinh(a*x)^4, x)
```

Sympy [N/A]

Not integrable

Time = 5.76 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int x^m \text{arcsinh}(ax)^4 \, dx = \int x^m \operatorname{asinh}^4\left(a x\right) dx$$

```
[In] integrate(x**m*asinh(a*x)**4,x)
[Out] Integral(x**m*asinh(a*x)**4, x)
```

Maxima [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 134, normalized size of antiderivative = 13.40

$$\int x^m \text{arcsinh}(ax)^4 \, dx = \int x^m \operatorname{arsinh}(ax)^4 \, dx$$

```
[In] integrate(x^m*arcsinh(a*x)^4,x, algorithm="maxima")
```

```
[Out] x*x^m*log(a*x + sqrt(a^2*x^2 + 1))^4/(m + 1) - integrate(4*(sqrt(a^2*x^2 +
1)*a^2*x^2*x^m + (a^3*x^3 + a*x)*x^m)*log(a*x + sqrt(a^2*x^2 + 1))^3/(a^3*(
m + 1)*x^3 + a*(m + 1)*x + (a^2*(m + 1)*x^2 + m + 1)*sqrt(a^2*x^2 + 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \text{arcsinh}(ax)^4 \, dx = \int x^m \operatorname{arsinh}(ax)^4 \, dx$$

```
[In] integrate(x^m*arcsinh(a*x)^4,x, algorithm="giac")
```

```
[Out] integrate(x^m*arcsinh(a*x)^4, x)
```

Mupad [N/A]

Not integrable

Time = 2.56 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \text{arcsinh}(ax)^4 \, dx = \int x^m \,\mathrm{asinh}(a\,x)^4 \, dx$$

```
[In] int(x^m*asinh(a*x)^4,x)
```

```
[Out] int(x^m*asinh(a*x)^4, x)
```

3.118 $\int x^m \textbf{arcsinh}(ax)^3\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int x^m \operatorname{arcsinh}(ax)^3\,dx = \frac{x^{1+m}\operatorname{arcsinh}(ax)^3}{1+m} - \frac{3a\text{Int}\left(\frac{x^{1+m}\operatorname{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}},x\right)}{1+m}$$

```
[Out] x^(1+m)*arcsinh(a*x)^3/(1+m)-3*a*Unintegrable(x^(1+m)*arcsinh(a*x)^2/(a^2*x
^2+1)^(1/2),x)/(1+m)
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m \operatorname{arcsinh}(ax)^3\,dx = \int x^m \operatorname{arcsinh}(ax)^3\,dx$$

```
[In] Int[x^m*ArcSinh[a*x]^3,x]
```

```
[Out] (x^(1 + m)*ArcSinh[a*x]^3)/(1 + m) - (3*a*Defer[Int][(x^(1 + m)*ArcSinh[a*x
]^2)/Sqrt[1 + a^2*x^2], x])/(1 + m)
```

Rubi steps

$$\text{integral} = \frac{x^{1+m}\operatorname{arcsinh}(ax)^3}{1+m} - \frac{(3a)\int \frac{x^{1+m}\operatorname{arcsinh}(ax)^2}{\sqrt{1+a^2x^2}}\,dx}{1+m}$$

Mathematica [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \text{arcsinh}(ax)^3 \, dx = \int x^m \text{arcsinh}(ax)^3 \, dx$$

```
[In] Integrate[x^m*ArcSinh[a*x]^3,x]
[Out] Integrate[x^m*ArcSinh[a*x]^3, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.34 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int x^{m} \operatorname{arcsinh}\left(a x \right)^{3} d x$$

```
[In] int(x^m*arcsinh(a*x)^3,x)
[Out] int(x^m*arcsinh(a*x)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \text{arcsinh}(ax)^3 \, dx = \int { x^{m} \operatorname{arsinh}\left(a x\right)^{3} \,d x }$$

```
[In] integrate(x^m*arcsinh(a*x)^3,x, algorithm="fricas")
[Out] integral(x^m*arcsinh(a*x)^3, x)
```

Sympy [N/A]

Not integrable

Time = 3.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int x^m \text{arcsinh}(ax)^3 \, dx = \int x^{m} \operatorname{asinh}^{3}{\left(a x \right)}\, dx$$

```
[In] integrate(x**m*asinh(a*x)**3,x)
[Out] Integral(x**m*asinh(a*x)**3, x)
```

Maxima [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 134, normalized size of antiderivative = 13.40

$$\int x^m \operatorname{arcsinh}(ax)^3\,dx = \int x^m \operatorname{arsinh}(ax)^3\,dx$$

```
[In] integrate(x^m*arcsinh(a*x)^3,x, algorithm="maxima")
```

```
[Out] x*x^m*log(a*x + sqrt(a^2*x^2 + 1))^3/(m + 1) - integrate(3*(sqrt(a^2*x^2 +
1)*a^2*x^2*x^m + (a^3*x^3 + a*x)*x^m)*log(a*x + sqrt(a^2*x^2 + 1))^2/(a^3*(
m + 1)*x^3 + a*(m + 1)*x + (a^2*(m + 1)*x^2 + m + 1)*sqrt(a^2*x^2 + 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \operatorname{arcsinh}(ax)^3\,dx = \int x^m \operatorname{arsinh}(ax)^3\,dx$$

```
[In] integrate(x^m*arcsinh(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^m*arcsinh(a*x)^3, x)
```

Mupad [N/A]

Not integrable

Time = 2.88 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \operatorname{arcsinh}(ax)^3\,dx = \int x^m \operatorname{asinh}(a\,x)^3\,dx$$

```
[In] int(x^m*asinh(a*x)^3,x)
```

```
[Out] int(x^m*asinh(a*x)^3, x)
```

3.119 $\int x^m \text{arcsinh}(ax)^2 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 137

$$\int x^m \text{arcsinh}(ax)^2\,dx = \frac{x^{1+m}\text{arcsinh}(ax)^2}{1+m} - \frac{2ax^{2+m}\text{arcsinh}(ax)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},-a^2x^2\right)}{2+3m+m^2} + \frac{2a^2x^{3+m}\,{}_3F_2\left(1,\frac{3}{2}+\frac{m}{2},\frac{3}{2}+\frac{m}{2};2+\frac{m}{2},\frac{5}{2}+\frac{m}{2};-a^2x^2\right)}{6+11m+6m^2+m^3}$$

```
[Out] x^(1+m)*arcsinh(a*x)^2/(1+m)-2*a*x^(2+m)*arcsinh(a*x)*hypergeom([1/2, 1+1/2
*m],[2+1/2*m],-a^2*x^2)/(m^2+3*m+2)+2*a^2*x^(3+m)*hypergeom([1, 3/2+1/2*m,
3/2+1/2*m],[2+1/2*m, 5/2+1/2*m],-a^2*x^2)/(m^3+6*m^2+11*m+6)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5776, 5817}

$$\int x^m \text{arcsinh}(ax)^2\,dx = \frac{2a^2x^{m+3}\,{}_3F_2\left(1,\frac{m}{2}+\frac{3}{2},\frac{m}{2}+\frac{3}{2};\frac{m}{2}+2,\frac{m}{2}+\frac{5}{2};-a^2x^2\right)}{m^3+6m^2+11m+6} - \frac{2ax^{m+2}\text{arcsinh}(ax)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},-a^2x^2\right)}{m^2+3m+2} + \frac{x^{m+1}\text{arcsinh}(ax)^2}{m+1}$$

```
[In] Int[x^m*ArcSinh[a*x]^2,x]
```

```
[Out] (x^(1 + m)*ArcSinh[a*x]^2)/(1 + m) - (2*a*x^(2 + m)*ArcSinh[a*x]*Hypergeome
tric2F1[1/2, (2 + m)/2, (4 + m)/2, -(a^2*x^2)])/(2 + 3*m + m^2) + (2*a^2*x^
```

```
(3 + m)*HypergeometricPFQ[{1, 3/2 + m/2, 3/2 + m/2}, {2 + m/2, 5/2 + m/2},
-(a^2*x^2)])/(6 + 11*m + 6*m^2 + m^3)
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5817

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_
.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*Simp[Sqrt[1 + c^2
*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, (-c^2)*x^2], x] - Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2
)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2,
1 + m/2}, {3/2 + m/2, 2 + m/2}, (-c^2)*x^2], x] /; FreeQ[{a, b, c, d, e, f,
 m}, x] && EqQ[e, c^2*d] &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x^{1+m}\text{arcsinh}(ax)^2}{1+m} - \frac{(2a)\int \frac{x^{1+m}\text{arcsinh}(ax)}{\sqrt{1+a^2x^2}}\,dx}{1+m}\\ &= \frac{x^{1+m}\text{arcsinh}(ax)^2}{1+m} - \frac{2ax^{2+m}\text{arcsinh}(ax)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},-a^2x^2\right)}{2+3m+m^2}\\ &\quad + \frac{2a^2x^{3+m}\,{}_3F_2\left(1,\frac{3}{2}+\frac{m}{2},\frac{3}{2}+\frac{m}{2};2+\frac{m}{2},\frac{5}{2}+\frac{m}{2};-a^2x^2\right)}{6+11m+6m^2+m^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.90

$$\int x^m\text{arcsinh}(ax)^2\,dx = \frac{x^{1+m}\left((3+m)\text{arcsinh}(ax)\left((2+m)\text{arcsinh}(ax) - 2ax\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},-a^2x^2\right)\right) + 2a^2x}{(1+m)(2+m)(3+m)}$$

```
[In] Integrate[x^m*ArcSinh[a*x]^2,x]
```

```
[Out] (x^(1 + m)*((3 + m)*ArcSinh[a*x]*((2 + m)*ArcSinh[a*x] - 2*a*x*Hypergeometr
ic2F1[1/2, (2 + m)/2, (4 + m)/2, -(a^2*x^2)]) + 2*a^2*x^2*HypergeometricPFQ
[{1, 3/2 + m/2, 3/2 + m/2}, {2 + m/2, 5/2 + m/2}, -(a^2*x^2)]))/((1 + m)*(2
 + m)*(3 + m))
```

Maple [F]

$$\int x^m \operatorname{arcsinh}(ax)^2\,dx$$

```
[In] int(x^m*arcsinh(a*x)^2,x)
```

```
[Out] int(x^m*arcsinh(a*x)^2,x)
```

Fricas [F]

$$\int x^m \operatorname{arcsinh}(ax)^2\,dx = \int x^m \operatorname{arsinh}\left(ax\right)^2\,dx$$

```
[In] integrate(x^m*arcsinh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^m*arcsinh(a*x)^2, x)
```

Sympy [F]

$$\int x^m \operatorname{arcsinh}(ax)^2\,dx = \int x^m \operatorname{asinh}^2\left(ax\right)dx$$

```
[In] integrate(x**m*asinh(a*x)**2,x)
```

```
[Out] Integral(x**m*asinh(a*x)**2, x)
```

Maxima [F]

$$\int x^m \operatorname{arcsinh}(ax)^2\,dx = \int x^m \operatorname{arsinh}\left(ax\right)^2\,dx$$

```
[In] integrate(x^m*arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] x*x^m*log(a*x + sqrt(a^2*x^2 + 1))^2/(m + 1) - integrate(2*(sqrt(a^2*x^2 +
1)*a^2*x^2*x^m + (a^3*x^3 + a*x)*x^m)*log(a*x + sqrt(a^2*x^2 + 1))/(a^3*(m
+ 1)*x^3 + a*(m + 1)*x + (a^2*(m + 1)*x^2 + m + 1)*sqrt(a^2*x^2 + 1)), x)
```

Giac [F]

$$\int x^m \operatorname{arcsinh}(ax)^2 \, dx = \int x^m \operatorname{arsinh}\left(ax\right)^2 \, dx$$

```
[In] integrate(x^m*arcsinh(a*x)^2,x, algorithm="giac")
[Out] integrate(x^m*arcsinh(a*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \operatorname{arcsinh}(ax)^2 \, dx = \int x^m \operatorname{asinh}(a\,x)^2 \, dx$$

```
[In] int(x^m*asinh(a*x)^2,x)
[Out] int(x^m*asinh(a*x)^2, x)
```

3.120 $\int x^m \operatorname{arcsinh}(ax)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 60

$$\int x^m \operatorname{arcsinh}(ax)\,dx = \frac{x^{1+m}\operatorname{arcsinh}(ax)}{1+m} - \frac{ax^{2+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},-a^2x^2\right)}{2+3m+m^2}$$

```
[Out] x^(1+m)*arcsinh(a*x)/(1+m)-a*x^(2+m)*hypergeom([1/2, 1+1/2*m],[2+1/2*m],-a^
2*x^2)/(m^2+3*m+2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5776, 371}

$$\int x^m \operatorname{arcsinh}(ax)\,dx = \frac{x^{m+1}\operatorname{arcsinh}(ax)}{m+1} - \frac{ax^{m+2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},-a^2x^2\right)}{m^2+3m+2}$$

```
[In] Int[x^m*ArcSinh[a*x],x]
```

```
[Out] (x^(1 + m)*ArcSinh[a*x])/(1 + m) - (a*x^(2 + m)*Hypergeometric2F1[1/2, (2 +
 m)/2, (4 + m)/2, -(a^2*x^2)])/(2 + 3*m + m^2)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x^{1+m}\text{arcsinh}(ax)}{1+m} - \frac{a\int \frac{x^{1+m}}{\sqrt{1+a^2x^2}}\,dx}{1+m}\\ &= \frac{x^{1+m}\text{arcsinh}(ax)}{1+m} - \frac{ax^{2+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},-a^2x^2\right)}{2+3m+m^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.92

$$\int x^m\text{arcsinh}(ax)\,dx = \frac{x^{1+m}\left((2+m)\text{arcsinh}(ax) - ax\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},-a^2x^2\right)\right)}{(1+m)(2+m)}$$

```
[In] Integrate[x^m*ArcSinh[a*x],x]
[Out] (x^(1 + m)*((2 + m)*ArcSinh[a*x] - a*x*Hypergeometric2F1[1/2, (2 + m)/2, (4
 + m)/2, -(a^2*x^2)]))/((1 + m)*(2 + m))
```

Maple [F]

$$\int x^m \operatorname{arcsinh}(ax)\,dx$$

```
[In] int(x^m*arcsinh(a*x),x)
[Out] int(x^m*arcsinh(a*x),x)
```

Fricas [F]

$$\int x^m\text{arcsinh}(ax)\,dx = \int x^m \operatorname{arsinh}(ax)\;dx$$

```
[In] integrate(x^m*arcsinh(a*x),x, algorithm="fricas")
[Out] integral(x^m*arcsinh(a*x), x)
```

Sympy [F]

$$\int x^m \operatorname{arcsinh}(ax)\,dx = \int x^m \operatorname{asinh}(ax)\,dx$$

```
[In] integrate(x**m*asinh(a*x),x)
```

```
[Out] Integral(x**m*asinh(a*x), x)
```

Maxima [F]

$$\int x^m \operatorname{arcsinh}(ax)\,dx = \int x^m \operatorname{arsinh}(ax)\,dx$$

```
[In] integrate(x^m*arcsinh(a*x),x, algorithm="maxima")
```

```
[Out] -a^2*integrate(x^2*x^m/(a^2*(m + 1)*x^2 + m + 1), x) - a*integrate(x*x^m/(a
^3*(m + 1)*x^3 + a*(m + 1)*x + (a^2*(m + 1)*x^2 + m + 1)*sqrt(a^2*x^2 + 1))
, x) + x*x^m*log(a*x + sqrt(a^2*x^2 + 1))/(m + 1)
```

Giac [F]

$$\int x^m \operatorname{arcsinh}(ax)\,dx = \int x^m \operatorname{arsinh}(ax)\,dx$$

```
[In] integrate(x^m*arcsinh(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^m*arcsinh(a*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \operatorname{arcsinh}(ax)\,dx = \int x^m \operatorname{asinh}(a\,x)\,dx$$

```
[In] int(x^m*asinh(a*x),x)
```

```
[Out] int(x^m*asinh(a*x), x)
```

3.121 $\int \frac{x^m}{\mathbf{arcsinh}(ax)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{x^m}{\operatorname{arcsinh}(ax)}\,dx = \operatorname{Int}\left(\frac{x^m}{\operatorname{arcsinh}(ax)},x\right)$$

```
[Out] Unintegrable(x^m/arcsinh(a*x),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{x^m}{\operatorname{arcsinh}(ax)}\,dx$$

```
[In] Int[x^m/ArcSinh[a*x],x]
```

```
[Out] Defer[Int][x^m/ArcSinh[a*x], x]
```

Rubi steps

$$\text{integral} = \int \frac{x^m}{\operatorname{arcsinh}(ax)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{x^m}{\text{arcsinh}(ax)}\,dx=\int \frac{x^m}{\text{arcsinh}(ax)}\,dx$$

```
[In] Integrate[x^m/ArcSinh[a*x],x]
[Out] Integrate[x^m/ArcSinh[a*x], x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\text{arcsinh}\left(ax\right)}dx$$

```
[In] int(x^m/arcsinh(a*x),x)
[Out] int(x^m/arcsinh(a*x),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{x^m}{\text{arcsinh}(ax)}\,dx=\int \frac{x^m}{\text{arsinh}\left(ax\right)}\,dx$$

```
[In] integrate(x^m/arcsinh(a*x),x, algorithm="fricas")
[Out] integral(x^m/arcsinh(a*x), x)
```

Sympy [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{x^m}{\text{arcsinh}(ax)}\,dx=\int \frac{x^m}{\text{asinh}\left(ax\right)}\,dx$$

```
[In] integrate(x**m/asinh(a*x),x)
[Out] Integral(x**m/asinh(a*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{x^m}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{x^m}{\operatorname{arsinh}\left(ax\right)}\,dx$$

```
[In] integrate(x^m/arcsinh(a*x),x, algorithm="maxima")
[Out] integrate(x^m/arcsinh(a*x), x)
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{x^m}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{x^m}{\operatorname{arsinh}\left(ax\right)}\,dx$$

```
[In] integrate(x^m/arcsinh(a*x),x, algorithm="giac")
[Out] integrate(x^m/arcsinh(a*x), x)
```

Mupad [N/A]

Not integrable

Time = 2.41 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{x^m}{\operatorname{arcsinh}(ax)}\,dx = \int \frac{x^m}{\operatorname{asinh}\left(a\,x\right)}\,dx$$

```
[In] int(x^m/asinh(a*x),x)
[Out] int(x^m/asinh(a*x), x)
```

3.122 $\int \frac{x^m}{\operatorname{arcsinh}(ax)^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{x^m}{\operatorname{arcsinh}(ax)^2}\,dx = \operatorname{Int}\left(\frac{x^m}{\operatorname{arcsinh}(ax)^2}, x\right)$$

```
[Out] Unintegrable(x^m/arcsinh(a*x)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^m}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{x^m}{\operatorname{arcsinh}(ax)^2}\,dx$$

```
[In] Int[x^m/ArcSinh[a*x]^2,x]
```

```
[Out] Defer[Int][x^m/ArcSinh[a*x]^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{x^m}{\operatorname{arcsinh}(ax)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{x^m}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{x^m}{\operatorname{arcsinh}(ax)^2}\,dx$$

```
[In] Integrate[x^m/ArcSinh[a*x]^2,x]
[Out] Integrate[x^m/ArcSinh[a*x]^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\operatorname{arcsinh}\left(ax\right)^2}dx$$

```
[In] int(x^m/arcsinh(a*x)^2,x)
[Out] int(x^m/arcsinh(a*x)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{x^m}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{x^m}{\operatorname{arsinh}\left(ax\right)^2}\,dx$$

```
[In] integrate(x^m/arcsinh(a*x)^2,x, algorithm="fricas")
[Out] integral(x^m/arcsinh(a*x)^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{x^m}{\operatorname{asinh}^2\left(ax\right)}\,dx$$

```
[In] integrate(x**m/asinh(a*x)**2,x)
[Out] Integral(x**m/asinh(a*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 268, normalized size of antiderivative = 26.80

$$\int \frac{x^m}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{x^m}{\operatorname{arsinh}\left(ax\right)^2}\,dx$$

```
[In] integrate(x^m/arcsinh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -((a^2*x^2 + 1)^(3/2)*x^m + (a^3*x^3 + a*x)*x^m)/((a^3*x^2 + sqrt(a^2*x^2 +
 1)*a^2*x + a)*log(a*x + sqrt(a^2*x^2 + 1))) + integrate(((a^3*(m + 1)*x^3
+ a*(m - 1)*x)*(a^2*x^2 + 1)*x^m + (2*a^4*(m + 1)*x^4 + a^2*(3*m + 1)*x^2 +
 m)*sqrt(a^2*x^2 + 1)*x^m + (a^5*(m + 1)*x^5 + 2*a^3*(m + 1)*x^3 + a*(m + 1
)*x)*x^m)/((a^5*x^5 + (a^2*x^2 + 1)*a^3*x^3 + 2*a^3*x^3 + a*x + 2*(a^4*x^4
+ a^2*x^2)*sqrt(a^2*x^2 + 1))*log(a*x + sqrt(a^2*x^2 + 1))), x)
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{x^m}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{x^m}{\operatorname{arsinh}\left(ax\right)^2}\,dx$$

```
[In] integrate(x^m/arcsinh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m/arcsinh(a*x)^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.43 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{x^m}{\operatorname{arcsinh}(ax)^2}\,dx = \int \frac{x^m}{\operatorname{asinh}\left(a\,x\right)^2}\,dx$$

```
[In] int(x^m/asinh(a*x)^2,x)
```

```
[Out] int(x^m/asinh(a*x)^2, x)
```

3.123 $\int x^m \textbf{arcsinh}(ax)^{5/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int x^m \text{arcsinh}(ax)^{5/2}\,dx = \text{Int}\left(x^m \text{arcsinh}(ax)^{5/2}, x\right)$$

```
[Out] Unintegrable(x^m*arcsinh(a*x)^(5/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^m \text{arcsinh}(ax)^{5/2}\,dx = \int x^m \text{arcsinh}(ax)^{5/2}\,dx$$

```
[In] Int[x^m*ArcSinh[a*x]^(5/2),x]
[Out] Defer[Int][x^m*ArcSinh[a*x]^(5/2), x]
```

Rubi steps

$$\text{integral} = \int x^m \text{arcsinh}(ax)^{5/2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int x^m \operatorname{arcsinh}(ax)^{5/2} \, dx = \int x^m \operatorname{arcsinh}(ax)^{5/2} \, dx$$

```
[In] Integrate[x^m*ArcSinh[a*x]^(5/2),x]
[Out] Integrate[x^m*ArcSinh[a*x]^(5/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int x^m \operatorname{arcsinh}\left(a x \right)^{\frac{5}{2}} dx$$

```
[In] int(x^m*arcsinh(a*x)^(5/2),x)
[Out] int(x^m*arcsinh(a*x)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^m \operatorname{arcsinh}(ax)^{5/2} \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^m*arcsinh(a*x)^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int x^m \operatorname{arcsinh}(ax)^{5/2} \, dx = \text{Timed out}$$

```
[In] integrate(x**m*asinh(a*x)**(5/2),x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int x^m \operatorname{arcsinh}(ax)^{5/2}\,dx = \int x^m \operatorname{arsinh}\left(ax\right)^{\frac{5}{2}}\,dx$$

```
[In] integrate(x^m*arcsinh(a*x)^(5/2),x, algorithm="maxima")
[Out] integrate(x^m*arcsinh(a*x)^(5/2), x)
```

Giac [F(-1)]

Timed out.

$$\int x^m \operatorname{arcsinh}(ax)^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate(x^m*arcsinh(a*x)^(5/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [N/A]

Not integrable

Time = 2.45 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int x^m \operatorname{arcsinh}(ax)^{5/2}\,dx = \int x^m\,\operatorname{asinh}(a\,x)^{5/2}\,dx$$

```
[In] int(x^m*asinh(a*x)^(5/2),x)
[Out] int(x^m*asinh(a*x)^(5/2), x)
```

3.124 $\int x^m \textbf{arcsinh}(ax)^{3/2}\, dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int x^m \text{arcsinh}(ax)^{3/2}\, dx = \text{Int}\left(x^m \text{arcsinh}(ax)^{3/2}, x\right)$$

```
[Out] Unintegrable(x^m*arcsinh(a*x)^(3/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^m \text{arcsinh}(ax)^{3/2}\, dx = \int x^m \text{arcsinh}(ax)^{3/2}\, dx$$

```
[In] Int[x^m*ArcSinh[a*x]^(3/2),x]
```

```
[Out] Defer[Int][x^m*ArcSinh[a*x]^(3/2), x]
```

Rubi steps

$$\text{integral} = \int x^m \text{arcsinh}(ax)^{3/2}\, dx$$

Mathematica [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int x^m \text{arcsinh}(ax)^{3/2}\,dx = \int x^m \text{arcsinh}(ax)^{3/2}\,dx$$

```
[In] Integrate[x^m*ArcSinh[a*x]^(3/2),x]
[Out] Integrate[x^m*ArcSinh[a*x]^(3/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int x^m \operatorname{arcsinh}\left(a x\right)^{\frac{3}{2}}d x$$

```
[In] int(x^m*arcsinh(a*x)^(3/2),x)
[Out] int(x^m*arcsinh(a*x)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^m \text{arcsinh}(ax)^{3/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^m*arcsinh(a*x)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 64.65 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int x^m \text{arcsinh}(ax)^{3/2}\,dx = \int x^{m} \operatorname{asinh}^{\frac{3}{2}}{\left(a x \right)}\, dx$$

```
[In] integrate(x**m*asinh(a*x)**(3/2),x)
[Out] Integral(x**m*asinh(a*x)**(3/2), x)
```

Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int x^m \text{arcsinh}(ax)^{3/2}\,dx = \int x^m\,\text{arsinh}\left(ax\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x^m*arcsinh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^m*arcsinh(a*x)^(3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int x^m \text{arcsinh}(ax)^{3/2}\,dx = \text{Timed out}$$

```
[In] integrate(x^m*arcsinh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [N/A]

Not integrable

Time = 2.42 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int x^m \text{arcsinh}(ax)^{3/2}\,dx = \int x^m\,\mathrm{asinh}(a\,x)^{3/2}\,dx$$

```
[In] int(x^m*asinh(a*x)^(3/2),x)
```

```
[Out] int(x^m*asinh(a*x)^(3/2), x)
```

3.125 $\int x^m \sqrt{\text{arcsinh}(ax)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int x^m \sqrt{\text{arcsinh}(ax)}\,dx = \text{Int}\left(x^m \sqrt{\text{arcsinh}(ax)}, x\right)$$

```
[Out] Unintegrable(x^m*arcsinh(a*x)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^m \sqrt{\text{arcsinh}(ax)}\,dx = \int x^m \sqrt{\text{arcsinh}(ax)}\,dx$$

```
[In] Int[x^m*Sqrt[ArcSinh[a*x]],x]
[Out] Defer[Int][x^m*Sqrt[ArcSinh[a*x]], x]
```

Rubi steps

$$\text{integral} = \int x^m \sqrt{\text{arcsinh}(ax)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int x^m \sqrt{\operatorname{arcsinh}(ax)}\,dx = \int x^m \sqrt{\operatorname{arcsinh}(ax)}\,dx$$

```
[In] Integrate[x^m*Sqrt[ArcSinh[a*x]],x]
[Out] Integrate[x^m*Sqrt[ArcSinh[a*x]], x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int x^m \sqrt{\operatorname{arcsinh}\left(ax\right)}dx$$

```
[In] int(x^m*arcsinh(a*x)^(1/2),x)
[Out] int(x^m*arcsinh(a*x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^m \sqrt{\operatorname{arcsinh}(ax)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^m*arcsinh(a*x)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int x^m \sqrt{\operatorname{arcsinh}(ax)}\,dx = \int x^m \sqrt{\operatorname{asinh}\left(ax\right)}\,dx$$

```
[In] integrate(x**m*asinh(a*x)**(1/2),x)
[Out] Integral(x**m*sqrt(asinh(a*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int x^m \sqrt{\operatorname{arcsinh}(ax)}\,dx = \int x^m \sqrt{\operatorname{arsinh}\left(ax\right)}\,dx$$

```
[In] integrate(x^m*arcsinh(a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(x^m*sqrt(arcsinh(a*x)), x)
```

Giac [F(-1)]

Timed out.

$$\int x^m \sqrt{\operatorname{arcsinh}(ax)}\,dx = \text{Timed out}$$

```
[In] integrate(x^m*arcsinh(a*x)^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [N/A]

Not integrable

Time = 2.43 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int x^m \sqrt{\operatorname{arcsinh}(ax)}\,dx = \int x^m\,\sqrt{\operatorname{asinh}\left(a\,x\right)}\,dx$$

```
[In] int(x^m*asinh(a*x)^(1/2),x)
[Out] int(x^m*asinh(a*x)^(1/2), x)
```

3.126 $\int \frac{x^m}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{x^m}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx=\operatorname{Int}\left(\frac{x^m}{\sqrt{\operatorname{arcsinh}(ax)}},x\right)$$

```
[Out] Unintegrable(x^m/arcsinh(a*x)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx=\int \frac{x^m}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx$$

```
[In] Int[x^m/Sqrt[ArcSinh[a*x]],x]
```

```
[Out] Defer[Int][x^m/Sqrt[ArcSinh[a*x]], x]
```

Rubi steps

$$\text{integral}=\int \frac{x^m}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{x^m}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{x^m}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx$$

```
[In] Integrate[x^m/Sqrt[ArcSinh[a*x]],x]
[Out] Integrate[x^m/Sqrt[ArcSinh[a*x]], x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x^m}{\sqrt{\operatorname{arcsinh}\left(ax\right)}}dx$$

```
[In] int(x^m/arcsinh(a*x)^(1/2),x)
[Out] int(x^m/arcsinh(a*x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^m}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^m/arcsinh(a*x)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{x^m}{\sqrt{\operatorname{asinh}\left(ax\right)}}\,dx$$

```
[In] integrate(x**m/asinh(a*x)**(1/2),x)
[Out] Integral(x**m/sqrt(asinh(a*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{x^m}{\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

```
[In] integrate(x^m/arcsinh(a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(x^m/sqrt(arcsinh(a*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{x^m}{\sqrt{\operatorname{arsinh}(ax)}}\,dx$$

```
[In] integrate(x^m/arcsinh(a*x)^(1/2),x, algorithm="giac")
[Out] integrate(x^m/sqrt(arcsinh(a*x)), x)
```

Mupad [N/A]

Not integrable

Time = 2.44 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{\operatorname{arcsinh}(ax)}}\,dx = \int \frac{x^m}{\sqrt{\operatorname{asinh}(a\,x)}}\,dx$$

```
[In] int(x^m/asinh(a*x)^(1/2),x)
[Out] int(x^m/asinh(a*x)^(1/2), x)
```

3.127 $\int \frac{x^m}{\operatorname{arcsinh}(ax)^{3/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{x^m}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \operatorname{Int}\left(\frac{x^m}{\operatorname{arcsinh}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable(x^m/arcsinh(a*x)^(3/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{x^m}{\operatorname{arcsinh}(ax)^{3/2}}\,dx$$

```
[In] Int[x^m/ArcSinh[a*x]^(3/2),x]
```

```
[Out] Defer[Int][x^m/ArcSinh[a*x]^(3/2), x]
```

Rubi steps

$$\text{integral} = \int \frac{x^m}{\operatorname{arcsinh}(ax)^{3/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{x^m}{\operatorname{arcsinh}(ax)^{3/2}}\,dx=\int \frac{x^m}{\operatorname{arcsinh}(ax)^{3/2}}\,dx$$

```
[In] Integrate[x^m/ArcSinh[a*x]^(3/2),x]
[Out] Integrate[x^m/ArcSinh[a*x]^(3/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x^m}{\operatorname{arcsinh}\left(ax\right)^{\frac{3}{2}}}dx$$

```
[In] int(x^m/arcsinh(a*x)^(3/2),x)
[Out] int(x^m/arcsinh(a*x)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^m}{\operatorname{arcsinh}(ax)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(x^m/arcsinh(a*x)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 4.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\operatorname{arcsinh}(ax)^{3/2}}\,dx=\int \frac{x^m}{\operatorname{asinh}^{\frac{3}{2}}\left(ax\right)}\,dx$$

```
[In] integrate(x**m/asinh(a*x)**(3/2),x)
[Out] Integral(x**m/asinh(a*x)**(3/2), x)
```

Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{x^m}{\operatorname{arsinh}\left(ax\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^m/arcsinh(a*x)^(3/2),x, algorithm="maxima")
[Out] integrate(x^m/arcsinh(a*x)^(3/2), x)
```

Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{x^m}{\operatorname{arsinh}\left(ax\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^m/arcsinh(a*x)^(3/2),x, algorithm="giac")
[Out] integrate(x^m/arcsinh(a*x)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 2.46 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\operatorname{arcsinh}(ax)^{3/2}}\,dx = \int \frac{x^m}{{\mathrm{asinh}\left(a\,x\right)}^{3/2}}\,dx$$

```
[In] int(x^m/asinh(a*x)^(3/2),x)
[Out] int(x^m/asinh(a*x)^(3/2), x)
```

3.128 $\int (bx)^m \textbf{arcsinh}(ax)^n \, dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int (bx)^m \text{arcsinh}(ax)^n \, dx = \text{Int}((bx)^m \text{arcsinh}(ax)^n, x)$$

```
[Out] Unintegrable((b*x)^m*arcsinh(a*x)^n,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (bx)^m \text{arcsinh}(ax)^n \, dx = \int (bx)^m \text{arcsinh}(ax)^n \, dx$$

```
[In] Int[(b*x)^m*ArcSinh[a*x]^n,x]
```

```
[Out] Defer[Int][(b*x)^m*ArcSinh[a*x]^n, x]
```

Rubi steps

$$\text{integral} = \int (bx)^m \text{arcsinh}(ax)^n \, dx$$

Mathematica [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int (bx)^m \text{arcsinh}(ax)^n\,dx = \int (bx)^m \text{arcsinh}(ax)^n\,dx$$

```
[In] Integrate[(b*x)^m*ArcSinh[a*x]^n,x]
[Out] Integrate[(b*x)^m*ArcSinh[a*x]^n, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int (bx)^m \operatorname{arcsinh}(ax)^n\,dx$$

```
[In] int((b*x)^m*arcsinh(a*x)^n,x)
[Out] int((b*x)^m*arcsinh(a*x)^n,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int (bx)^m \text{arcsinh}(ax)^n\,dx = \int (bx)^m \operatorname{arsinh}(ax)^n\,dx$$

```
[In] integrate((b*x)^m*arcsinh(a*x)^n,x, algorithm="fricas")
[Out] integral((b*x)^m*arcsinh(a*x)^n, x)
```

Sympy [N/A]

Not integrable

Time = 3.98 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int (bx)^m \text{arcsinh}(ax)^n\,dx = \int (bx)^m \operatorname{asinh}^n(ax)\,dx$$

```
[In] integrate((b*x)**m*asinh(a*x)**n,x)
[Out] Integral((b*x)**m*asinh(a*x)**n, x)
```

Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int (bx)^m \text{arcsinh}(ax)^n \, dx = \int (bx)^m \operatorname{arsinh}\left(ax\right)^n \, dx$$

```
[In] integrate((b*x)^m*arcsinh(a*x)^n,x, algorithm="maxima")
[Out] integrate((b*x)^m*arcsinh(a*x)^n, x)
```

Giac [F(-1)]

Timed out.

$$\int (bx)^m \text{arcsinh}(ax)^n \, dx = \text{Timed out}$$

```
[In] integrate((b*x)^m*arcsinh(a*x)^n,x, algorithm="giac")
[Out] Timed out
```

Mupad [N/A]

Not integrable

Time = 2.44 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int (bx)^m \text{arcsinh}(ax)^n \, dx = \int \operatorname{asinh}(a\,x)^n\,(b\,x)^m \, dx$$

```
[In] int(asinh(a*x)^n*(b*x)^m,x)
[Out] int(asinh(a*x)^n*(b*x)^m, x)
```

3.129 $\int x^4 \text{arcsinh}(ax)^n \, dx$

Optimal result

Integrand size = 10, antiderivative size = 173

$$\begin{aligned}\int x^4 \text{arcsinh}(ax)^n \, dx = {} & \frac{5^{-1-n}(-\text{arcsinh}(ax))^{-n}\text{arcsinh}(ax)^n\Gamma(1+n,-5\text{arcsinh}(ax))}{32a^5} \\ & - \frac{3^{-n}(-\text{arcsinh}(ax))^{-n}\text{arcsinh}(ax)^n\Gamma(1+n,-3\text{arcsinh}(ax))}{32a^5} \\ & + \frac{(-\text{arcsinh}(ax))^{-n}\text{arcsinh}(ax)^n\Gamma(1+n,-\text{arcsinh}(ax))}{16a^5} \\ & - \frac{\Gamma(1+n,\text{arcsinh}(ax))}{16a^5} + \frac{3^{-n}\Gamma(1+n,3\text{arcsinh}(ax))}{32a^5} \\ & - \frac{5^{-1-n}\Gamma(1+n,5\text{arcsinh}(ax))}{32a^5}\end{aligned}$$

```
[Out] 1/32*5^(-1-n)*arcsinh(a*x)^n*GAMMA(1+n,-5*arcsinh(a*x))/a^5/((-arcsinh(a*x)
)^n)-1/32*arcsinh(a*x)^n*GAMMA(1+n,-3*arcsinh(a*x))/(3^n)/a^5/((-arcsinh(a*
x))^n)+1/16*arcsinh(a*x)^n*GAMMA(1+n,-arcsinh(a*x))/a^5/((-arcsinh(a*x))^n)
-1/16*GAMMA(1+n,arcsinh(a*x))/a^5+1/32*GAMMA(1+n,3*arcsinh(a*x))/(3^n)/a^5-
1/32*5^(-1-n)*GAMMA(1+n,5*arcsinh(a*x))/a^5
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used

$= \{5780, 5556, 3388, 2212\}$

$$\begin{aligned}\int x^4\text{arcsinh}(ax)^n\,dx = &\frac{5^{-n-1}\text{arcsinh}(ax)^n(-\text{arcsinh}(ax))^{-n}\Gamma(n+1,-5\text{arcsinh}(ax))}{32a^5}\\ &-\frac{3^{-n}\text{arcsinh}(ax)^n(-\text{arcsinh}(ax))^{-n}\Gamma(n+1,-3\text{arcsinh}(ax))}{32a^5}\\ &+\frac{\text{arcsinh}(ax)^n(-\text{arcsinh}(ax))^{-n}\Gamma(n+1,-\text{arcsinh}(ax))}{16a^5}\\ &-\frac{\Gamma(n+1,\text{arcsinh}(ax))}{16a^5}+\frac{3^{-n}\Gamma(n+1,3\text{arcsinh}(ax))}{32a^5}\\ &-\frac{5^{-n-1}\Gamma(n+1,5\text{arcsinh}(ax))}{32a^5}\end{aligned}$$

```
[In] Int[x^4*ArcSinh[a*x]^n,x]
```

```
[Out] (5^(-1 - n)*ArcSinh[a*x]^n*Gamma[1 + n, -5*ArcSinh[a*x]])/(32*a^5*(-ArcSinh
[a*x])^n) - (ArcSinh[a*x]^n*Gamma[1 + n, -3*ArcSinh[a*x]])/(32*3^n*a^5*(-Ar
cSinh[a*x])^n) + (ArcSinh[a*x]^n*Gamma[1 + n, -ArcSinh[a*x]])/(16*a^5*(-Arc
Sinh[a*x])^n) - Gamma[1 + n, ArcSinh[a*x]]/(16*a^5) + Gamma[1 + n, 3*ArcSin
h[a*x]]/(32*3^n*a^5) - (5^(-1 - n)*Gamma[1 + n, 5*ArcSinh[a*x]])/(32*a^5)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
```

```
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int x^n\cosh(x)\sinh^4(x)\,dx, x, \operatorname{arcsinh}(ax)\right)}{a^5}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{8}x^n\cosh(x)-\frac{3}{16}x^n\cosh(3x)+\frac{1}{16}x^n\cosh(5x)\right)\,dx, x, \operatorname{arcsinh}(ax)\right)}{a^5}\\
&= \frac{\text{Subst}\left(\int x^n\cosh(5x)\,dx, x, \operatorname{arcsinh}(ax)\right)}{16a^5}+\frac{\text{Subst}\left(\int x^n\cosh(x)\,dx, x, \operatorname{arcsinh}(ax)\right)}{8a^5}\\
&\quad-\frac{3\text{Subst}\left(\int x^n\cosh(3x)\,dx, x, \operatorname{arcsinh}(ax)\right)}{16a^5}\\
&= \frac{\text{Subst}\left(\int e^{-5x}x^n\,dx, x, \operatorname{arcsinh}(ax)\right)}{32a^5}+\frac{\text{Subst}\left(\int e^{5x}x^n\,dx, x, \operatorname{arcsinh}(ax)\right)}{32a^5}\\
&\quad+\frac{\text{Subst}\left(\int e^{-x}x^n\,dx, x, \operatorname{arcsinh}(ax)\right)}{16a^5}+\frac{\text{Subst}\left(\int e^{x}x^n\,dx, x, \operatorname{arcsinh}(ax)\right)}{16a^5}\\
&\quad-\frac{3\text{Subst}\left(\int e^{-3x}x^n\,dx, x, \operatorname{arcsinh}(ax)\right)}{32a^5}-\frac{3\text{Subst}\left(\int e^{3x}x^n\,dx, x, \operatorname{arcsinh}(ax)\right)}{32a^5}\\
&= \frac{5^{-1-n}(-\operatorname{arcsinh}(ax))^{-n}\operatorname{arcsinh}(ax)^n\Gamma(1+n,-5\operatorname{arcsinh}(ax))}{32a^5}\\
&\quad-\frac{3^{-n}(-\operatorname{arcsinh}(ax))^{-n}\operatorname{arcsinh}(ax)^n\Gamma(1+n,-3\operatorname{arcsinh}(ax))}{32a^5}\\
&\quad+\frac{(-\operatorname{arcsinh}(ax))^{-n}\operatorname{arcsinh}(ax)^n\Gamma(1+n,-\operatorname{arcsinh}(ax))}{16a^5}-\frac{\Gamma(1+n,\operatorname{arcsinh}(ax))}{16a^5}\\
&\quad+\frac{3^{-n}\Gamma(1+n,3\operatorname{arcsinh}(ax))}{32a^5}-\frac{5^{-1-n}\Gamma(1+n,5\operatorname{arcsinh}(ax))}{32a^5}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.84

$$\int x^4\operatorname{arcsinh}(ax)^n\,dx$$

$$= \frac{5^{-n}(-\operatorname{arcsinh}(ax))^{-n}\operatorname{arcsinh}(ax)^n\Gamma(1+n,-5\operatorname{arcsinh}(ax))-5\;3^{-n}(-\operatorname{arcsinh}(ax))^{-n}\operatorname{arcsinh}(ax)^n\Gamma(1+}{}$$

```
[In] Integrate[x^4*ArcSinh[a*x]^n,x]
```

```
[Out] ((ArcSinh[a*x]^n*Gamma[1 + n, -5*ArcSinh[a*x]])/(5^n*(-ArcSinh[a*x])^n) - (
5*ArcSinh[a*x]^n*Gamma[1 + n, -3*ArcSinh[a*x]])/(3^n*(-ArcSinh[a*x])^n) + (
10*ArcSinh[a*x]^n*Gamma[1 + n, -ArcSinh[a*x]])/(-ArcSinh[a*x])^n - 10*Gamma
[1 + n, ArcSinh[a*x]] + (5*Gamma[1 + n, 3*ArcSinh[a*x]])/3^n - Gamma[1 + n,
 5*ArcSinh[a*x]]/5^n)/(160*a^5)
```

Maple [F]

$$\int x^4 \operatorname{arcsinh}(ax)^n \, dx$$

```
[In] int(x^4*arcsinh(a*x)^n,x)
```

```
[Out] int(x^4*arcsinh(a*x)^n,x)
```

Fricas [F]

$$\int x^4 \operatorname{arcsinh}(ax)^n \, dx = \int x^4 \operatorname{arsinh}(ax)^n \, dx$$

```
[In] integrate(x^4*arcsinh(a*x)^n,x, algorithm="fricas")
```

```
[Out] integral(x^4*arcsinh(a*x)^n, x)
```

Sympy [F]

$$\int x^4 \operatorname{arcsinh}(ax)^n \, dx = \int x^4 \operatorname{asinh}^n(ax) \, dx$$

```
[In] integrate(x**4*asinh(a*x)**n,x)
```

```
[Out] Integral(x**4*asinh(a*x)**n, x)
```

Maxima [F]

$$\int x^4 \operatorname{arcsinh}(ax)^n \, dx = \int x^4 \operatorname{arsinh}(ax)^n \, dx$$

```
[In] integrate(x^4*arcsinh(a*x)^n,x, algorithm="maxima")
```

```
[Out] integrate(x^4*arcsinh(a*x)^n, x)
```

Giac [F]

$$\int x^4 \text{arcsinh}(ax)^n \, dx = \int x^4 \,\text{arsinh}\,(ax)^n \, dx$$

```
[In] integrate(x^4*arcsinh(a*x)^n,x, algorithm="giac")
[Out] integrate(x^4*arcsinh(a*x)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \text{arcsinh}(ax)^n \, dx = \int x^4 \,\text{asinh}(a\,x)^n \, dx$$

```
[In] int(x^4*asinh(a*x)^n,x)
[Out] int(x^4*asinh(a*x)^n, x)
```

3.130 $\int x^3\mathbf{arcsinh}(ax)^n\,dx$

Optimal result

Integrand size = 10, antiderivative size = 119

$$\int x^3\operatorname{arcsinh}(ax)^n\,dx=\frac{2^{-2(3+n)}(-\operatorname{arcsinh}(ax))^{-n}\operatorname{arcsinh}(ax)^n\Gamma(1+n,-4\operatorname{arcsinh}(ax))}{a^4}-\frac{2^{-4-n}(-\operatorname{arcsinh}(ax))^{-n}\operatorname{arcsinh}(ax)^n\Gamma(1+n,-2\operatorname{arcsinh}(ax))}{a^4}-\frac{2^{-4-n}\Gamma(1+n,2\operatorname{arcsinh}(ax))}{a^4}+\frac{2^{-2(3+n)}\Gamma(1+n,4\operatorname{arcsinh}(ax))}{a^4}$$

```
[Out] arcsinh(a*x)^n*GAMMA(1+n,-4*arcsinh(a*x))/(2^(6+2*n))/a^4/((-arcsinh(a*x))^
n)-2^(-4-n)*arcsinh(a*x)^n*GAMMA(1+n,-2*arcsinh(a*x))/a^4/((-arcsinh(a*x))^
n)-2^(-4-n)*GAMMA(1+n,2*arcsinh(a*x))/a^4+GAMMA(1+n,4*arcsinh(a*x))/(2^(6+2
*n))/a^4
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5780, 5556, 3389, 2212}

$$\int x^3\operatorname{arcsinh}(ax)^n\,dx=\frac{2^{-2(n+3)}\operatorname{arcsinh}(ax)^n(-\operatorname{arcsinh}(ax))^{-n}\Gamma(n+1,-4\operatorname{arcsinh}(ax))}{a^4}-\frac{2^{-n-4}\operatorname{arcsinh}(ax)^n(-\operatorname{arcsinh}(ax))^{-n}\Gamma(n+1,-2\operatorname{arcsinh}(ax))}{a^4}-\frac{2^{-n-4}\Gamma(n+1,2\operatorname{arcsinh}(ax))}{a^4}+\frac{2^{-2(n+3)}\Gamma(n+1,4\operatorname{arcsinh}(ax))}{a^4}$$

```
[In] Int[x^3*ArcSinh[a*x]^n,x]
```

```
[Out] (ArcSinh[a*x]^n*Gamma[1 + n, -4*ArcSinh[a*x]])/(2^(2*(3 + n))*a^4*(-ArcSinh
[a*x])^n) - (2^(-4 - n)*ArcSinh[a*x]^n*Gamma[1 + n, -2*ArcSinh[a*x]])/(a^4*
(-ArcSinh[a*x])^n) - (2^(-4 - n)*Gamma[1 + n, 2*ArcSinh[a*x]])/a^4 + Gamma[
1 + n, 4*ArcSinh[a*x]]/(2^(2*(3 + n))*a^4)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int x^n \cosh(x)\sinh^3(x)\,dx, x, \operatorname{arcsinh}(ax)\right)}{a^4} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{1}{4}x^n \sinh(2x) + \frac{1}{8}x^n \sinh(4x)\right)\,dx, x, \operatorname{arcsinh}(ax)\right)}{a^4} \\
&= \frac{\text{Subst}\left(\int x^n \sinh(4x)\,dx, x, \operatorname{arcsinh}(ax)\right)}{8a^4} - \frac{\text{Subst}\left(\int x^n \sinh(2x)\,dx, x, \operatorname{arcsinh}(ax)\right)}{4a^4} \\
&= -\frac{\text{Subst}\left(\int e^{-4x}x^n\,dx, x, \operatorname{arcsinh}(ax)\right)}{16a^4} + \frac{\text{Subst}\left(\int e^{4x}x^n\,dx, x, \operatorname{arcsinh}(ax)\right)}{16a^4} \\
&\quad + \frac{\text{Subst}\left(\int e^{-2x}x^n\,dx, x, \operatorname{arcsinh}(ax)\right)}{8a^4} - \frac{\text{Subst}\left(\int e^{2x}x^n\,dx, x, \operatorname{arcsinh}(ax)\right)}{8a^4}
\end{aligned}
$$

$$= \frac{4^{-3-n}(-\text{arcsinh}(ax))^{-n}\text{arcsinh}(ax)^n\Gamma(1+n,-4\text{arcsinh}(ax))}{a^4} - \frac{2^{-4-n}(-\text{arcsinh}(ax))^{-n}\text{arcsinh}(ax)^n\Gamma(1+n,-2\text{arcsinh}(ax))}{a^4} - \frac{2^{-4-n}\Gamma(1+n,2\text{arcsinh}(ax))}{a^4} + \frac{4^{-3-n}\Gamma(1+n,4\text{arcsinh}(ax))}{a^4}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.83

$$\int x^3\text{arcsinh}(ax)^n\,dx = \frac{4^{-3-n}(-\text{arcsinh}(ax))^{-n}\left(\text{arcsinh}(ax)^n\Gamma(1+n,-4\text{arcsinh}(ax)) - 2^{2+n}\text{arcsinh}(ax)^n\Gamma(1+n,-2\text{arcsinh}(ax)\right.}{a^4}$$

```
[In] Integrate[x^3*ArcSinh[a*x]^n,x]
```

```
[Out] (4^(-3 - n)*(ArcSinh[a*x]^n*Gamma[1 + n, -4*ArcSinh[a*x]] - 2^(2 + n)*ArcSi
nh[a*x]^n*Gamma[1 + n, -2*ArcSinh[a*x]] + (-ArcSinh[a*x])^n*(-(2^(2 + n)*Ga
mma[1 + n, 2*ArcSinh[a*x]]) + Gamma[1 + n, 4*ArcSinh[a*x]])))/(a^4*(-ArcSin
h[a*x])^n)
```

Maple [F]

$$\int x^3\,\text{arcsinh}\,(ax)^n\,dx$$

```
[In] int(x^3*arcsinh(a*x)^n,x)
```

```
[Out] int(x^3*arcsinh(a*x)^n,x)
```

Fricas [F]

$$\int x^3\text{arcsinh}(ax)^n\,dx = \int x^3\,\text{arsinh}\,(ax)^n\,dx$$

```
[In] integrate(x^3*arcsinh(a*x)^n,x, algorithm="fricas")
```

```
[Out] integral(x^3*arcsinh(a*x)^n, x)
```

Sympy [F]

$$\int x^3 \text{arcsinh}(ax)^n \, dx = \int x^3 \operatorname{asinh}^n(ax) \, dx$$

```
[In] integrate(x**3*asinh(a*x)**n,x)
```

```
[Out] Integral(x**3*asinh(a*x)**n, x)
```

Maxima [F]

$$\int x^3 \text{arcsinh}(ax)^n \, dx = \int x^3 \operatorname{arsinh}(ax)^n \, dx$$

```
[In] integrate(x^3*arcsinh(a*x)^n,x, algorithm="maxima")
```

```
[Out] integrate(x^3*arcsinh(a*x)^n, x)
```

Giac [F(-2)]

Exception generated.

$$\int x^3 \text{arcsinh}(ax)^n \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3*arcsinh(a*x)^n,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \text{arcsinh}(ax)^n \, dx = \int x^3 \operatorname{asinh}(a\,x)^n \, dx$$

```
[In] int(x^3*asinh(a*x)^n,x)
```

```
[Out] int(x^3*asinh(a*x)^n, x)
```

3.131 $\int x^2\mathbf{arcsinh}(ax)^n\,dx$

Optimal result

Integrand size = 10, antiderivative size = 113

$$\int x^2\text{arcsinh}(ax)^n\,dx = \frac{3^{-1-n}(-\text{arcsinh}(ax))^{-n}\text{arcsinh}(ax)^n\Gamma(1+n,-3\text{arcsinh}(ax))}{8a^3} - \frac{(-\text{arcsinh}(ax))^{-n}\text{arcsinh}(ax)^n\Gamma(1+n,-\text{arcsinh}(ax))}{8a^3} + \frac{\Gamma(1+n,\text{arcsinh}(ax))}{8a^3} - \frac{3^{-1-n}\Gamma(1+n,3\text{arcsinh}(ax))}{8a^3}$$

```
[Out] 1/8*3^(-1-n)*arcsinh(a*x)^n*GAMMA(1+n,-3*arcsinh(a*x))/a^3/((-arcsinh(a*x))
^n)-1/8*arcsinh(a*x)^n*GAMMA(1+n,-arcsinh(a*x))/a^3/((-arcsinh(a*x))^n)+1/8
*GAMMA(1+n,arcsinh(a*x))/a^3-1/8*3^(-1-n)*GAMMA(1+n,3*arcsinh(a*x))/a^3
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5780, 5556, 3388, 2212}

$$\int x^2\text{arcsinh}(ax)^n\,dx = \frac{3^{-n-1}\text{arcsinh}(ax)^n(-\text{arcsinh}(ax))^{-n}\Gamma(n+1,-3\text{arcsinh}(ax))}{8a^3} - \frac{\text{arcsinh}(ax)^n(-\text{arcsinh}(ax))^{-n}\Gamma(n+1,-\text{arcsinh}(ax))}{8a^3} + \frac{\Gamma(n+1,\text{arcsinh}(ax))}{8a^3} - \frac{3^{-n-1}\Gamma(n+1,3\text{arcsinh}(ax))}{8a^3}$$

```
[In] Int[x^2*ArcSinh[a*x]^n,x]
```

```
[Out] (3^(-1 - n)*ArcSinh[a*x]^n*Gamma[1 + n, -3*ArcSinh[a*x]])/(8*a^3*(-ArcSinh[
a*x])^n) - (ArcSinh[a*x]^n*Gamma[1 + n, -ArcSinh[a*x]])/(8*a^3*(-ArcSinh[a*
```

```
x])^n) + Gamma[1 + n, ArcSinh[a*x]]/(8*a^3) - (3^(-1 - n)*Gamma[1 + n, 3*Ar
cSinh[a*x]])/(8*a^3)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\operatorname{Subst}\left(\int x^n \cosh(x)\sinh^2(x)\,dx, x, \operatorname{arcsinh}(ax)\right)}{a^3}\\
&= \frac{\operatorname{Subst}\left(\int \left(-\frac{1}{4}x^n\cosh(x)+\frac{1}{4}x^n\cosh(3x)\right)\,dx, x, \operatorname{arcsinh}(ax)\right)}{a^3}\\
&= -\frac{\operatorname{Subst}\left(\int x^n\cosh(x)\,dx, x, \operatorname{arcsinh}(ax)\right)}{4a^3}+\frac{\operatorname{Subst}\left(\int x^n\cosh(3x)\,dx, x, \operatorname{arcsinh}(ax)\right)}{4a^3}\\
&= \frac{\operatorname{Subst}\left(\int e^{-3x}x^n\,dx, x, \operatorname{arcsinh}(ax)\right)}{8a^3}-\frac{\operatorname{Subst}\left(\int e^{-x}x^n\,dx, x, \operatorname{arcsinh}(ax)\right)}{8a^3}\\
&\quad -\frac{\operatorname{Subst}\left(\int e^{x}x^n\,dx, x, \operatorname{arcsinh}(ax)\right)}{8a^3}+\frac{\operatorname{Subst}\left(\int e^{3x}x^n\,dx, x, \operatorname{arcsinh}(ax)\right)}{8a^3}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{3^{-1-n}(-\operatorname{arcsinh}(ax))^{-n}\operatorname{arcsinh}(ax)^n\Gamma(1+n,-3\operatorname{arcsinh}(ax))}{8a^3} \\
&\quad - \frac{(-\operatorname{arcsinh}(ax))^{-n}\operatorname{arcsinh}(ax)^n\Gamma(1+n,-\operatorname{arcsinh}(ax))}{8a^3} \\
&\quad + \frac{\Gamma(1+n,\operatorname{arcsinh}(ax))}{8a^3} - \frac{3^{-1-n}\Gamma(1+n,3\operatorname{arcsinh}(ax))}{8a^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.86

$$
\begin{aligned}
&\int x^2\operatorname{arcsinh}(ax)^n\,dx \\
&= \frac{3^{-1-n}(-\operatorname{arcsinh}(ax))^{-n}\operatorname{arcsinh}(ax)^n\Gamma(1+n,-3\operatorname{arcsinh}(ax)) - (-\operatorname{arcsinh}(ax))^{-n}\operatorname{arcsinh}(ax)^n\Gamma(1+n,-}{8a^3}
\end{aligned}
$$

```
[In] Integrate[x^2*ArcSinh[a*x]^n,x]
```

```
[Out] ((3^(-1 - n)*ArcSinh[a*x]^n*Gamma[1 + n, -3*ArcSinh[a*x]])/(-ArcSinh[a*x])^
n - (ArcSinh[a*x]^n*Gamma[1 + n, -ArcSinh[a*x]])/(-ArcSinh[a*x])^n + Gamma[
1 + n, ArcSinh[a*x]] - 3^(-1 - n)*Gamma[1 + n, 3*ArcSinh[a*x]])/(8*a^3)
```

Maple [F]

$$\int x^2 \operatorname{arcsinh}(ax)^n\,dx$$

```
[In] int(x^2*arcsinh(a*x)^n,x)
```

```
[Out] int(x^2*arcsinh(a*x)^n,x)
```

Fricas [F]

$$\int x^2\operatorname{arcsinh}(ax)^n\,dx = \int x^2 \operatorname{arsinh}(ax)^n\,dx$$

```
[In] integrate(x^2*arcsinh(a*x)^n,x, algorithm="fricas")
```

```
[Out] integral(x^2*arcsinh(a*x)^n, x)
```

Sympy [F]

$$\int x^2\text{arcsinh}(ax)^n\,dx = \int x^2\,\text{asinh}^n\,(ax)\,dx$$

```
[In] integrate(x**2*asinh(a*x)**n,x)
[Out] Integral(x**2*asinh(a*x)**n, x)
```

Maxima [F]

$$\int x^2\text{arcsinh}(ax)^n\,dx = \int x^2\,\text{arsinh}\,(ax)^n\;dx$$

```
[In] integrate(x^2*arcsinh(a*x)^n,x, algorithm="maxima")
[Out] integrate(x^2*arcsinh(a*x)^n, x)
```

Giac [F]

$$\int x^2\text{arcsinh}(ax)^n\,dx = \int x^2\,\text{arsinh}\,(ax)^n\;dx$$

```
[In] integrate(x^2*arcsinh(a*x)^n,x, algorithm="giac")
[Out] integrate(x^2*arcsinh(a*x)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\text{arcsinh}(ax)^n\,dx = \int x^2\,\text{asinh}(a\,x)^n\,dx$$

```
[In] int(x^2*asinh(a*x)^n,x)
[Out] int(x^2*asinh(a*x)^n, x)
```

3.132 $\int x\text{arcsinh}(ax)^n\,dx$

Optimal result

Integrand size = 8, antiderivative size = 59

$$\int x\text{arcsinh}(ax)^n\,dx = \frac{2^{-3-n}(-\text{arcsinh}(ax))^{-n}\text{arcsinh}(ax)^n\Gamma(1+n,-2\text{arcsinh}(ax))}{a^2} + \frac{2^{-3-n}\Gamma(1+n,2\text{arcsinh}(ax))}{a^2}$$

```
[Out] 2^(-3-n)*arcsinh(a*x)^n*GAMMA(1+n,-2*arcsinh(a*x))/a^2/((-arcsinh(a*x))^n)+
2^(-3-n)*GAMMA(1+n,2*arcsinh(a*x))/a^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5780, 5556, 12, 3389, 2212}

$$\int x\text{arcsinh}(ax)^n\,dx = \frac{2^{-n-3}\text{arcsinh}(ax)^n(-\text{arcsinh}(ax))^{-n}\Gamma(n+1,-2\text{arcsinh}(ax))}{a^2} + \frac{2^{-n-3}\Gamma(n+1,2\text{arcsinh}(ax))}{a^2}$$

```
[In] Int[x*ArcSinh[a*x]^n,x]
```

```
[Out] (2^(-3 - n)*ArcSinh[a*x]^n*Gamma[1 + n, -2*ArcSinh[a*x]])/(a^2*(-ArcSinh[a*
x])^n) + (2^(-3 - n)*Gamma[1 + n, 2*ArcSinh[a*x]])/a^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int x^n \cosh(x)\sinh(x)\,dx, x, \text{arcsinh}(ax)\right)}{a^2} \\
&= \frac{\text{Subst}\left(\int \frac{1}{2}x^n \sinh(2x)\,dx, x, \text{arcsinh}(ax)\right)}{a^2} \\
&= \frac{\text{Subst}\left(\int x^n \sinh(2x)\,dx, x, \text{arcsinh}(ax)\right)}{2a^2} \\
&= -\frac{\text{Subst}\left(\int e^{-2x}x^n\,dx, x, \text{arcsinh}(ax)\right)}{4a^2} + \frac{\text{Subst}\left(\int e^{2x}x^n\,dx, x, \text{arcsinh}(ax)\right)}{4a^2} \\
&= \frac{2^{-3-n}(-\text{arcsinh}(ax))^{-n}\text{arcsinh}(ax)^n\Gamma(1+n, -2\text{arcsinh}(ax))}{a^2} + \frac{2^{-3-n}\Gamma(1+n, 2\text{arcsinh}(ax))}{a^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97

$$\int x\operatorname{arcsinh}(ax)^n\,dx = \frac{2^{-3-n}(-\operatorname{arcsinh}(ax))^{-n}\operatorname{arcsinh}(ax)^n\Gamma(1+n,-2\operatorname{arcsinh}(ax))+2^{-3-n}\Gamma(1+n,2\operatorname{arcsinh}(ax))}{a^2}$$

```
[In] Integrate[x*ArcSinh[a*x]^n,x]
```

```
[Out] ((2^(-3 - n)*ArcSinh[a*x]^n*Gamma[1 + n, -2*ArcSinh[a*x]])/(-ArcSinh[a*x])^
n + 2^(-3 - n)*Gamma[1 + n, 2*ArcSinh[a*x]])/a^2
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.64

method	result	size
default	$\frac{\operatorname{arcsinh}(ax)^{2+n}\operatorname{hypergeom}\left([1+\frac{n}{2}],[\frac{3}{2},2+\frac{n}{2}],\operatorname{arcsinh}(ax)^2\right)}{a^2(2+n)}$	38

```
[In] int(x*arcsinh(a*x)^n,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2/(2+n)*arcsinh(a*x)^(2+n)*hypergeom([1+1/2*n],[3/2,2+1/2*n],arcsinh(a*
x)^2)
```

Fricas [F]

$$\int x\operatorname{arcsinh}(ax)^n\,dx = \int x\operatorname{arsinh}(ax)^n\,dx$$

```
[In] integrate(x*arcsinh(a*x)^n,x, algorithm="fricas")
```

```
[Out] integral(x*arcsinh(a*x)^n, x)
```

Sympy [F]

$$\int x\text{arcsinh}(ax)^n\,dx=\int x\operatorname{asinh}^{n}(ax)\,dx$$

```
[In] integrate(x*asinh(a*x)**n,x)
```

```
[Out] Integral(x*asinh(a*x)**n, x)
```

Maxima [F]

$$\int x\text{arcsinh}(ax)^n\,dx=\int x\operatorname{arsinh}(ax)^{n}\,dx$$

```
[In] integrate(x*arcsinh(a*x)^n,x, algorithm="maxima")
```

```
[Out] integrate(x*arcsinh(a*x)^n, x)
```

Giac [F]

$$\int x\text{arcsinh}(ax)^n\,dx=\int x\operatorname{arsinh}(ax)^{n}\,dx$$

```
[In] integrate(x*arcsinh(a*x)^n,x, algorithm="giac")
```

```
[Out] integrate(x*arcsinh(a*x)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\text{arcsinh}(ax)^n\,dx=\int x\operatorname{asinh}(a\,x)^n\,dx$$

```
[In] int(x*asinh(a*x)^n,x)
```

```
[Out] int(x*asinh(a*x)^n, x)
```

3.133 $\int \operatorname{arcsinh}(ax)^n \, dx$

Optimal result

Integrand size = 6, antiderivative size = 49

$$\int \operatorname{arcsinh}(ax)^n \, dx = \frac{(-\operatorname{arcsinh}(ax))^{-n} \operatorname{arcsinh}(ax)^n \Gamma(1+n, -\operatorname{arcsinh}(ax))}{2a} - \frac{\Gamma(1+n, \operatorname{arcsinh}(ax))}{2a}$$

```
[Out] 1/2*arcsinh(a*x)^n*GAMMA(1+n,-arcsinh(a*x))/a/((-arcsinh(a*x))^n)-1/2*GAMMA
(1+n,arcsinh(a*x))/a
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5774, 3388, 2212}

$$\int \operatorname{arcsinh}(ax)^n \, dx = \frac{(-\operatorname{arcsinh}(ax))^{-n} \operatorname{arcsinh}(ax)^n \Gamma(n+1, -\operatorname{arcsinh}(ax))}{2a} - \frac{\Gamma(n+1, \operatorname{arcsinh}(ax))}{2a}$$

```
[In] Int[ArcSinh[a*x]^n,x]
```

```
[Out] (ArcSinh[a*x]^n*Gamma[1 + n, -ArcSinh[a*x]])/(2*a*(-ArcSinh[a*x])^n) - Gamm
a[1 + n, ArcSinh[a*x]]/(2*a)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
```

```
((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int x^n \cosh(x)\,dx, x, \text{arcsinh}(ax)\right)}{a}\\ &= \frac{\text{Subst}\left(\int e^{-x}x^n\,dx, x, \text{arcsinh}(ax)\right)}{2a} + \frac{\text{Subst}\left(\int e^{x}x^n\,dx, x, \text{arcsinh}(ax)\right)}{2a}\\ &= \frac{(-\text{arcsinh}(ax))^{-n}\text{arcsinh}(ax)^n\Gamma(1+n, -\text{arcsinh}(ax))}{2a} - \frac{\Gamma(1+n, \text{arcsinh}(ax))}{2a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.92

$$\int \text{arcsinh}(ax)^n\,dx = \frac{(-\text{arcsinh}(ax))^{-n}\text{arcsinh}(ax)^n\Gamma(1+n, -\text{arcsinh}(ax)) - \Gamma(1+n, \text{arcsinh}(ax))}{2a}$$

```
[In] Integrate[ArcSinh[a*x]^n,x]
[Out] ((ArcSinh[a*x]^n*Gamma[1 + n, -ArcSinh[a*x]])/(-ArcSinh[a*x])^n - Gamma[1 +
 n, ArcSinh[a*x]])/(2*a)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{\operatorname{arcsinh}(ax)^{1+n} \operatorname{hypergeom}\left(\left[\frac{1}{2}+\frac{n}{2}\right],\left[\frac{1}{2},\frac{3}{2}+\frac{n}{2}\right],\frac{\operatorname{arcsinh}(ax)^2}{4}\right)}{a(1+n)}$	40

```
[In] int(arcsinh(a*x)^n,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a/(1+n)*arcsinh(a*x)^(1+n)*hypergeom([1/2+1/2*n],[1/2,3/2+1/2*n],1/4*arcs
inh(a*x)^2)
```

Fricas [F]

$$\int \operatorname{arcsinh}(ax)^n\,dx = \int \operatorname{arsinh}\left(ax\right)^n\,dx$$

```
[In] integrate(arcsinh(a*x)^n,x, algorithm="fricas")
```

```
[Out] integral(arcsinh(a*x)^n, x)
```

Sympy [F]

$$\int \operatorname{arcsinh}(ax)^n\,dx = \int \operatorname{asinh}^{n}\left(ax\right)dx$$

```
[In] integrate(asinh(a*x)**n,x)
```

```
[Out] Integral(asinh(a*x)**n, x)
```

Maxima [F]

$$\int \operatorname{arcsinh}(ax)^n\,dx = \int \operatorname{arsinh}\left(ax\right)^n\,dx$$

```
[In] integrate(arcsinh(a*x)^n,x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^n, x)
```

Giac [F]

$$\int \operatorname{arcsinh}(ax)^n\,dx = \int \operatorname{arsinh}\left(ax\right)^n\,dx$$

```
[In] integrate(arcsinh(a*x)^n,x, algorithm="giac")
[Out] integrate(arcsinh(a*x)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{arcsinh}(ax)^n\,dx = \int \operatorname{asinh}(a\,x)^n\,dx$$

```
[In] int(asinh(a*x)^n,x)
[Out] int(asinh(a*x)^n, x)
```

3.134 $\int \frac{\mathbf{arcsinh}(ax)^n}{x}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\operatorname{arcsinh}(ax)^n}{x}\,dx = \operatorname{Int}\left(\frac{\operatorname{arcsinh}(ax)^n}{x}, x\right)$$

```
[Out] Unintegrable(arcsinh(a*x)^n/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\operatorname{arcsinh}(ax)^n}{x}\,dx = \int \frac{\operatorname{arcsinh}(ax)^n}{x}\,dx$$

```
[In] Int[ArcSinh[a*x]^n/x,x]
```

```
[Out] Defer[Int][ArcSinh[a*x]^n/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\operatorname{arcsinh}(ax)^n}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{arcsinh}(ax)^n}{x}\,dx=\int \frac{\operatorname{arcsinh}(ax)^n}{x}\,dx$$

```
[In] Integrate[ArcSinh[a*x]^n/x,x]
[Out] Integrate[ArcSinh[a*x]^n/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}\left(ax\right)^{n}}{x}dx$$

```
[In] int(arcsinh(a*x)^n/x,x)
[Out] int(arcsinh(a*x)^n/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{arcsinh}(ax)^n}{x}\,dx=\int \frac{\operatorname{arsinh}\left(ax\right)^{n}}{x}\,dx$$

```
[In] integrate(arcsinh(a*x)^n/x,x, algorithm="fricas")
[Out] integral(arcsinh(a*x)^n/x, x)
```

Sympy [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\operatorname{arcsinh}(ax)^n}{x}\,dx=\int \frac{\operatorname{asinh}^{n}\left(ax\right)}{x}\,dx$$

```
[In] integrate(asinh(a*x)**n/x,x)
[Out] Integral(asinh(a*x)**n/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{arcsinh}(ax)^n}{x}\,dx=\int \frac{\operatorname{arsinh}\left(ax\right)^n}{x}\,dx$$

```
[In] integrate(arcsinh(a*x)^n/x,x, algorithm="maxima")
```

```
[Out] integrate(arcsinh(a*x)^n/x, x)
```

Giac [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{arcsinh}(ax)^n}{x}\,dx=\int \frac{\operatorname{arsinh}\left(ax\right)^n}{x}\,dx$$

```
[In] integrate(arcsinh(a*x)^n/x,x, algorithm="giac")
```

```
[Out] integrate(arcsinh(a*x)^n/x, x)
```

Mupad [N/A]

Not integrable

Time = 2.58 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{arcsinh}(ax)^n}{x}\,dx=\int \frac{\operatorname{asinh}(a\,x)^n}{x}\,dx$$

```
[In] int(asinh(a*x)^n/x,x)
```

```
[Out] int(asinh(a*x)^n/x, x)
```

3.135 $\int \frac{\textbf{arcsinh}(ax)^n}{x^2} dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\operatorname{arcsinh}(ax)^n}{x^2} \, dx = \operatorname{Int}\left(\frac{\operatorname{arcsinh}(ax)^n}{x^2}, x\right)$$

```
[Out] Unintegrable(arcsinh(a*x)^n/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\operatorname{arcsinh}(ax)^n}{x^2} \, dx = \int \frac{\operatorname{arcsinh}(ax)^n}{x^2} \, dx$$

```
[In] Int[ArcSinh[a*x]^n/x^2,x]
```

```
[Out] Defer[Int][ArcSinh[a*x]^n/x^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{\operatorname{arcsinh}(ax)^n}{x^2} \, dx$$

Mathematica [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{arcsinh}(ax)^n}{x^2}\,dx = \int \frac{\operatorname{arcsinh}(ax)^n}{x^2}\,dx$$

```
[In] Integrate[ArcSinh[a*x]^n/x^2,x]
[Out] Integrate[ArcSinh[a*x]^n/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}\left(ax\right)^n}{x^2}dx$$

```
[In] int(arcsinh(a*x)^n/x^2,x)
[Out] int(arcsinh(a*x)^n/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{arcsinh}(ax)^n}{x^2}\,dx = \int \frac{\operatorname{arsinh}\left(ax\right)^n}{x^2}\,dx$$

```
[In] integrate(arcsinh(a*x)^n/x^2,x, algorithm="fricas")
[Out] integral(arcsinh(a*x)^n/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arcsinh}(ax)^n}{x^2}\,dx = \int \frac{\operatorname{asinh}^n\left(ax\right)}{x^2}\,dx$$

```
[In] integrate(asinh(a*x)**n/x**2,x)
[Out] Integral(asinh(a*x)**n/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{arcsinh}(ax)^n}{x^2}\,dx = \int \frac{\operatorname{arsinh}(ax)^n}{x^2}\,dx$$

```
[In] integrate(arcsinh(a*x)^n/x^2,x, algorithm="maxima")
[Out] integrate(arcsinh(a*x)^n/x^2, x)
```

Giac [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{arcsinh}(ax)^n}{x^2}\,dx = \int \frac{\operatorname{arsinh}(ax)^n}{x^2}\,dx$$

```
[In] integrate(arcsinh(a*x)^n/x^2,x, algorithm="giac")
[Out] integrate(arcsinh(a*x)^n/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.47 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{arcsinh}(ax)^n}{x^2}\,dx = \int \frac{\operatorname{asinh}(a\,x)^n}{x^2}\,dx$$

```
[In] int(asinh(a*x)^n/x^2,x)
[Out] int(asinh(a*x)^n/x^2, x)
```

3.136 $\int x^2\sqrt{a+b\text{arcsinh}(cx)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 213

$$\int x^2\sqrt{a+b\text{arcsinh}(cx)}\,dx=\frac{1}{3}x^3\sqrt{a+b\text{arcsinh}(cx)}-\frac{\sqrt{b}e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16c^3}+\frac{\sqrt{b}e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{48c^3}+\frac{\sqrt{b}e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16c^3}-\frac{\sqrt{b}e^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{48c^3}$$

```
[Out] 1/144*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*b^(1/2)*3^(1
/2)*Pi^(1/2)/c^3-1/144*erfi(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*b^(1/
2)*3^(1/2)*Pi^(1/2)/c^3/exp(3*a/b)-1/16*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/
2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c^3+1/16*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2)
)*b^(1/2)*Pi^(1/2)/c^3/exp(a/b)+1/3*x^3*(a+b*arcsinh(c*x))^(1/2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {5777, 5819, 3393, 3389, 2211, 2236, 2235}

$$\int x^2\sqrt{a+b\text{arcsinh}(cx)}\,dx = -\frac{\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16c^3} + \frac{\sqrt{\frac{\pi}{3}}\sqrt{b}e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{48c^3} + \frac{\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16c^3} - \frac{\sqrt{\frac{\pi}{3}}\sqrt{b}e^{-\frac{3a}{b}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{48c^3} + \frac{1}{3}x^3\sqrt{a+b\text{arcsinh}(cx)}$$

```
[In] Int[x^2*Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] (x^3*Sqrt[a + b*ArcSinh[c*x]])/3 - (Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b
*ArcSinh[c*x]]/Sqrt[b]])/(16*c^3) + (Sqrt[b]*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sq
rt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(48*c^3) + (Sqrt[b]*Sqrt[Pi]*Erfi
[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(16*c^3*E^(a/b)) - (Sqrt[b]*Sqrt[Pi/3]*
Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(48*c^3*E^((3*a)/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\sqrt{a+b\text{arcsinh}(cx)} - \frac{1}{6}(bc)\int \frac{x^3}{\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}\,dx \\
&= \frac{1}{3}x^3\sqrt{a+b\text{arcsinh}(cx)} + \frac{\text{Subst}\left(\int \frac{\sinh^3\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{6c^3} \\
&= \frac{1}{3}x^3\sqrt{a+b\text{arcsinh}(cx)} + \frac{i\text{Subst}\left(\int \left(-\frac{i\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{4\sqrt{x}} + \frac{3i\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{4\sqrt{x}}\right)dx, x, a+b\text{arcsinh}(cx)\right)}{6c^3} \\
&= \frac{1}{3}x^3\sqrt{a+b\text{arcsinh}(cx)} + \frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{24c^3} \\
&\quad - \frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{8c^3}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{3}x^3\sqrt{a+b\text{arcsinh}(cx)} + \frac{\text{Subst}\left(\int \frac{e^{-i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{48c^3} \\
&\quad - \frac{\text{Subst}\left(\int \frac{e^{i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{48c^3} \\
&\quad - \frac{\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16c^3} \\
&\quad + \frac{\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16c^3}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1}{3}x^3\sqrt{a+b\text{arcsinh}(cx)} + \frac{\text{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{24c^3} \\
&\quad - \frac{\text{Subst}\left(\int e^{-\frac{3a}{b}+\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{24c^3} \\
&\quad - \frac{\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{8c^3} \\
&\quad + \frac{\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{8c^3}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1}{3}x^3\sqrt{a+b\text{arcsinh}(cx)} - \frac{\sqrt{b}e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16c^3} \\
&\quad + \frac{\sqrt{b}e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{48c^3} + \frac{\sqrt{b}e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16c^3} \\
&\quad - \frac{\sqrt{b}e^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\,\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{48c^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.01

$$\int x^2\sqrt{a+b\text{arcsinh}(cx)}\,dx$$

$$= \frac{e^{-\frac{3a}{b}}\sqrt{a+b\text{arcsinh}(cx)}\,\Big(9e^{\frac{4a}{b}}\sqrt{-\frac{a+b\text{arcsinh}(cx)}{b}}\,\Gamma\big(\frac{3}{2}, \frac{a}{b}+\text{arcsinh}(cx)\big)+\sqrt{3}\sqrt{\frac{a}{b}+\text{arcsinh}(cx)}\,\Gamma\Big(\frac{3}{2}, -\frac{3(a+}{}}{72c^3\sqrt{-}}$$

```
[In] Integrate[x^2*Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] (Sqrt[a + b*ArcSinh[c*x]]*(9*E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Ga
mma[3/2, a/b + ArcSinh[c*x]] + Sqrt[3]*Sqrt[a/b + ArcSinh[c*x]]*Gamma[3/2,
(-3*(a + b*ArcSinh[c*x]))/b] - 9*E^((2*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma
[3/2, -((a + b*ArcSinh[c*x])/b)] - Sqrt[3]*E^((6*a)/b)*Sqrt[-((a + b*ArcSin
h[c*x])/b)]*Gamma[3/2, (3*(a + b*ArcSinh[c*x]))/b]))/(72*c^3*E^((3*a)/b)*Sq
rt[-((a + b*ArcSinh[c*x])^2/b^2)])
```

Maple [F]

$$\int x^2\sqrt{a+b\ \operatorname{arcsinh}\left(cx\right)}dx$$

```
[In] int(x^2*(a+b*arcsinh(c*x))^(1/2),x)
```

```
[Out] int(x^2*(a+b*arcsinh(c*x))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^2\sqrt{a+b\text{arcsinh}(cx)}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x^2\sqrt{a+b\text{arcsinh}(cx)}\,dx=\int x^2\sqrt{a+b\operatorname{asinh}\left(cx\right)}\,dx$$

```
[In] integrate(x**2*(a+b*asinh(c*x))**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a + b*asinh(c*x)), x)
```

Maxima [F]

$$\int x^2\sqrt{a+b\text{arcsinh}(cx)}\,dx=\int\sqrt{b\,\text{arsinh}\left(cx\right)+a}x^2\,dx$$

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*arcsinh(c*x) + a)*x^2, x)
```

Giac [F]

$$\int x^2\sqrt{a+b\text{arcsinh}(cx)}\,dx=\int\sqrt{b\,\text{arsinh}\left(cx\right)+a}x^2\,dx$$

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*arcsinh(c*x) + a)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{a+b\text{arcsinh}(cx)}\,dx=\int x^2\,\sqrt{a+b\,\text{asinh}\left(c\,x\right)}\,dx$$

```
[In] int(x^2*(a + b*asinh(c*x))^(1/2),x)
[Out] int(x^2*(a + b*asinh(c*x))^(1/2), x)
```

3.137 $\int x\sqrt{a+b\,\mathrm{arcsinh}(cx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 145

$$\int x\sqrt{a+b\,\mathrm{arcsinh}(cx)}\,dx=\frac{\sqrt{a+b\,\mathrm{arcsinh}(cx)}}{4c^2}+\frac{1}{2}x^2\sqrt{a+b\,\mathrm{arcsinh}(cx)}-\frac{\sqrt{b}e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\mathrm{erf}\left(\frac{\sqrt{2}\sqrt{a+b\,\mathrm{arcsinh}(cx)}}{\sqrt{b}}\right)}{16c^2}-\frac{\sqrt{b}e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\mathrm{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\,\mathrm{arcsinh}(cx)}}{\sqrt{b}}\right)}{16c^2}$$

```
[Out] -1/32*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*b^(1/2)*2^(1
/2)*Pi^(1/2)/c^2-1/32*erfi(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*b^(1/2
)*2^(1/2)*Pi^(1/2)/c^2/exp(2*a/b)+1/4*(a+b*arcsinh(c*x))^(1/2)/c^2+1/2*x^2*
(a+b*arcsinh(c*x))^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5777, 5819, 3393, 3388, 2211, 2236, 2235}

$$\int x\sqrt{a+b\,\mathrm{arcsinh}(cx)}\,dx=-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2a}{b}}\,\mathrm{erf}\left(\frac{\sqrt{2}\sqrt{a+b\,\mathrm{arcsinh}(cx)}}{\sqrt{b}}\right)}{16c^2}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}e^{-\frac{2a}{b}}\,\mathrm{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\,\mathrm{arcsinh}(cx)}}{\sqrt{b}}\right)}{16c^2}+\frac{\sqrt{a+b\,\mathrm{arcsinh}(cx)}}{4c^2}+\frac{1}{2}x^2\sqrt{a+b\,\mathrm{arcsinh}(cx)}$$

```
[In] Int[x*Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] Sqrt[a + b*ArcSinh[c*x]]/(4*c^2) + (x^2*Sqrt[a + b*ArcSinh[c*x]])/2 - (Sqrt
[b]*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])
/(16*c^2) - (Sqrt[b]*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqr
t[b]])/(16*c^2*E^((2*a)/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
```

```
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}x^2\sqrt{a+b\text{arcsinh}(cx)}-\frac{1}{4}(bc)\int\frac{x^2}{\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}\,dx\\
&= \frac{1}{2}x^2\sqrt{a+b\text{arcsinh}(cx)}-\frac{\text{Subst}\left(\int\frac{\sinh^2\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx,x,a+b\text{arcsinh}(cx)\right)}{4c^2}\\
&= \frac{1}{2}x^2\sqrt{a+b\text{arcsinh}(cx)}+\frac{\text{Subst}\left(\int\left(\frac{1}{2\sqrt{x}}-\frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{2\sqrt{x}}\right)\,dx,x,a+b\text{arcsinh}(cx)\right)}{4c^2}\\
&= \frac{\sqrt{a+b\text{arcsinh}(cx)}}{4c^2}+\frac{1}{2}x^2\sqrt{a+b\text{arcsinh}(cx)}-\frac{\text{Subst}\left(\int\frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{\sqrt{x}}\,dx,x,a+b\text{arcsinh}(cx)\right)}{8c^2}\\
&= \frac{\sqrt{a+b\text{arcsinh}(cx)}}{4c^2}+\frac{1}{2}x^2\sqrt{a+b\text{arcsinh}(cx)}\\
&\quad-\frac{\text{Subst}\left(\int\frac{e^{-i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx,x,a+b\text{arcsinh}(cx)\right)}{16c^2}\\
&\quad-\frac{\text{Subst}\left(\int\frac{e^{i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx,x,a+b\text{arcsinh}(cx)\right)}{16c^2}\\
&= \frac{\sqrt{a+b\text{arcsinh}(cx)}}{4c^2}+\frac{1}{2}x^2\sqrt{a+b\text{arcsinh}(cx)}\\
&\quad-\frac{\text{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx,x,\sqrt{a+b\text{arcsinh}(cx)}\right)}{8c^2}\\
&\quad-\frac{\text{Subst}\left(\int e^{-\frac{2a}{b}+\frac{2x^2}{b}}\,dx,x,\sqrt{a+b\text{arcsinh}(cx)}\right)}{8c^2}\\
&= \frac{\sqrt{a+b\text{arcsinh}(cx)}}{4c^2}+\frac{1}{2}x^2\sqrt{a+b\text{arcsinh}(cx)}\\
&\quad-\frac{\sqrt{b}e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16c^2}-\frac{\sqrt{b}e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16c^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.77

$$\int x\sqrt{a+b\text{arcsinh}(cx)}\,dx = \frac{e^{-\frac{2a}{b}}\left(-b\sqrt{-\frac{a+b\text{arcsinh}(cx)}{b}}\Gamma\left(\frac{3}{2},-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)+be^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\text{arcsinh}(cx)}\Gamma\left(\frac{3}{2},\frac{2(a+b\text{arcsinh}(cx))}{b}\right)\right)}{8\sqrt{2}c^2\sqrt{a+b\text{arcsinh}(cx)}}$$

```
[In] Integrate[x*Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] (-(b*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[3/2, (-2*(a + b*ArcSinh[c*x]))/b
]) + b*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[3/2, (2*(a + b*ArcSinh[c*
x]))/b])/(8*Sqrt[2]*c^2*E^((2*a)/b)*Sqrt[a + b*ArcSinh[c*x]])
```

Maple [F]

$$\int x\sqrt{a+b\ \text{arcsinh}\,(cx)}dx$$

```
[In] int(x*(a+b*arcsinh(c*x))^(1/2),x)
```

```
[Out] int(x*(a+b*arcsinh(c*x))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x\sqrt{a+b\text{arcsinh}(cx)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x\sqrt{a+b\mathrm{arcsinh}(cx)}\,dx=\int x\sqrt{a+b\,\mathrm{asinh}\,(cx)}\,dx$$

```
[In] integrate(x*(a+b*asinh(c*x))**(1/2),x)
[Out] Integral(x*sqrt(a + b*asinh(c*x)), x)
```

Maxima [F]

$$\int x\sqrt{a+b\mathrm{arcsinh}(cx)}\,dx=\int\sqrt{b\,\mathrm{arsinh}\,(cx)+a}x\,dx$$

```
[In] integrate(x*(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*arcsinh(c*x) + a)*x, x)
```

Giac [F]

$$\int x\sqrt{a+b\mathrm{arcsinh}(cx)}\,dx=\int\sqrt{b\,\mathrm{arsinh}\,(cx)+a}x\,dx$$

```
[In] integrate(x*(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*arcsinh(c*x) + a)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{a+b\mathrm{arcsinh}(cx)}\,dx=\int x\,\sqrt{a+b\,\mathrm{asinh}\,(c\,x)}\,dx$$

```
[In] int(x*(a + b*asinh(c*x))^(1/2),x)
[Out] int(x*(a + b*asinh(c*x))^(1/2), x)
```

3.138 $\int \sqrt{a+b\text{arcsinh}(cx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 102

$$\int \sqrt{a+b\text{arcsinh}(cx)}\,dx=x\sqrt{a+b\text{arcsinh}(cx)}+\frac{\sqrt{b}e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4c}-\frac{\sqrt{b}e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4c}$$

```
[Out] 1/4*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c-1/4*e
rfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c/exp(a/b)+x*(a+b*ar
csinh(c*x))^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5772, 5819, 3389, 2211, 2236, 2235}

$$\int \sqrt{a+b\text{arcsinh}(cx)}\,dx=\frac{\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4c}-\frac{\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4c}+x\sqrt{a+b\text{arcsinh}(cx)}$$

```
[In] Int[Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] x*Sqrt[a + b*ArcSinh[c*x]] + (Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSi
nh[c*x]]/Sqrt[b]])/(4*c) - (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/
Sqrt[b]])/(4*c*E^(a/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = x\sqrt{a + b\text{arcsinh}(cx)} - \frac{1}{2}(bc)\int \frac{x}{\sqrt{1+c^2x^2}\sqrt{a + b\text{arcsinh}(cx)}}\,dx$$

$$= x\sqrt{a+b\text{arcsinh}(cx)} + \frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{2c}$$

$$= x\sqrt{a+b\text{arcsinh}(cx)} + \frac{\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{4c} - \frac{\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{4c}$$

$$= x\sqrt{a+b\text{arcsinh}(cx)} + \frac{\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{2c} - \frac{\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{2c}$$

$$= x\sqrt{a+b\text{arcsinh}(cx)} + \frac{\sqrt{b}e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4c} - \frac{\sqrt{b}e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4c}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.99

$$\int \sqrt{a+b\text{arcsinh}(cx)}\,dx$$

$$= \frac{e^{-\frac{a}{b}}\sqrt{a+b\text{arcsinh}(cx)}\left(-\frac{e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\text{arcsinh}(cx)\right)}{\sqrt{\frac{a}{b}+\text{arcsinh}(cx)}} + \frac{\Gamma\left(\frac{3}{2},-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{-\frac{a+b\text{arcsinh}(cx)}{b}}}\right)}{2c}$$

```
[In] Integrate[Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] (Sqrt[a + b*ArcSinh[c*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSinh[c*x]])/S
qrt[a/b + ArcSinh[c*x]]) + Gamma[3/2, -((a + b*ArcSinh[c*x])/b)]/Sqrt[-((a
+ b*ArcSinh[c*x])/b)]))/(2*c*E^(a/b))
```

Maple [F]

$$\int \sqrt{a+b\ \mathrm{arcsinh}\left(cx\right)}dx$$

```
[In] int((a+b*arcsinh(c*x))^(1/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{a+b\mathrm{arcsinh}(cx)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \sqrt{a+b\mathrm{arcsinh}(cx)}\,dx = \int \sqrt{a+b\,\mathrm{asinh}\left(cx\right)}\,dx$$

```
[In] integrate((a+b*asinh(c*x))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*asinh(c*x)), x)
```

Maxima [F]

$$\int \sqrt{a+b\mathrm{arcsinh}(cx)}\,dx = \int \sqrt{b\,\mathrm{arsinh}\left(cx\right)+a}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*arcsinh(c*x) + a), x)
```

Giac [F]

$$\int \sqrt{a + b\text{arcsinh}(cx)}\,dx = \int \sqrt{b\,\text{arsinh}\left(cx\right) + a}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*arcsinh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a + b\text{arcsinh}(cx)}\,dx = \int \sqrt{a + b\,\text{asinh}\left(c\,x\right)}\,dx$$

```
[In] int((a + b*asinh(c*x))^(1/2),x)
[Out] int((a + b*asinh(c*x))^(1/2), x)
```

3.139 $\int x^2(a+b\text{arcsinh}(cx))^{3/2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 282

$$\begin{aligned}\int x^2(a+b\text{arcsinh}(cx))^{3/2}\,dx &= \frac{b\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}{3c^3}\\ &- \frac{bx^2\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}{6c} + \frac{1}{3}x^3(a+b\text{arcsinh}(cx))^{3/2}\\ &- \frac{3b^{3/2}e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32c^3} + \frac{b^{3/2}e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{96c^3}\\ &- \frac{3b^{3/2}e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32c^3} + \frac{b^{3/2}e^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{96c^3}\end{aligned}$$

```
[Out] 1/3*x^3*(a+b*arcsinh(c*x))^(3/2)+1/288*b^(3/2)*exp(3*a/b)*erf(3^(1/2)*(a+b*
arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/c^3+1/288*b^(3/2)*erfi(3^(1/2
)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/c^3/exp(3*a/b)-3/32*b^
(3/2)*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/c^3-3/32*b^(3
/2)*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/c^3/exp(a/b)+1/3*b*(c^2
*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^(1/2)/c^3-1/6*b*x^2*(c^2*x^2+1)^(1/2)*(a+b
*arcsinh(c*x))^(1/2)/c
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5777, 5812, 5798, 5774, 3388, 2211, 2236, 2235, 5780, 5556}

$$\begin{aligned}\int x^2(a+b\text{arcsinh}(cx))^{3/2}\,dx &= -\frac{3\sqrt{\pi}b^{3/2}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32c^3} \\ &+ \frac{\sqrt{\frac{\pi}{3}}b^{3/2}e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{96c^3} - \frac{3\sqrt{\pi}b^{3/2}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{32c^3} \\ &+ \frac{\sqrt{\frac{\pi}{3}}b^{3/2}e^{-\frac{3a}{b}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{96c^3} - \frac{bx^2\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{6c} \\ &+ \frac{b\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{3c^3} + \frac{1}{3}x^3(a+b\text{arcsinh}(cx))^{3/2}\end{aligned}$$

```
[In] Int[x^2*(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (b*Sqrt[1 + c^2*x^2]*Sqrt[a + b*ArcSinh[c*x]])/(3*c^3) - (b*x^2*Sqrt[1 + c^
2*x^2]*Sqrt[a + b*ArcSinh[c*x]])/(6*c) + (x^3*(a + b*ArcSinh[c*x])^(3/2))/3
 - (3*b^(3/2)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(32*c
^3) + (b^(3/2)*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]]
)/Sqrt[b]])/(96*c^3) - (3*b^(3/2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sq
rt[b]])/(32*c^3*E^(a/b)) + (b^(3/2)*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*Arc
Sinh[c*x]])/Sqrt[b]])/(96*c^3*E^((3*a)/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
```

```
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}x^3(a+b\operatorname{arcsinh}(cx))^{3/2}-\frac{1}{2}(bc)\int\frac{x^3\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{1+c^2x^2}}\,dx\\
&= -\frac{bx^2\sqrt{1+c^2x^2}\sqrt{a+b\operatorname{arcsinh}(cx)}}{6c}+\frac{1}{3}x^3(a+b\operatorname{arcsinh}(cx))^{3/2}\\
&\quad+\frac{1}{12}b^2\int\frac{x^2}{\sqrt{a+b\operatorname{arcsinh}(cx)}}\,dx+\frac{b\int\frac{x\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{1+c^2x^2}}\,dx}{3c}\\
&= \frac{b\sqrt{1+c^2x^2}\sqrt{a+b\operatorname{arcsinh}(cx)}}{3c^3}\\
&\quad-\frac{bx^2\sqrt{1+c^2x^2}\sqrt{a+b\operatorname{arcsinh}(cx)}}{6c}+\frac{1}{3}x^3(a+b\operatorname{arcsinh}(cx))^{3/2}\\
&\quad+\frac{b\operatorname{Subst}\left(\int\frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx,x,a+b\operatorname{arcsinh}(cx)\right)}{12c^3}-\frac{b^2\int\frac{1}{\sqrt{a+b\operatorname{arcsinh}(cx)}}\,dx}{6c^2}\\
&= \frac{b\sqrt{1+c^2x^2}\sqrt{a+b\operatorname{arcsinh}(cx)}}{3c^3}\\
&\quad-\frac{bx^2\sqrt{1+c^2x^2}\sqrt{a+b\operatorname{arcsinh}(cx)}}{6c}+\frac{1}{3}x^3(a+b\operatorname{arcsinh}(cx))^{3/2}\\
&\quad+\frac{b\operatorname{Subst}\left(\int\left(\frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{4\sqrt{x}}-\frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{4\sqrt{x}}\right)dx,x,a+b\operatorname{arcsinh}(cx)\right)}{12c^3}\\
&\quad-\frac{b\operatorname{Subst}\left(\int\frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx,x,a+b\operatorname{arcsinh}(cx)\right)}{6c^3}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{b\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}{3c^3} - \frac{bx^2\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}{6c} \\
&\quad + \frac{1}{3}x^3(a+b\text{arcsinh}(cx))^{3/2} + \frac{b\text{Subst}\left(\int \frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{48c^3} \\
&\quad - \frac{b\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{48c^3} \\
&\quad - \frac{b\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{12c^3} \\
&\quad - \frac{b\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{12c^3}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{b\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}{3c^3} - \frac{bx^2\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}{6c} \\
&\quad + \frac{1}{3}x^3(a+b\text{arcsinh}(cx))^{3/2} - \frac{b\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{96c^3} \\
&\quad - \frac{b\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{96c^3} \\
&\quad + \frac{b\text{Subst}\left(\int \frac{e^{-i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{96c^3} \\
&\quad + \frac{b\text{Subst}\left(\int \frac{e^{i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{96c^3} \\
&\quad - \frac{b\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{6c^3} \\
&\quad - \frac{b\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{6c^3}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{b\sqrt{1+c^2x^2}\sqrt{a+b\operatorname{arcsinh}(cx)}}{3c^3} - \frac{bx^2\sqrt{1+c^2x^2}\sqrt{a+b\operatorname{arcsinh}(cx)}}{6c} \\
&\quad + \frac{1}{3}x^3(a+b\operatorname{arcsinh}(cx))^{3/2} - \frac{b^{3/2}e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{12c^3} \\
&\quad - \frac{b^{3/2}e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{12c^3} \\
&\quad + \frac{b\text{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{48c^3} \\
&\quad - \frac{b\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{48c^3} \\
&\quad - \frac{b\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{48c^3} \\
&\quad + \frac{b\text{Subst}\left(\int e^{-\frac{3a}{b}+\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{48c^3} \\
&= \frac{b\sqrt{1+c^2x^2}\sqrt{a+b\operatorname{arcsinh}(cx)}}{3c^3} - \frac{bx^2\sqrt{1+c^2x^2}\sqrt{a+b\operatorname{arcsinh}(cx)}}{6c} \\
&\quad + \frac{1}{3}x^3(a+b\operatorname{arcsinh}(cx))^{3/2} - \frac{3b^{3/2}e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{32c^3} \\
&\quad + \frac{b^{3/2}e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{96c^3} - \frac{3b^{3/2}e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{32c^3} \\
&\quad + \frac{b^{3/2}e^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{96c^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.76

$$
\int x^2(a+b\operatorname{arcsinh}(cx))^{3/2}\,dx =
$$

$$
-\frac{be^{-\frac{3a}{b}}\sqrt{a+b\operatorname{arcsinh}(cx)}\left(-27e^{\frac{4a}{b}}\sqrt{-\frac{a+b\operatorname{arcsinh}(cx)}{b}}\Gamma\left(\frac{5}{2},\frac{a}{b}+\operatorname{arcsinh}(cx)\right)+\sqrt{3}\sqrt{\frac{a}{b}+\operatorname{arcsinh}(cx)}\Gamma\left(\frac{5}{2},-\right.\right.}{216c^3}
$$

```
[In] Integrate[x^2*(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] -1/216*(b*Sqrt[a + b*ArcSinh[c*x]]*(-27*E^((4*a)/b)*Sqrt[-((a + b*ArcSinh[c
*x])/b)]*Gamma[5/2, a/b + ArcSinh[c*x]] + Sqrt[3]*Sqrt[a/b + ArcSinh[c*x]]*
Gamma[5/2, (-3*(a + b*ArcSinh[c*x]))/b] - 27*E^((2*a)/b)*Sqrt[a/b + ArcSinh
[c*x]]*Gamma[5/2, -((a + b*ArcSinh[c*x])/b)] + Sqrt[3]*E^((6*a)/b)*Sqrt[-((
a + b*ArcSinh[c*x])/b)]*Gamma[5/2, (3*(a + b*ArcSinh[c*x]))/b]))/(c^3*E^((3
*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])^2/b^2)])
```

Maple [F]

$$\int x^2(a+b\ \text{arcsinh}\,(cx))^{\frac{3}{2}}\,dx$$

```
[In] int(x^2*(a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int(x^2*(a+b*arcsinh(c*x))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^2(a+b\text{arcsinh}(cx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x^2(a+b\text{arcsinh}(cx))^{3/2}\,dx = \int x^2(a+b\,\text{asinh}\,(cx))^{\frac{3}{2}}\,dx$$

```
[In] integrate(x**2*(a+b*asinh(c*x))**(3/2),x)
```

```
[Out] Integral(x**2*(a + b*asinh(c*x))**(3/2), x)
```

Maxima [F]

$$\int x^2(a+b\operatorname{arcsinh}(cx))^{3/2}\,dx=\int(b\operatorname{arsinh}(cx)+a)^{\frac{3}{2}}x^2\,dx$$

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^(3/2)*x^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int x^2(a+b\operatorname{arcsinh}(cx))^{3/2}\,dx=\text{Exception raised: RuntimeError}$$

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a+b\operatorname{arcsinh}(cx))^{3/2}\,dx=\int x^2\,(a+b\operatorname{asinh}(c\,x))^{3/2}\,dx$$

```
[In] int(x^2*(a + b*asinh(c*x))^(3/2),x)
```

```
[Out] int(x^2*(a + b*asinh(c*x))^(3/2), x)
```

3.140 $\int x(a+b\operatorname{arcsinh}(cx))^{3/2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 179

$$\begin{aligned}\int x(a+b\operatorname{arcsinh}(cx))^{3/2}\,dx &= -\frac{3bx\sqrt{1+c^2x^2}\sqrt{a+b\operatorname{arcsinh}(cx)}}{8c}\\ &\quad+\frac{(a+b\operatorname{arcsinh}(cx))^{3/2}}{4c^2}+\frac{1}{2}x^2(a+b\operatorname{arcsinh}(cx))^{3/2}\\ &\quad-\frac{3b^{3/2}e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{64c^2}+\frac{3b^{3/2}e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{64c^2}\end{aligned}$$

```
[Out] 1/4*(a+b*arcsinh(c*x))^(3/2)/c^2+1/2*x^2*(a+b*arcsinh(c*x))^(3/2)-3/128*b^(
3/2)*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1
/2)/c^2+3/128*b^(3/2)*erfi(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(1/2
)*Pi^(1/2)/c^2/exp(2*a/b)-3/8*b*x*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^(1/2
)/c
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {5777, 5812, 5783, 5780, 5556, 12, 3389, 2211, 2236, 2235}

$$\begin{aligned}\int x(a+b\operatorname{arcsinh}(cx))^{3/2}\,dx &= -\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}e^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{64c^2}\\ &\quad+\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}e^{-\frac{2a}{b}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{64c^2}-\frac{3bx\sqrt{c^2x^2+1}\sqrt{a+b\operatorname{arcsinh}(cx)}}{8c}\\ &\quad+\frac{(a+b\operatorname{arcsinh}(cx))^{3/2}}{4c^2}+\frac{1}{2}x^2(a+b\operatorname{arcsinh}(cx))^{3/2}\end{aligned}$$

```
[In] Int[x*(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (-3*b*x*Sqrt[1 + c^2*x^2]*Sqrt[a + b*ArcSinh[c*x]])/(8*c) + (a + b*ArcSinh[
c*x])^(3/2)/(4*c^2) + (x^2*(a + b*ArcSinh[c*x])^(3/2))/2 - (3*b^(3/2)*E^((2
*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(64*c^2)
 + (3*b^(3/2)*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/
(64*c^2*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
```

```
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}x^2(a+b\text{arcsinh}(cx))^{3/2} - \frac{1}{4}(3bc)\int \frac{x^2\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{1+c^2x^2}}\,dx \\
&= -\frac{3bx\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}{8c} + \frac{1}{2}x^2(a+b\text{arcsinh}(cx))^{3/2} \\
&\quad + \frac{1}{16}\left(3b^2\right)\int \frac{x}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx + \frac{(3b)\int \frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{1+c^2x^2}}\,dx}{8c} \\
&= -\frac{3bx\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}{8c} \\
&\quad + \frac{(a+b\text{arcsinh}(cx))^{3/2}}{4c^2} + \frac{1}{2}x^2(a+b\text{arcsinh}(cx))^{3/2} \\
&\quad - \frac{(3b)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{16c^2}
\end{aligned}
$$

$$= -\frac{3bx\sqrt{1+c^2x^2}\sqrt{a+b\operatorname{arcsinh}(cx)}}{8c} + \frac{(a+b\operatorname{arcsinh}(cx))^{3/2}}{4c^2} + \frac{1}{2}x^2(a+b\operatorname{arcsinh}(cx))^{3/2} - \frac{(3b)\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{2\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{16c^2}$$

$$= -\frac{3bx\sqrt{1+c^2x^2}\sqrt{a+b\operatorname{arcsinh}(cx)}}{8c} + \frac{(a+b\operatorname{arcsinh}(cx))^{3/2}}{4c^2} + \frac{1}{2}x^2(a+b\operatorname{arcsinh}(cx))^{3/2} - \frac{(3b)\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{32c^2}$$

$$= -\frac{3bx\sqrt{1+c^2x^2}\sqrt{a+b\operatorname{arcsinh}(cx)}}{8c} + \frac{(a+b\operatorname{arcsinh}(cx))^{3/2}}{4c^2} + \frac{1}{2}x^2(a+b\operatorname{arcsinh}(cx))^{3/2} - \frac{(3b)\text{Subst}\left(\int \frac{e^{-i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{64c^2} + \frac{(3b)\text{Subst}\left(\int \frac{e^{i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{64c^2}$$

$$= -\frac{3bx\sqrt{1+c^2x^2}\sqrt{a+b\operatorname{arcsinh}(cx)}}{8c} + \frac{(a+b\operatorname{arcsinh}(cx))^{3/2}}{4c^2} + \frac{1}{2}x^2(a+b\operatorname{arcsinh}(cx))^{3/2} - \frac{(3b)\text{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{32c^2} + \frac{(3b)\text{Subst}\left(\int e^{-\frac{2a}{b}+\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{32c^2}$$

$$= -\frac{3bx\sqrt{1+c^2x^2}\sqrt{a+b\operatorname{arcsinh}(cx)}}{8c} + \frac{(a+b\operatorname{arcsinh}(cx))^{3/2}}{4c^2} + \frac{1}{2}x^2(a+b\operatorname{arcsinh}(cx))^{3/2} - \frac{3b^{3/2}e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{64c^2} + \frac{3b^{3/2}e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{64c^2}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.64

$$\int x(a+b\operatorname{arcsinh}(cx))^{3/2}\,dx = \frac{e^{-\frac{2a}{b}}\left(b^2\sqrt{-\frac{a+b\operatorname{arcsinh}(cx)}{b}}\Gamma\left(\frac{5}{2},-\frac{2(a+b\operatorname{arcsinh}(cx))}{b}\right)+b^2e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\operatorname{arcsinh}(cx)}\Gamma\left(\frac{5}{2},\right.\right.}{16\sqrt{2}c^2\sqrt{a+b\operatorname{arcsinh}(cx)}}$$

```
[In] Integrate[x*(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (b^2*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[5/2, (-2*(a + b*ArcSinh[c*x]))/b
] + b^2*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[5/2, (2*(a + b*ArcSinh[c
*x]))/b])/(16*Sqrt[2]*c^2*E^((2*a)/b)*Sqrt[a + b*ArcSinh[c*x]])
```

Maple [F]

$$\int x(a + b\,\operatorname{arcsinh}\left(cx\right))^{\frac{3}{2}}\,dx$$

```
[In] int(x*(a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int(x*(a+b*arcsinh(c*x))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x(a + b\operatorname{arcsinh}(cx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x(a + b\operatorname{arcsinh}(cx))^{3/2}\,dx = \int x\left(a + b\,\operatorname{asinh}\left(cx\right)\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x*(a+b*asinh(c*x))**(3/2),x)
```

```
[Out] Integral(x*(a + b*asinh(c*x))**(3/2), x)
```

Maxima [F]

$$\int x(a + b\operatorname{arcsinh}(cx))^{3/2}\,dx = \int (b\,\operatorname{arsinh}\left(cx\right) + a)^{\frac{3}{2}}x\,dx$$

```
[In] integrate(x*(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^(3/2)*x, x)
```

Giac [F]

$$\int x(a+b\text{arcsinh}(cx))^{3/2}\,dx=\int (b\,\text{arsinh}\,(cx)+a)^{\frac{3}{2}}x\,dx$$

```
[In] integrate(x*(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^(3/2)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(a+b\text{arcsinh}(cx))^{3/2}\,dx=\int x\,(a+b\,\text{asinh}(c\,x))^{3/2}\,dx$$

```
[In] int(x*(a + b*asinh(c*x))^(3/2),x)
[Out] int(x*(a + b*asinh(c*x))^(3/2), x)
```

3.141 $\int (a+b\text{arcsinh}(cx))^{3/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 135

$$\int (a+b\text{arcsinh}(cx))^{3/2}\,dx=-\frac{3b\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}{2c}+x(a+b\text{arcsinh}(cx))^{3/2}+\frac{3b^{3/2}e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8c}+\frac{3b^{3/2}e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8c}$$

```
[Out] x*(a+b*arcsinh(c*x))^(3/2)+3/8*b^(3/2)*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2
)/b^(1/2))*Pi^(1/2)/c+3/8*b^(3/2)*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi
^(1/2)/c/exp(a/b)-3/2*b*(c^2*x^2+1)^(1/2)*(a+b*arcsinh(c*x))^(1/2)/c
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5772, 5798, 5774, 3388, 2211, 2236, 2235}

$$\int (a+b\text{arcsinh}(cx))^{3/2}\,dx=\frac{3\sqrt{\pi}b^{3/2}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8c}+\frac{3\sqrt{\pi}b^{3/2}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8c}-\frac{3b\sqrt{c^2x^2+1}\sqrt{a+b\text{arcsinh}(cx)}}{2c}+x(a+b\text{arcsinh}(cx))^{3/2}$$

```
[In] Int[(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (-3*b*Sqrt[1 + c^2*x^2]*Sqrt[a + b*ArcSinh[c*x]])/(2*c) + x*(a + b*ArcSinh[
c*x])^(3/2) + (3*b^(3/2)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt
[b]])/(8*c) + (3*b^(3/2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(
8*c*E^(a/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
```

```
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= x(a+b\text{arcsinh}(cx))^{3/2}-\frac{1}{2}(3bc)\int\frac{x\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{1+c^2x^2}}\,dx\\
&=-\frac{3b\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}{2c}+x(a+b\text{arcsinh}(cx))^{3/2}\\
&\quad+\frac{1}{4}\left(3b^2\right)\int\frac{1}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx\\
&=-\frac{3b\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}{2c}+x(a+b\text{arcsinh}(cx))^{3/2}\\
&\quad+\frac{(3b)\text{Subst}\left(\int\frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx,x,a+b\text{arcsinh}(cx)\right)}{4c}\\
&=-\frac{3b\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}{2c}+x(a+b\text{arcsinh}(cx))^{3/2}\\
&\quad+\frac{(3b)\text{Subst}\left(\int\frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx,x,a+b\text{arcsinh}(cx)\right)}{8c}\\
&\quad+\frac{(3b)\text{Subst}\left(\int\frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx,x,a+b\text{arcsinh}(cx)\right)}{8c}\\
&=-\frac{3b\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}{2c}+x(a+b\text{arcsinh}(cx))^{3/2}\\
&\quad+\frac{(3b)\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx,x,\sqrt{a+b\text{arcsinh}(cx)}\right)}{4c}\\
&\quad+\frac{(3b)\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+b\text{arcsinh}(cx)}\right)}{4c}\\
&=-\frac{3b\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}{2c}+x(a+b\text{arcsinh}(cx))^{3/2}\\
&\quad+\frac{3b^{3/2}e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8c}+\frac{3b^{3/2}e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8c}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.86

$$\int (a+b\text{arcsinh}(cx))^{3/2}\,dx=\frac{ae^{-\frac{a}{b}}\sqrt{a+b\text{arcsinh}(cx)}\left(-\frac{e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\text{arcsinh}(cx)\right)}{\sqrt{\frac{a}{b}+\text{arcsinh}(cx)}}+\frac{\Gamma\left(\frac{3}{2},-\frac{a+b\text{arcsinh}(cx)}{b}\right)}{\sqrt{-\frac{a+b\text{arcsinh}(cx)}{b}}}\right)}{2c}+\frac{\sqrt{b}\left(4\sqrt{b}\sqrt{a+b\text{arcsinh}(cx)}\left(-3\sqrt{1+c^2x^2}+2cx\text{arcsinh}(cx)\right)+(2a+3b)\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)\left(\cosh\right.\right.}{8c}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (a*Sqrt[a + b*ArcSinh[c*x]]*(-((E^((2*a)/b)*Gamma[3/2, a/b + ArcSinh[c*x]])
/Sqrt[a/b + ArcSinh[c*x]]) + Gamma[3/2, -((a + b*ArcSinh[c*x])/b)]/Sqrt[-((
a + b*ArcSinh[c*x])/b)]))/(2*c*E^(a/b)) + (Sqrt[b]*(4*Sqrt[b]*Sqrt[a + b*Ar
cSinh[c*x]]*(-3*Sqrt[1 + c^2*x^2] + 2*c*x*ArcSinh[c*x]) + (2*a + 3*b)*Sqrt[
Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]) + (-2*a
+ 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b
])))/(8*c)
```

Maple [F]

$$\int (a+b\,\text{arcsinh}\,(cx))^{\frac{3}{2}}\,dx$$

```
[In] int((a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (a+b\text{arcsinh}(cx))^{3/2}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int (a+b\text{arcsinh}(cx))^{3/2}\,dx=\int \left(a+b\operatorname{asinh}\left(cx\right)\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*asinh(c*x))**(3/2),x)
[Out] Integral((a + b*asinh(c*x))**(3/2), x)
```

Maxima [F]

$$\int (a+b\text{arcsinh}(cx))^{3/2}\,dx=\int (b\,\text{arsinh}\left(cx\right)+a)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
[Out] integrate((b*arcsinh(c*x) + a)^(3/2), x)
```

Giac [F]

$$\int (a+b\text{arcsinh}(cx))^{3/2}\,dx=\int (b\,\text{arsinh}\left(cx\right)+a)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\text{arcsinh}(cx))^{3/2}\,dx=\int (a+b\,\text{asinh}(c\,x))^{3/2}\,dx$$

```
[In] int((a + b*asinh(c*x))^(3/2),x)
[Out] int((a + b*asinh(c*x))^(3/2), x)
```

3.142 $\int x^2(a+b\text{arcsinh}(cx))^{5/2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 327

$$\int x^2(a+b\text{arcsinh}(cx))^{5/2}\,dx = -\frac{5b^2x\sqrt{a+b\text{arcsinh}(cx)}}{6c^2}+\frac{5}{36}b^2x^3\sqrt{a+b\text{arcsinh}(cx)}$$

$$+\frac{5b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^{3/2}}{9c^3}-\frac{5bx^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^{3/2}}{18c}$$

$$+\frac{1}{3}x^3(a+b\text{arcsinh}(cx))^{5/2}-\frac{15b^{5/2}e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{64c^3}+\frac{5b^{5/2}e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{576c^3}+ 15b$$

```
[Out] 1/3*x^3*(a+b*arcsinh(c*x))^(5/2)+5/1728*b^(5/2)*exp(3*a/b)*erf(3^(1/2)*(a+b
*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/c^3-5/1728*b^(5/2)*erfi(3^(1
/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/c^3/exp(3*a/b)-15/64
*b^(5/2)*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/c^3+15/64*
b^(5/2)*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/c^3/exp(a/b)+5/9*b*
(a+b*arcsinh(c*x))^(3/2)*(c^2*x^2+1)^(1/2)/c^3-5/18*b*x^2*(a+b*arcsinh(c*x)
)^(3/2)*(c^2*x^2+1)^(1/2)/c-5/6*b^2*x*(a+b*arcsinh(c*x))^(1/2)/c^2+5/36*b^2
*x^3*(a+b*arcsinh(c*x))^(1/2)
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules

used = {5777, 5812, 5798, 5772, 5819, 3389, 2211, 2236, 2235, 3393}

$$\begin{aligned}\int x^2(a+b\operatorname{arcsinh}(cx))^{5/2}\,dx = &-\frac{15\sqrt{\pi}b^{5/2}e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{64c^3}\\ &+\frac{5\sqrt{\frac{\pi}{3}}b^{5/2}e^{\frac{3a}{b}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{576c^3}+\frac{15\sqrt{\pi}b^{5/2}e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{64c^3}\\ &-\frac{5\sqrt{\frac{\pi}{3}}b^{5/2}e^{-\frac{3a}{b}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{576c^3}-\frac{5b^2x\sqrt{a+b\operatorname{arcsinh}(cx)}}{6c^2}\\ &+\frac{5}{36}b^2x^3\sqrt{a+b\operatorname{arcsinh}(cx)}-\frac{5bx^2\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))^{3/2}}{18c}\\ &+\frac{5b\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))^{3/2}}{9c^3}+\frac{1}{3}x^3(a+b\operatorname{arcsinh}(cx))^{5/2}\end{aligned}$$

```
[In] Int[x^2*(a + b*ArcSinh[c*x])^(5/2),x]
```

```
[Out] (-5*b^2*x*Sqrt[a + b*ArcSinh[c*x]])/(6*c^2) + (5*b^2*x^3*Sqrt[a + b*ArcSinh
[c*x]])/36 + (5*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(3/2))/(9*c^3) - (
5*b*x^2*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(3/2))/(18*c) + (x^3*(a + b*
ArcSinh[c*x])^(5/2))/3 - (15*b^(5/2)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSin
h[c*x]]/Sqrt[b]])/(64*c^3) + (5*b^(5/2)*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]
*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(576*c^3) + (15*b^(5/2)*Sqrt[Pi]*Erfi[
Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(64*c^3*E^(a/b)) - (5*b^(5/2)*Sqrt[Pi/3]
*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(576*c^3*E^((3*a)/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
```

```
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}x^3(a+b\text{arcsinh}(cx))^{5/2} - \frac{1}{6}(5bc)\int \frac{x^3(a+b\text{arcsinh}(cx))^{3/2}}{\sqrt{1+c^2x^2}}\,dx$$

$$= -\frac{5bx^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^{3/2}}{18c} + \frac{1}{3}x^3(a+b\text{arcsinh}(cx))^{5/2} + \frac{1}{12}\left(5b^2\right)\int x^2\sqrt{a+b\text{arcsinh}(cx)}\,dx + \frac{(5b)\int \frac{x(a+b\text{arcsinh}(cx))^{3/2}}{\sqrt{1+c^2x^2}}\,dx}{9c}$$

$$= \frac{5}{36}b^2x^3\sqrt{a+b\text{arcsinh}(cx)} + \frac{5b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^{3/2}}{9c^3} - \frac{5bx^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^{3/2}}{18c} + \frac{1}{3}x^3(a+b\text{arcsinh}(cx))^{5/2} - \frac{\left(5b^2\right)\int \sqrt{a+b\text{arcsinh}(cx)}\,dx}{6c^2} - \frac{1}{72}\left(5b^3c\right)\int \frac{x^3}{\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}\,dx$$

$$= -\frac{5b^2x\sqrt{a+b\text{arcsinh}(cx)}}{6c^2} + \frac{5}{36}b^2x^3\sqrt{a+b\text{arcsinh}(cx)} + \frac{5b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^{3/2}}{9c^3} - \frac{5bx^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^{3/2}}{18c} + \frac{1}{3}x^3(a+b\text{arcsinh}(cx))^{5/2} + \frac{\left(5b^2\right)\text{Subst}\left(\int \frac{\sinh^3\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{72c^3} + \frac{\left(5b^3\right)\int \frac{x}{\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}\,dx}{12c}$$

$$= -\frac{5b^2x\sqrt{a+b\text{arcsinh}(cx)}}{6c^2} + \frac{5}{36}b^2x^3\sqrt{a+b\text{arcsinh}(cx)} + \frac{5b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^{3/2}}{9c^3} - \frac{5bx^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^{3/2}}{18c} + \frac{1}{3}x^3(a+b\text{arcsinh}(cx))^{5/2} + \frac{\left(5ib^2\right)\text{Subst}\left(\int \left(-\frac{i\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{4\sqrt{x}} + \frac{3i\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{4\sqrt{x}}\right)dx, x, a+b\text{arcsinh}(cx)\right)}{72c^3} - \frac{\left(5b^2\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{12c^3}$$

$$\begin{aligned}
&= -\frac{5b^2x\sqrt{a+b\text{arcsinh}(cx)}}{6c^2} + \frac{5}{36}b^2x^3\sqrt{a+b\text{arcsinh}(cx)} \\
&\quad + \frac{5b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^{3/2}}{9c^3} - \frac{5bx^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^{3/2}}{18c} \\
&\quad + \frac{1}{3}x^3(a+b\text{arcsinh}(cx))^{5/2} + \frac{(5b^2)\,\text{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{288c^3} \\
&\quad - \frac{(5b^2)\,\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{96c^3} \\
&\quad - \frac{(5b^2)\,\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{24c^3} \\
&\quad + \frac{(5b^2)\,\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{24c^3}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{5b^2x\sqrt{a+b\text{arcsinh}(cx)}}{6c^2} + \frac{5}{36}b^2x^3\sqrt{a+b\text{arcsinh}(cx)} \\
&\quad + \frac{5b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^{3/2}}{9c^3} - \frac{5bx^2\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^{3/2}}{18c} \\
&\quad + \frac{1}{3}x^3(a+b\text{arcsinh}(cx))^{5/2} + \frac{(5b^2)\,\text{Subst}\left(\int \frac{e^{-i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{576c^3} \\
&\quad - \frac{(5b^2)\,\text{Subst}\left(\int \frac{e^{i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{576c^3} \\
&\quad - \frac{(5b^2)\,\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{192c^3} \\
&\quad + \frac{(5b^2)\,\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{192c^3} \\
&\quad - \frac{(5b^2)\,\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{12c^3} \\
&\quad + \frac{(5b^2)\,\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{12c^3}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{5b^2x\sqrt{a+b\operatorname{arcsinh}(cx)}}{6c^2}+\frac{5}{36}b^2x^3\sqrt{a+b\operatorname{arcsinh}(cx)} \\
&\quad+\frac{5b\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^{3/2}}{9c^3}-\frac{5bx^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^{3/2}}{18c} \\
&\quad+\frac{1}{3}x^3(a+b\operatorname{arcsinh}(cx))^{5/2}-\frac{5b^{5/2}e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{24c^3} \\
&\quad+\frac{5b^{5/2}e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{24c^3} \\
&\quad+\frac{(5b^2)\operatorname{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx,x,\sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{288c^3} \\
&\quad-\frac{(5b^2)\operatorname{Subst}\left(\int e^{-\frac{3a}{b}+\frac{3x^2}{b}}\,dx,x,\sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{288c^3} \\
&\quad-\frac{(5b^2)\operatorname{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx,x,\sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{96c^3} \\
&\quad+\frac{(5b^2)\operatorname{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{96c^3}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{5b^2x\sqrt{a+b\operatorname{arcsinh}(cx)}}{6c^2}+\frac{5}{36}b^2x^3\sqrt{a+b\operatorname{arcsinh}(cx)} \\
&\quad+\frac{5b\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^{3/2}}{9c^3}-\frac{5bx^2\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^{3/2}}{18c} \\
&\quad+\frac{1}{3}x^3(a+b\operatorname{arcsinh}(cx))^{5/2}-\frac{15b^{5/2}e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{64c^3} \\
&\quad+\frac{5b^{5/2}e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{576c^3}+\frac{15b^{5/2}e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{64c^3} \\
&\quad-\frac{5b^{5/2}e^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{576c^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.61

$$\int x^2(a+b\text{arcsinh}(cx))^{5/2}\,dx =$$
$$-\frac{b^3e^{-\frac{3a}{b}}\left(-81e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\text{arcsinh}(cx)}\Gamma\left(\frac{7}{2},\frac{a}{b}+\text{arcsinh}(cx)\right)+\sqrt{3}\sqrt{-\frac{a+b\text{arcsinh}(cx)}{b}}\Gamma\left(\frac{7}{2},-\frac{3(a+b\text{arcsinh}(cx))}{b}\right)\right.}{648c^3\sqrt{a+ba}}$$

```
[In] Integrate[x^2*(a + b*ArcSinh[c*x])^(5/2),x]
```

```
[Out] -1/648*(b^3*(-81*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[7/2, a/b + ArcS
inh[c*x]] + Sqrt[3]*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[7/2, (-3*(a + b*A
rcSinh[c*x]))/b] - 81*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[7/2
, -((a + b*ArcSinh[c*x])/b)] + Sqrt[3]*E^((6*a)/b)*Sqrt[a/b + ArcSinh[c*x]]
*Gamma[7/2, (3*(a + b*ArcSinh[c*x]))/b]))/(c^3*E^((3*a)/b)*Sqrt[a + b*ArcSi
nh[c*x]])
```

Maple [F]

$$\int x^2(a+b\,\text{arcsinh}\,(cx))^{\frac{5}{2}}\,dx$$

```
[In] int(x^2*(a+b*arcsinh(c*x))^(5/2),x)
```

```
[Out] int(x^2*(a+b*arcsinh(c*x))^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^2(a+b\text{arcsinh}(cx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x^2(a+b\text{arcsinh}(cx))^{5/2}\,dx=\int x^2(a+b\operatorname{asinh}{(cx)})^{\frac{5}{2}}\,dx$$

```
[In] integrate(x**2*(a+b*asinh(c*x))**(5/2),x)
```

```
[Out] Integral(x**2*(a + b*asinh(c*x))**(5/2), x)
```

Maxima [F]

$$\int x^2(a+b\text{arcsinh}(cx))^{5/2}\,dx=\int (b\,\text{arsinh}\,(cx)+a)^{\frac{5}{2}}x^2\,dx$$

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^(5/2)*x^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int x^2(a+b\text{arcsinh}(cx))^{5/2}\,dx=\text{Exception raised: RuntimeError}$$

```
[In] integrate(x^2*(a+b*arcsinh(c*x))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a+b\text{arcsinh}(cx))^{5/2}\,dx=\int x^2\,{\left(a+b\,\mathrm{asinh}(c\,x)\right)}^{5/2}\,dx$$

```
[In] int(x^2*(a + b*asinh(c*x))^(5/2),x)
```

```
[Out] int(x^2*(a + b*asinh(c*x))^(5/2), x)
```

3.143 $\int x(a+b\text{arcsinh}(cx))^{5/2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 223

$$\int x(a+b\text{arcsinh}(cx))^{5/2}\,dx=\frac{15b^2\sqrt{a+b\text{arcsinh}(cx)}}{64c^2}+\frac{15}{32}b^2x^2\sqrt{a+b\text{arcsinh}(cx)}$$
$$-\frac{5bx\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^{3/2}}{8c}+\frac{(a+b\text{arcsinh}(cx))^{5/2}}{4c^2}$$
$$+\frac{1}{2}x^2(a+b\text{arcsinh}(cx))^{5/2}-\frac{15b^{5/2}e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{256c^2}-\frac{15b^{5/2}e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{256c^2}$$

```
[Out] 1/4*(a+b*arcsinh(c*x))^(5/2)/c^2+1/2*x^2*(a+b*arcsinh(c*x))^(5/2)-15/512*b^
(5/2)*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(1/2)*Pi^(
1/2)/c^2-15/512*b^(5/2)*erfi(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(1
/2)*Pi^(1/2)/c^2/exp(2*a/b)-5/8*b*x*(a+b*arcsinh(c*x))^(3/2)*(c^2*x^2+1)^(1
/2)/c+15/64*b^2*(a+b*arcsinh(c*x))^(1/2)/c^2+15/32*b^2*x^2*(a+b*arcsinh(c*x
))^(1/2)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used

$= \{5777, 5812, 5783, 5819, 3393, 3388, 2211, 2236, 2235\}$

$$\int x(a+b\operatorname{arcsinh}(cx))^{5/2}\,dx = -\frac{15\sqrt{\frac{\pi}{2}}b^{5/2}e^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{256c^2} - \frac{15\sqrt{\frac{\pi}{2}}b^{5/2}e^{-\frac{2a}{b}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{256c^2} + \frac{15b^2\sqrt{a+b\operatorname{arcsinh}(cx)}}{64c^2} + \frac{15}{32}b^2x^2\sqrt{a+b\operatorname{arcsinh}(cx)} - \frac{5bx\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))^{3/2}}{8c} + \frac{(a+b\operatorname{arcsinh}(cx))^{5/2}}{4c^2} + \frac{1}{2}x^2(a+b\operatorname{arcsinh}(cx))^{5/2}$$

```
[In] Int[x*(a + b*ArcSinh[c*x])^(5/2),x]
```

```
[Out] (15*b^2*Sqrt[a + b*ArcSinh[c*x]])/(64*c^2) + (15*b^2*x^2*Sqrt[a + b*ArcSinh
[c*x]])/32 - (5*b*x*Sqrt[1 + c^2*x^2]*(a + b*ArcSinh[c*x])^(3/2))/(8*c) + (
a + b*ArcSinh[c*x])^(5/2)/(4*c^2) + (x^2*(a + b*ArcSinh[c*x])^(5/2))/2 - (1
5*b^(5/2)*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqr
t[b]])/(256*c^2) - (15*b^(5/2)*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[
c*x]])/Sqrt[b]])/(256*c^2*E^((2*a)/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5777

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^(m + 1)*((a + b*ArcSinh[c*x])^n/(m + 1)), x] - Dist[b*c*(n/(m + 1)), Int[
x^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5812

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a
+ b*ArcSinh[c*x])^n/(e*(m + 2*p + 1))), x] + (-Dist[f^2*((m - 1)/(c^2*(m +
2*p + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcSinh[c*x])^n, x], x]
- Dist[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p], Int[(
f*x)^(m - 1)*(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && IGtQ[m,
 1] && NeQ[m + 2*p + 1, 0]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{2}x^2(a+b\text{arcsinh}(cx))^{5/2} - \frac{1}{4}(5bc)\int \frac{x^2(a+b\text{arcsinh}(cx))^{3/2}}{\sqrt{1+c^2x^2}}\,dx$$

$$= -\frac{5bx\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^{3/2}}{8c} + \frac{1}{2}x^2(a+b\text{arcsinh}(cx))^{5/2} + \frac{1}{16}(15b^2)\int x\sqrt{a+b\text{arcsinh}(cx)}\,dx + \frac{(5b)\int \frac{(a+b\text{arcsinh}(cx))^{3/2}}{\sqrt{1+c^2x^2}}\,dx}{8c}$$

$$= \frac{15}{32}b^2x^2\sqrt{a+b\text{arcsinh}(cx)} - \frac{5bx\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^{3/2}}{8c} + \frac{(a+b\text{arcsinh}(cx))^{5/2}}{4c^2} + \frac{1}{2}x^2(a+b\text{arcsinh}(cx))^{5/2} - \frac{1}{64}(15b^3c)\int \frac{x^2}{\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}\,dx$$

$$= \frac{15}{32}b^2x^2\sqrt{a+b\text{arcsinh}(cx)} - \frac{5bx\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^{3/2}}{8c} + \frac{(a+b\text{arcsinh}(cx))^{5/2}}{4c^2} + \frac{1}{2}x^2(a+b\text{arcsinh}(cx))^{5/2} - \frac{(15b^2)\,\text{Subst}\left(\int \frac{\sinh^2\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{64c^2}$$

$$= \frac{15}{32}b^2x^2\sqrt{a+b\text{arcsinh}(cx)} - \frac{5bx\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^{3/2}}{8c} + \frac{(a+b\text{arcsinh}(cx))^{5/2}}{4c^2} + \frac{1}{2}x^2(a+b\text{arcsinh}(cx))^{5/2} + \frac{(15b^2)\,\text{Subst}\left(\int \left(\frac{1}{2\sqrt{x}} - \frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{2\sqrt{x}}\right)dx, x, a+b\text{arcsinh}(cx)\right)}{64c^2}$$

$$= \frac{15b^2\sqrt{a+b\text{arcsinh}(cx)}}{64c^2} + \frac{15}{32}b^2x^2\sqrt{a+b\text{arcsinh}(cx)} - \frac{5bx\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^{3/2}}{8c} + \frac{(a+b\text{arcsinh}(cx))^{5/2}}{4c^2} + \frac{1}{2}x^2(a+b\text{arcsinh}(cx))^{5/2} - \frac{(15b^2)\,\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{128c^2}$$

$$= \frac{15b^2\sqrt{a+b\text{arcsinh}(cx)}}{64c^2} + \frac{15}{32}b^2x^2\sqrt{a+b\text{arcsinh}(cx)} - \frac{5bx\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^{3/2}}{8c} + \frac{(a+b\text{arcsinh}(cx))^{5/2}}{4c^2} + \frac{1}{2}x^2(a+b\text{arcsinh}(cx))^{5/2} - \frac{(15b^2)\,\text{Subst}\left(\int \frac{e^{-i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{256c^2} - \frac{(15b^2)\,\text{Subst}\left(\int \frac{e^{i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{256c^2}$$

$$\begin{aligned}
&= \frac{15b^2\sqrt{a+b\text{arcsinh}(cx)}}{64c^2} + \frac{15}{32}b^2x^2\sqrt{a+b\text{arcsinh}(cx)} \\
&\quad - \frac{5bx\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^{3/2}}{8c} + \frac{(a+b\text{arcsinh}(cx))^{5/2}}{4c^2} \\
&\quad + \frac{1}{2}x^2(a+b\text{arcsinh}(cx))^{5/2} - \frac{(15b^2)\,\text{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{128c^2} \\
&\quad - \frac{(15b^2)\,\text{Subst}\left(\int e^{-\frac{2a}{b}+\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{128c^2}
\end{aligned}$$

$$\begin{aligned}
&= \frac{15b^2\sqrt{a+b\text{arcsinh}(cx)}}{64c^2} + \frac{15}{32}b^2x^2\sqrt{a+b\text{arcsinh}(cx)} \\
&\quad - \frac{5bx\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^{3/2}}{8c} + \frac{(a+b\text{arcsinh}(cx))^{5/2}}{4c^2} \\
&\quad + \frac{1}{2}x^2(a+b\text{arcsinh}(cx))^{5/2} - \frac{15b^{5/2}e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{256c^2} \\
&\quad - \frac{15b^{5/2}e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{256c^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.52

$$\int x(a+b\text{arcsinh}(cx))^{5/2}\,dx = \frac{e^{-\frac{2a}{b}}\Bigg(-b^3\sqrt{-\frac{a+b\text{arcsinh}(cx)}{b}}\,\Gamma\Big(\frac{7}{2},-\frac{2(a+b\text{arcsinh}(cx))}{b}\Big)+b^3e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\text{arcsinh}(cx)}\,\Gamma\Big(\frac{7}{2},}{32\sqrt{2}c^2\sqrt{a+b\text{arcsinh}(cx)}}$$

```
[In] Integrate[x*(a + b*ArcSinh[c*x])^(5/2),x]
```

```
[Out] (-(b^3*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[7/2, (-2*(a + b*ArcSinh[c*x]))
/b]) + b^3*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[7/2, (2*(a + b*ArcSin
h[c*x]))/b])/(32*Sqrt[2]*c^2*E^((2*a)/b)*Sqrt[a + b*ArcSinh[c*x]])
```

Maple [F]

$$\int x(a+b\ \mathrm{arcsinh}\,(cx))^{\frac{5}{2}}\,dx$$

```
[In] int(x*(a+b*arcsinh(c*x))^(5/2),x)
```

```
[Out] int(x*(a+b*arcsinh(c*x))^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x(a+b\mathrm{arcsinh}(cx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*(a+b*arcsinh(c*x))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x(a+b\mathrm{arcsinh}(cx))^{5/2}\,dx = \int x(a+b\,\mathrm{asinh}\,(cx))^{\frac{5}{2}}\,dx$$

```
[In] integrate(x*(a+b*asinh(c*x))**(5/2),x)
```

```
[Out] Integral(x*(a + b*asinh(c*x))**(5/2), x)
```

Maxima [F]

$$\int x(a+b\mathrm{arcsinh}(cx))^{5/2}\,dx = \int (b\,\mathrm{arsinh}\,(cx)+a)^{\frac{5}{2}}x\,dx$$

```
[In] integrate(x*(a+b*arcsinh(c*x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^(5/2)*x, x)
```

Giac [F(-2)]

Exception generated.

$$\int x(a+b\text{arcsinh}(cx))^{5/2}\,dx=\text{Exception raised: RuntimeError}$$

```
[In] integrate(x*(a+b*arcsinh(c*x))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int x(a+b\text{arcsinh}(cx))^{5/2}\,dx=\int x\,\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^{5/2}\,dx$$

```
[In] int(x*(a + b*asinh(c*x))^(5/2),x)
```

```
[Out] int(x*(a + b*asinh(c*x))^(5/2), x)
```

3.144 $\int (a+b\operatorname{arcsinh}(cx))^{5/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 155

$$\begin{aligned}\int (a+b\operatorname{arcsinh}(cx))^{5/2}\,dx &= \frac{15}{4}b^2x\sqrt{a+b\operatorname{arcsinh}(cx)} \\ &\quad - \frac{5b\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^{3/2}}{2c} + x(a+b\operatorname{arcsinh}(cx))^{5/2} \\ &\quad + \frac{15b^{5/2}e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{16c} - \frac{15b^{5/2}e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{16c}\end{aligned}$$

```
[Out] x*(a+b*arcsinh(c*x))^(5/2)+15/16*b^(5/2)*exp(a/b)*erf((a+b*arcsinh(c*x))^(1
/2)/b^(1/2))*Pi^(1/2)/c-15/16*b^(5/2)*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2)
)*Pi^(1/2)/c/exp(a/b)-5/2*b*(a+b*arcsinh(c*x))^(3/2)*(c^2*x^2+1)^(1/2)/c+15
/4*b^2*x*(a+b*arcsinh(c*x))^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5772, 5798, 5819, 3389, 2211, 2236, 2235}

$$\begin{aligned}\int (a+b\operatorname{arcsinh}(cx))^{5/2}\,dx &= \frac{15\sqrt{\pi}b^{5/2}e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{16c} \\ &\quad - \frac{15\sqrt{\pi}b^{5/2}e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{16c} + \frac{15}{4}b^2x\sqrt{a+b\operatorname{arcsinh}(cx)} \\ &\quad - \frac{5b\sqrt{c^2x^2+1}(a+b\operatorname{arcsinh}(cx))^{3/2}}{2c} + x(a+b\operatorname{arcsinh}(cx))^{5/2}\end{aligned}$$

```
[In] Int[(a + b*ArcSinh[c*x])^(5/2),x]
```

```
[Out] (15*b^2*x*Sqrt[a + b*ArcSinh[c*x]])/4 - (5*b*Sqrt[1 + c^2*x^2]*(a + b*ArcSi
nh[c*x])^(3/2))/(2*c) + x*(a + b*ArcSinh[c*x])^(5/2) + (15*b^(5/2)*E^(a/b)*
Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(16*c) - (15*b^(5/2)*Sqrt[P
i]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(16*c*E^(a/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5798

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])^n/(2*e*(p
+ 1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*x^2)^p],
Int[(1 + c^2*x^2)^(p + 1/2)*(a + b*ArcSinh[c*x])^(n - 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
```

```
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x(a+b\text{arcsinh}(cx))^{5/2}-\frac{1}{2}(5bc)\int\frac{x(a+b\text{arcsinh}(cx))^{3/2}}{\sqrt{1+c^2x^2}}\,dx\\
&= -\frac{5b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^{3/2}}{2c}\\
&\quad +x(a+b\text{arcsinh}(cx))^{5/2}+\frac{1}{4}\left(15b^2\right)\int\sqrt{a+b\text{arcsinh}(cx)}\,dx\\
&= \frac{15}{4}b^2x\sqrt{a+b\text{arcsinh}(cx)}-\frac{5b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^{3/2}}{2c}\\
&\quad +x(a+b\text{arcsinh}(cx))^{5/2}-\frac{1}{8}\left(15b^3c\right)\int\frac{x}{\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}\,dx\\
&= \frac{15}{4}b^2x\sqrt{a+b\text{arcsinh}(cx)}-\frac{5b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^{3/2}}{2c}\\
&\quad +x(a+b\text{arcsinh}(cx))^{5/2}+\frac{\left(15b^2\right)\text{Subst}\left(\int\frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx,x,a+b\text{arcsinh}(cx)\right)}{8c}\\
&= \frac{15}{4}b^2x\sqrt{a+b\text{arcsinh}(cx)}-\frac{5b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^{3/2}}{2c}\\
&\quad +x(a+b\text{arcsinh}(cx))^{5/2}+\frac{\left(15b^2\right)\text{Subst}\left(\int\frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx,x,a+b\text{arcsinh}(cx)\right)}{16c}\\
&\quad -\frac{\left(15b^2\right)\text{Subst}\left(\int\frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx,x,a+b\text{arcsinh}(cx)\right)}{16c}\\
&= \frac{15}{4}b^2x\sqrt{a+b\text{arcsinh}(cx)}-\frac{5b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^{3/2}}{2c}\\
&\quad +x(a+b\text{arcsinh}(cx))^{5/2}+\frac{\left(15b^2\right)\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx,x,\sqrt{a+b\text{arcsinh}(cx)}\right)}{8c}\\
&\quad -\frac{\left(15b^2\right)\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+b\text{arcsinh}(cx)}\right)}{8c}
\end{aligned}$$

$$= \frac{15}{4}b^2x\sqrt{a+b\text{arcsinh}(cx)} - \frac{5b\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^{3/2}}{2c} + x(a + b\text{arcsinh}(cx))^{5/2} + \frac{15b^{5/2}e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16c} - \frac{15b^{5/2}e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{16c}$$

Mathematica [A] (verified)

Time = 2.06 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.82

$$\int (a+b\text{arcsinh}(cx))^{5/2}\,dx = \frac{\sqrt{b}e^{-\frac{a}{b}}\left(-\left((4a^2-15b^2)\,e^{\frac{2a}{b}}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)\right)+(4a^2-15b^2)\sqrt{\pi}\text{erfi}\left(\sqrt{a+}\right.\right.}{}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])^(5/2),x]
```

```
[Out] (Sqrt[b]*(-((4*a^2 - 15*b^2)*E^((2*a)/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*
x]]/Sqrt[b]]) + (4*a^2 - 15*b^2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqr
t[b]] + (4*Sqrt[b]*(E^(a/b)*(a + b*ArcSinh[c*x])*(5*(3*b*c*x - 2*a*Sqrt[1 +
 c^2*x^2]) + 2*(4*a*c*x - 5*b*Sqrt[1 + c^2*x^2])*ArcSinh[c*x] + 4*b*c*x*Arc
Sinh[c*x]^2) - 2*a^2*E^((2*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[3/2, a/b +
ArcSinh[c*x]] - 2*a^2*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[3/2, -((a + b*A
rcSinh[c*x])/b)]))/Sqrt[a + b*ArcSinh[c*x]]))/(16*c*E^(a/b))
```

Maple [F]

$$\int (a + b\ \text{arcsinh}\,(cx))^{\frac{5}{2}}\,dx$$

```
[In] int((a+b*arcsinh(c*x))^(5/2),x)
```

```
[Out] int((a+b*arcsinh(c*x))^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (a+b\operatorname{arcsinh}(cx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arcsinh(c*x))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int (a+b\operatorname{arcsinh}(cx))^{5/2}\,dx = \int \left(a+b\operatorname{asinh}\left(cx\right)\right)^{\frac{5}{2}}\,dx$$

```
[In] integrate((a+b*asinh(c*x))**(5/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**(5/2), x)
```

Maxima [F]

$$\int (a+b\operatorname{arcsinh}(cx))^{5/2}\,dx = \int \left(b\operatorname{arsinh}\left(cx\right)+a\right)^{\frac{5}{2}}\,dx$$

```
[In] integrate((a+b*arcsinh(c*x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int (a+b\operatorname{arcsinh}(cx))^{5/2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((a+b*arcsinh(c*x))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (a + b\text{arcsinh}(cx))^{5/2}\,dx = \int (a + b\,\text{asinh}(c\,x))^{5/2}\,dx$$

```
[In] int((a + b*asinh(c*x))^(5/2),x)
[Out] int((a + b*asinh(c*x))^(5/2), x)
```

3.145 $\int \frac{x^2}{\sqrt{a+b\,\mathbf{arcsinh}(cx)}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 194

$$\int \frac{x^2}{\sqrt{a+b\,\text{arcsinh}(cx)}}\,dx = -\frac{e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3} + \frac{e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3} - \frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3} + \frac{e^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\,\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3}$$

```
[Out] 1/24*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1
/2)/c^3/b^(1/2)+1/24*erfi(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)
*Pi^(1/2)/c^3/exp(3*a/b)/b^(1/2)-1/8*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/
b^(1/2))*Pi^(1/2)/c^3/b^(1/2)-1/8*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi
^(1/2)/c^3/exp(a/b)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5780, 5556, 3388, 2211, 2236, 2235}

$$\int \frac{x^2}{\sqrt{a+b\operatorname{arcsinh}(cx)}}\,dx = -\frac{\sqrt{\pi}e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3} + \frac{\sqrt{\frac{\pi}{3}}e^{\frac{3a}{b}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3} - \frac{\sqrt{\pi}e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3} + \frac{\sqrt{\frac{\pi}{3}}e^{-\frac{3a}{b}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3}$$

```
[In] Int[x^2/Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] -1/8*(E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(Sqrt[b]*c^3)
 + (E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])
/(8*Sqrt[b]*c^3) - (Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(8*Sqr
t[b]*c^3*E^(a/b)) + (Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqr
t[b]])/(8*Sqrt[b]*c^3*E^((3*a)/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{bc^3}\\
&= \frac{\text{Subst}\left(\int \left(\frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{4\sqrt{x}}-\frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{4\sqrt{x}}\right)dx, x, a+b\text{arcsinh}(cx)\right)}{bc^3}\\
&= \frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{4bc^3}-\frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{4bc^3}\\
&= -\frac{\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{8bc^3}\\
&\quad-\frac{\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{8bc^3}\\
&\quad+\frac{\text{Subst}\left(\int \frac{e^{-i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{8bc^3}\\
&\quad+\frac{\text{Subst}\left(\int \frac{e^{i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{8bc^3}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{\operatorname{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{4bc^3} \\
&\quad - \frac{\operatorname{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{4bc^3} \\
&\quad - \frac{\operatorname{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{4bc^3} \\
&\quad + \frac{\operatorname{Subst}\left(\int e^{-\frac{3a}{b}+\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{4bc^3} \\
&= -\frac{e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3} + \frac{e^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3} \\
&\quad - \frac{e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3} + \frac{e^{-\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}c^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.01

$$
\int \frac{x^2}{\sqrt{a+b\operatorname{arcsinh}(cx)}}\,dx
$$

$$
= \frac{e^{-\frac{3a}{b}}\left(3e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\operatorname{arcsinh}(cx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\operatorname{arcsinh}(cx)\right)+\sqrt{3}\sqrt{-\frac{a+b\operatorname{arcsinh}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{3(a+b\operatorname{arcsinh}(cx))}{b}\right)-3e\right.}{24c^3\sqrt{a+b\operatorname{arcsi}}}
$$

```
[In] Integrate[x^2/Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] (3*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, a/b + ArcSinh[c*x]] + Sq
rt[3]*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-3*(a + b*ArcSinh[c*x]))/
b] - 3*E^((2*a)/b)*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, -((a + b*ArcS
inh[c*x])/b)] - Sqrt[3]*E^((6*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (3*
(a + b*ArcSinh[c*x]))/b])/(24*c^3*E^((3*a)/b)*Sqrt[a + b*ArcSinh[c*x]])
```

Maple [F]

$$\int \frac{x^2}{\sqrt{a+b \text{ arcsinh}\left(cx\right)}}dx$$

```
[In] int(x^2/(a+b*arcsinh(c*x))^(1/2),x)
```

```
[Out] int(x^2/(a+b*arcsinh(c*x))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2/(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{x^2}{\sqrt{a+b\,\text{asinh}\left(cx\right)}}\,dx$$

```
[In] integrate(x**2/(a+b*asinh(c*x))**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a + b*asinh(c*x)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{x^2}{\sqrt{b\,\text{arsinh}\left(cx\right)+a}}\,dx$$

```
[In] integrate(x^2/(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(b*arcsinh(c*x) + a), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{a + b\text{arcsinh}(cx)}}\,dx = \int \frac{x^2}{\sqrt{b\,\text{arsinh}\,(cx) + a}}\,dx$$

```
[In] integrate(x^2/(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
[Out] integrate(x^2/sqrt(b*arcsinh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a + b\text{arcsinh}(cx)}}\,dx = \int \frac{x^2}{\sqrt{a + b\,\text{asinh}\,(c\,x)}}\,dx$$

```
[In] int(x^2/(a + b*asinh(c*x))^(1/2),x)
[Out] int(x^2/(a + b*asinh(c*x))^(1/2), x)
```

3.146 $\int \frac{x}{\sqrt{a+b\operatorname{arcsinh}(cx)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 107

$$\int \frac{x}{\sqrt{a+b\operatorname{arcsinh}(cx)}}\,dx = -\frac{e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^2} + \frac{e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^2}$$

```
[Out] -1/8*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1
/2)/c^2/b^(1/2)+1/8*erfi(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(1/2)*
Pi^(1/2)/c^2/exp(2*a/b)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5780, 5556, 12, 3389, 2211, 2236, 2235}

$$\int \frac{x}{\sqrt{a+b\operatorname{arcsinh}(cx)}}\,dx$$
$$= \frac{\sqrt{\frac{\pi}{2}}e^{-\frac{2a}{b}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^2} - \frac{\sqrt{\frac{\pi}{2}}e^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^2}$$

```
[In] Int[x/Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] -1/4*(E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]
])/(Sqrt[b]*c^2) + (Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt
[b]])/(4*Sqrt[b]*c^2*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{bc^2}\\
&= -\frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{2\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{bc^2}\\
&= -\frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{2bc^2}\\
&= -\frac{\text{Subst}\left(\int \frac{e^{-i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{4bc^2}\\
&\quad+\frac{\text{Subst}\left(\int \frac{e^{i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{4bc^2}\\
&= -\frac{\text{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{2bc^2}+\frac{\text{Subst}\left(\int e^{-\frac{2a}{b}+\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{2bc^2}\\
&= -\frac{e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^2}+\frac{e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}c^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.01

$$\int \frac{x}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \frac{e^{-\frac{2a}{b}}\left(\sqrt{-\frac{a+b\text{arcsinh}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{2(a+b\text{arcsinh}(cx))}{b}\right)+e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\text{arcsinh}(cx)}\,\Gamma\left(\frac{1}{2},\frac{2(a+b\text{arcsinh}(cx))}{b}\right)\right)}{4\sqrt{2}c^2\sqrt{a+b\text{arcsinh}(cx)}}$$

```
[In] Integrate[x/Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] (Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-2*(a + b*ArcSinh[c*x]))/b] +
E^((4*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (2*(a + b*ArcSinh[c*x]))/b]
)/(4*Sqrt[2]*c^2*E^((2*a)/b)*Sqrt[a + b*ArcSinh[c*x]])
```

Maple [F]

$$\int \frac{x}{\sqrt{a+b\operatorname{arcsinh}(cx)}}dx$$

```
[In] int(x/(a+b*arcsinh(c*x))^(1/2),x)
```

```
[Out] int(x/(a+b*arcsinh(c*x))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x/(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{x}{\sqrt{a+b\operatorname{asinh}(cx)}}\,dx$$

```
[In] integrate(x/(a+b*asinh(c*x))**(1/2),x)
```

```
[Out] Integral(x/sqrt(a + b*asinh(c*x)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \int \frac{x}{\sqrt{b\operatorname{arsinh}(cx)+a}}\,dx$$

```
[In] integrate(x/(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(b*arcsinh(c*x) + a), x)
```

Giac [F]

$$\int \frac{x}{\sqrt{a + b\text{arcsinh}(cx)}}\,dx = \int \frac{x}{\sqrt{b\,\operatorname{arsinh}\left(cx\right) + a}}\,dx$$

```
[In] integrate(x/(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(b*arcsinh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a + b\text{arcsinh}(cx)}}\,dx = \int \frac{x}{\sqrt{a + b\,\operatorname{asinh}\left(c\,x\right)}}\,dx$$

```
[In] int(x/(a + b*asinh(c*x))^(1/2),x)
```

```
[Out] int(x/(a + b*asinh(c*x))^(1/2), x)
```

3.147 $\int \frac{1}{\sqrt{a+b\mathbf{arcsinh}(cx)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 88

$$\int \frac{1}{\sqrt{a+b\operatorname{arcsinh}(cx)}}\,dx=\frac{e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}+\frac{e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}$$

```
[Out] 1/2*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/c/b^(1/2)+1/2*e
rfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/c/exp(a/b)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5774, 3388, 2211, 2236, 2235}

$$\int \frac{1}{\sqrt{a+b\operatorname{arcsinh}(cx)}}\,dx=\frac{\sqrt{\pi}e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}+\frac{\sqrt{\pi}e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}$$

```
[In] Int[1/Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] (E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(2*Sqrt[b]*c) + (S
qrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(2*Sqrt[b]*c*E^(a/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
```

```
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{bc}\\
&= \frac{\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{2bc}+\frac{\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{2bc}\\
&= \frac{\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{bc}+\frac{\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{bc}\\
&= \frac{e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}+\frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.15

$$\int \frac{1}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \frac{e^{-\frac{a}{b}}\left(-e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\text{arcsinh}(cx)}\Gamma\left(\frac{1}{2},\frac{a}{b}+\text{arcsinh}(cx)\right)+\sqrt{-\frac{a+b\text{arcsinh}(cx)}{b}}\Gamma\left(\frac{1}{2},-\frac{a+b\text{arcsinh}(cx)}{b}\right)\right)}{2c\sqrt{a+b\text{arcsinh}(cx)}}$$

```
[In] Integrate[1/Sqrt[a + b*ArcSinh[c*x]],x]
```

```
[Out] (-(E^((2*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, a/b + ArcSinh[c*x]]) + S
qrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c*x])/b)])/(2*c*
E^(a/b)*Sqrt[a + b*ArcSinh[c*x]])
```

Maple [F]

$$\int \frac{1}{\sqrt{a+b\,\text{arcsinh}\,(cx)}}dx$$

```
[In] int(1/(a+b*arcsinh(c*x))^(1/2),x)
```

```
[Out] int(1/(a+b*arcsinh(c*x))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*arcsinh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+b\operatorname{arcsinh}(cx)}}\,dx=\int \frac{1}{\sqrt{a+b\operatorname{asinh}{\left(cx \right)}}}\,dx$$

```
[In] integrate(1/(a+b*asinh(c*x))**(1/2),x)
[Out] Integral(1/sqrt(a + b*asinh(c*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+b\operatorname{arcsinh}(cx)}}\,dx=\int \frac{1}{\sqrt{b\operatorname{arsinh}\left(cx\right)+a}}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(c*x))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(b*arcsinh(c*x) + a), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+b\operatorname{arcsinh}(cx)}}\,dx=\int \frac{1}{\sqrt{b\operatorname{arsinh}\left(cx\right)+a}}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(c*x))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(b*arcsinh(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\operatorname{arcsinh}(cx)}}\,dx=\int \frac{1}{\sqrt{a+b\,\mathrm{asinh}\left(c\,x\right)}}\,dx$$

```
[In] int(1/(a + b*asinh(c*x))^(1/2),x)
[Out] int(1/(a + b*asinh(c*x))^(1/2), x)
```

3.148 $\int \frac{x^2}{(a+b\,\mathbf{arcsinh}(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 226

$$\begin{aligned}\int \frac{x^2}{(a+b\operatorname{arcsinh}(cx))^{3/2}}\,dx &= -\frac{2x^2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\operatorname{arcsinh}(cx)}} \\ &+ \frac{e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3} - \frac{e^{\frac{3a}{b}}\sqrt{3\pi}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3} \\ &- \frac{e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3} + \frac{e^{-\frac{3a}{b}}\sqrt{3\pi}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}\end{aligned}$$

```
[Out] 1/4*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c^3-1/4
*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c^3/exp(a/b)-1/4*e
xp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/b^
(3/2)/c^3+1/4*erfi(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/
2)/b^(3/2)/c^3/exp(3*a/b)-2*x^2*(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))^(1
/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used

$= \{5778, 3389, 2211, 2236, 2235\}$

$$\int \frac{x^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \frac{\sqrt{\pi}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3} - \frac{\sqrt{3\pi}e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3} - \frac{\sqrt{\pi}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3} + \frac{\sqrt{3\pi}e^{-\frac{3a}{b}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3} - \frac{2x^2\sqrt{c^2x^2+1}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

```
[In] Int[x^2/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (-2*x^2*Sqrt[1 + c^2*x^2])/(b*c*Sqrt[a + b*ArcSinh[c*x]]) + (E^(a/b)*Sqrt[P
i]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(4*b^(3/2)*c^3) - (E^((3*a)/b)*Sq
rt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(4*b^(3/2)*c^3) -
 (Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(4*b^(3/2)*c^3*E^(a/b))
+ (Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(4*b^(3/2)*
c^3*E^((3*a)/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2x^2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} \\
&\quad + \frac{2\text{Subst}\left(\int\left(-\frac{3\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{4\sqrt{x}}+\frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{4\sqrt{x}}\right)dx,x,a+b\text{arcsinh}(cx)\right)}{b^2c^3} \\
&= -\frac{2x^2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}+\frac{\text{Subst}\left(\int\frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx,x,a+b\text{arcsinh}(cx)\right)}{2b^2c^3} \\
&\quad -\frac{3\text{Subst}\left(\int\frac{\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{\sqrt{x}}\,dx,x,a+b\text{arcsinh}(cx)\right)}{2b^2c^3} \\
&= -\frac{2x^2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}+\frac{\text{Subst}\left(\int\frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx,x,a+b\text{arcsinh}(cx)\right)}{4b^2c^3} \\
&\quad -\frac{\text{Subst}\left(\int\frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx,x,a+b\text{arcsinh}(cx)\right)}{4b^2c^3} \\
&\quad -\frac{3\text{Subst}\left(\int\frac{e^{-i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx,x,a+b\text{arcsinh}(cx)\right)}{4b^2c^3} \\
&\quad +\frac{3\text{Subst}\left(\int\frac{e^{i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx,x,a+b\text{arcsinh}(cx)\right)}{4b^2c^3} \\
&= -\frac{2x^2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\text{arcsinh}(cx)}}+\frac{\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx,x,\sqrt{a+b\text{arcsinh}(cx)}\right)}{2b^2c^3} \\
&\quad -\frac{\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+b\text{arcsinh}(cx)}\right)}{2b^2c^3} \\
&\quad -\frac{3\text{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx,x,\sqrt{a+b\text{arcsinh}(cx)}\right)}{2b^2c^3} \\
&\quad +\frac{3\text{Subst}\left(\int e^{-\frac{3a}{b}+\frac{3x^2}{b}}\,dx,x,\sqrt{a+b\text{arcsinh}(cx)}\right)}{2b^2c^3}
\end{aligned}$$

$$= -\frac{2x^2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\mathrm{arcsinh}(cx)}} + \frac{e^{a/b}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{a+b\mathrm{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}$$

$$- \frac{e^{\frac{3a}{b}}\sqrt{3\pi}\,\mathrm{erf}\left(\frac{\sqrt{3}\sqrt{a+b\mathrm{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3} - \frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{a+b\mathrm{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}$$

$$+ \frac{e^{-\frac{3a}{b}}\sqrt{3\pi}\,\mathrm{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\mathrm{arcsinh}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.28

$$\int \frac{x^2}{(a+b\mathrm{arcsinh}(cx))^{3/2}}\,dx = \frac{e^{-3\left(\frac{a}{b}+\mathrm{arcsinh}(cx)\right)}\left(-e^{\frac{3a}{b}} + e^{\frac{3a}{b}+2\mathrm{arcsinh}(cx)} + e^{\frac{3a}{b}+4\mathrm{arcsinh}(cx)} - e^{\frac{3a}{b}+6\mathrm{arcsinh}(cx)} - e^{\frac{4a}{b}}\right.}{}$$

```
[In] Integrate[x^2/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (-E^((3*a)/b) + E^((3*a)/b + 2*ArcSinh[c*x]) + E^((3*a)/b + 4*ArcSinh[c*x])
 - E^((3*a)/b + 6*ArcSinh[c*x]) - E^((4*a)/b + 3*ArcSinh[c*x])*Sqrt[a/b + A
rcSinh[c*x]]*Gamma[1/2, a/b + ArcSinh[c*x]] + Sqrt[3]*E^(3*ArcSinh[c*x])*Sq
rt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-3*(a + b*ArcSinh[c*x]))/b] - E^(
(2*a)/b + 3*ArcSinh[c*x])*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, -((a +
 b*ArcSinh[c*x])/b)] + Sqrt[3]*E^((6*a)/b + 3*ArcSinh[c*x])*Sqrt[a/b + ArcS
inh[c*x]]*Gamma[1/2, (3*(a + b*ArcSinh[c*x]))/b])/(4*b*c^3*E^(3*(a/b + ArcS
inh[c*x]))*Sqrt[a + b*ArcSinh[c*x]])
```

Maple [F]

$$\int \frac{x^2}{(a+b\ \mathrm{arcsinh}\,(cx))^{\frac{3}{2}}}dx$$

```
[In] int(x^2/(a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int(x^2/(a+b*arcsinh(c*x))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{(a+b\operatorname{arcsinh}(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^2}{(a+b\operatorname{arcsinh}(cx))^{3/2}}\,dx = \int \frac{x^2}{(a+b\operatorname{asinh}(cx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x**2/(a+b*asinh(c*x))**(3/2),x)
```

```
[Out] Integral(x**2/(a + b*asinh(c*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^2}{(a+b\operatorname{arcsinh}(cx))^{3/2}}\,dx = \int \frac{x^2}{(b\operatorname{arsinh}(cx)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b*arcsinh(c*x) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^2}{(a+b\operatorname{arcsinh}(cx))^{3/2}}\,dx = \int \frac{x^2}{(b\operatorname{arsinh}(cx)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*arcsinh(c*x) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\int \frac{x^2}{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)^{3/2}} \,d x$$

```
[In] int(x^2/(a + b*asinh(c*x))^(3/2),x)
```

```
[Out] int(x^2/(a + b*asinh(c*x))^(3/2), x)
```

3.149 $\int \frac{x}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 135

$$\int \frac{x}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = -\frac{2x\sqrt{1+c^2x^2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} + \frac{e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^2} + \frac{e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^2}$$

```
[Out] 1/2*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/
2)/b^(3/2)/c^2+1/2*erfi(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(1/2)*P
i^(1/2)/b^(3/2)/c^2/exp(2*a/b)-2*x*(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))
^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5778, 3388, 2211, 2236, 2235}

$$\int \frac{x}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \frac{\sqrt{\frac{\pi}{2}}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^2} + \frac{\sqrt{\frac{\pi}{2}}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^2} - \frac{2x\sqrt{c^2x^2+1}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

```
[In] Int[x/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (-2*x*Sqrt[1 + c^2*x^2])/(b*c*Sqrt[a + b*ArcSinh[c*x]]) + (E^((2*a)/b)*Sqrt
[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(b^(3/2)*c^2) + (Sq
rt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(b^(3/2)*c^2*E^(
(2*a)/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rubi steps

$$\text{integral} = -\frac{2x\sqrt{1+c^2x^2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} + \frac{2\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{b^2c^2}$$

$$= -\frac{2x\sqrt{1+c^2x^2}}{bc\sqrt{a+b\operatorname{arcsinh}(cx)}} + \frac{\text{Subst}\left(\int \frac{e^{-i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{b^2c^2} + \frac{\text{Subst}\left(\int \frac{e^{i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{b^2c^2}$$

$$= -\frac{2x\sqrt{1+c^2x^2}}{bc\sqrt{a+b\operatorname{arcsinh}(cx)}} + \frac{2\text{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{b^2c^2} + \frac{2\text{Subst}\left(\int e^{-\frac{2a}{b}+\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{b^2c^2}$$

$$= -\frac{2x\sqrt{1+c^2x^2}}{bc\sqrt{a+b\operatorname{arcsinh}(cx)}} + \frac{e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^2} + \frac{e^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^2}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.99

$$\int \frac{x}{(a+b\operatorname{arcsinh}(cx))^{3/2}}\,dx = \frac{e^{-\frac{2a}{b}}\left(\sqrt{2}\sqrt{-\frac{a+b\operatorname{arcsinh}(cx)}{b}}\Gamma\left(\frac{1}{2},-\frac{2(a+b\operatorname{arcsinh}(cx))}{b}\right) - \sqrt{2}e^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\operatorname{arcsinh}(}\right.}{2bc^2\sqrt{a+b\operatorname{arcsinh}(cx)}}$$

```
[In] Integrate[x/(a + b*ArcSinh[c*x])^(3/2),x]
```

```
[Out] (Sqrt[2]*Sqrt[-((a + b*ArcSinh[c*x])/b)]*Gamma[1/2, (-2*(a + b*ArcSinh[c*x]
))/b] - Sqrt[2]*E^((4*a)/b)*Sqrt[a/b + ArcSinh[c*x]]*Gamma[1/2, (2*(a + b*A
rcSinh[c*x]))/b] - 2*E^((2*a)/b)*Sinh[2*ArcSinh[c*x]])/(2*b*c^2*E^((2*a)/b)
*Sqrt[a + b*ArcSinh[c*x]])
```

Maple [F]

$$\int \frac{x}{(a+b\ \operatorname{arcsinh}\,(cx))^{\frac{3}{2}}}dx$$

```
[In] int(x/(a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int(x/(a+b*arcsinh(c*x))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{x}{(a+b\operatorname{asinh}(cx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/(a+b*asinh(c*x))**(3/2),x)
```

```
[Out] Integral(x/(a + b*asinh(c*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{x}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{x}{(b\operatorname{arsinh}(cx)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(b*arcsinh(c*x) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{x}{(b\operatorname{arsinh}(cx)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/(b*arcsinh(c*x) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\int \frac{x}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^{3/2}} \,d x$$

```
[In] int(x/(a + b*asinh(c*x))^(3/2),x)
[Out] int(x/(a + b*asinh(c*x))^(3/2), x)
```

3.150 $\int \frac{1}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 116

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = -\frac{2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} - \frac{e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c} + \frac{e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}$$

```
[Out] -exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c+erfi((a+
b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c/exp(a/b)-2*(c^2*x^2+1)^(1
/2)/b/c/(a+b*arcsinh(c*x))^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5773, 5819, 3389, 2211, 2236, 2235}

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = -\frac{\sqrt{\pi}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c} + \frac{\sqrt{\pi}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c} - \frac{2\sqrt{c^2x^2+1}}{bc\sqrt{a+b\text{arcsinh}(cx)}}$$

```
[In] Int[(a + b*ArcSinh[c*x])^(-3/2),x]
```

```
[Out] (-2*Sqrt[1 + c^2*x^2])/(b*c*Sqrt[a + b*ArcSinh[c*x]]) - (E^(a/b)*Sqrt[Pi]*E
rf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(b^(3/2)*c) + (Sqrt[Pi]*Erfi[Sqrt[a +
 b*ArcSinh[c*x]]/Sqrt[b]])/(b^(3/2)*c*E^(a/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = -\frac{2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} + \frac{(2c)\int \frac{x}{\sqrt{1+c^2x^2}\sqrt{a+b\text{arcsinh}(cx)}}\,dx}{b}$$

$$= -\frac{2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} - \frac{2\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{b^2c}$$

$$= -\frac{2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} - \frac{\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{b^2c} + \frac{\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{b^2c}$$

$$= -\frac{2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} - \frac{2\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{b^2c} + \frac{2\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{b^2c}$$

$$= -\frac{2\sqrt{1+c^2x^2}}{bc\sqrt{a+b\text{arcsinh}(cx)}} - \frac{e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c} + \frac{e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.18

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \frac{e^{-\frac{a+b\text{arcsinh}(cx)}{b}}\left(-e^{a/b}\left(1+e^{2\text{arcsinh}(cx)}\right)+e^{\frac{2a}{b}+\text{arcsinh}(cx)}\sqrt{\frac{a}{b}+\text{arcsinh}(cx)}\Gamma\left(\frac{1}{2},\frac{a}{b}\right.\right.}{bc\sqrt{a+b\text{arcsi}}}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])^(-3/2),x]
```

```
[Out] (-(E^(a/b)*(1 + E^(2*ArcSinh[c*x]))) + E^((2*a)/b + ArcSinh[c*x])*Sqrt[a/b
+ ArcSinh[c*x]]*Gamma[1/2, a/b + ArcSinh[c*x]] + E^ArcSinh[c*x]*Sqrt[-((a +
 b*ArcSinh[c*x])/b)]*Gamma[1/2, -((a + b*ArcSinh[c*x])/b)])/(b*c*E^((a + b*
ArcSinh[c*x])/b)*Sqrt[a + b*ArcSinh[c*x]])
```

Maple [F]

$$\int \frac{1}{(a+b\,\text{arcsinh}\,(cx))^{\frac{3}{2}}}dx$$

```
[In] int(1/(a+b*arcsinh(c*x))^(3/2),x)
```

```
[Out] int(1/(a+b*arcsinh(c*x))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*arcsinh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{1}{(a+b\operatorname{asinh}(cx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*asinh(c*x))**(3/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{1}{(b\operatorname{arsinh}(cx)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx = \int \frac{1}{(b\operatorname{arsinh}(cx)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^{3/2}}\,dx=\int \frac{1}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^{3/2}} \,d x$$

```
[In] int(1/(a + b*asinh(c*x))^(3/2),x)
[Out] int(1/(a + b*asinh(c*x))^(3/2), x)
```

3.151 $\int \frac{x^2}{(a+b\,\textbf{arcsinh}(cx))^{5/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 271

$$\begin{aligned}\int \frac{x^2}{(a+b\operatorname{arcsinh}(cx))^{5/2}}\,dx &= -\frac{2x^2\sqrt{1+c^2x^2}}{3bc(a+b\operatorname{arcsinh}(cx))^{3/2}} \\ &\quad - \frac{8x}{3b^2c^2\sqrt{a+b\operatorname{arcsinh}(cx)}} - \frac{4x^3}{b^2\sqrt{a+b\operatorname{arcsinh}(cx)}} \\ &\quad - \frac{e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{6b^{5/2}c^3} + \frac{e^{\frac{3a}{b}}\sqrt{3\pi}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{2b^{5/2}c^3} \\ &\quad - \frac{e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{6b^{5/2}c^3} + \frac{e^{-\frac{3a}{b}}\sqrt{3\pi}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{2b^{5/2}c^3}\end{aligned}$$

```
[Out] -1/6*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)/c^3-1/
6*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)/c^3/exp(a/b)+1/2*
exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/b
^(5/2)/c^3+1/2*erfi(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1
/2)/b^(5/2)/c^3/exp(3*a/b)-2/3*x^2*(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))
^(3/2)-8/3*x/b^2/c^2/(a+b*arcsinh(c*x))^(1/2)-4*x^3/b^2/(a+b*arcsinh(c*x))^
(1/2)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {5779, 5818, 5780, 5556, 3388, 2211, 2236, 2235, 5774}

$$\begin{aligned}\int \frac{x^2}{(a+b\operatorname{arcsinh}(cx))^{5/2}}\,dx = &-\frac{\sqrt{\pi}e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{6b^{5/2}c^3} \\ &+\frac{\sqrt{3\pi}e^{\frac{3a}{b}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{2b^{5/2}c^3}-\frac{\sqrt{\pi}e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{6b^{5/2}c^3} \\ &+\frac{\sqrt{3\pi}e^{-\frac{3a}{b}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{2b^{5/2}c^3}-\frac{8x}{3b^2c^2\sqrt{a+b\operatorname{arcsinh}(cx)}} \\ &-\frac{4x^3}{b^2\sqrt{a+b\operatorname{arcsinh}(cx)}}-\frac{2x^2\sqrt{c^2x^2+1}}{3bc(a+b\operatorname{arcsinh}(cx))^{3/2}}\end{aligned}$$

```
[In] Int[x^2/(a + b*ArcSinh[c*x])^(5/2),x]
```

```
[Out] (-2*x^2*Sqrt[1 + c^2*x^2])/(3*b*c*(a + b*ArcSinh[c*x])^(3/2)) - (8*x)/(3*b^
2*c^2*Sqrt[a + b*ArcSinh[c*x]]) - (4*x^3)/(b^2*Sqrt[a + b*ArcSinh[c*x]]) -
(E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(6*b^(5/2)*c^3) +
(E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(2
*b^(5/2)*c^3) - (Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(6*b^(5/2
)*c^3*E^(a/b)) + (Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b
]])/(2*b^(5/2)*c^3*E^((3*a)/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/S
qrt[1 + c^2*x^2]), x], x] - Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Sinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2x^2\sqrt{1+c^2x^2}}{3bc(a+b\text{arcsinh}(cx))^{3/2}} + \frac{4\int \frac{x}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^{3/2}}\,dx}{3bc} \\
&\quad + \frac{(2c)\int \frac{x^3}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^{3/2}}\,dx}{b} \\
&= -\frac{2x^2\sqrt{1+c^2x^2}}{3bc(a+b\text{arcsinh}(cx))^{3/2}} - \frac{8x}{3b^2c^2\sqrt{a+b\text{arcsinh}(cx)}} \\
&\quad - \frac{4x^3}{b^2\sqrt{a+b\text{arcsinh}(cx)}} + \frac{12\int \frac{x^2}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx}{b^2} + \frac{8\int \frac{1}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx}{3b^2c^2} \\
&= -\frac{2x^2\sqrt{1+c^2x^2}}{3bc(a+b\text{arcsinh}(cx))^{3/2}} - \frac{8x}{3b^2c^2\sqrt{a+b\text{arcsinh}(cx)}} \\
&\quad - \frac{4x^3}{b^2\sqrt{a+b\text{arcsinh}(cx)}} + \frac{8\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{3b^3c^3} \\
&\quad + \frac{12\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\sinh^2\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{b^3c^3} \\
&= -\frac{2x^2\sqrt{1+c^2x^2}}{3bc(a+b\text{arcsinh}(cx))^{3/2}} - \frac{8x}{3b^2c^2\sqrt{a+b\text{arcsinh}(cx)}} \\
&\quad - \frac{4x^3}{b^2\sqrt{a+b\text{arcsinh}(cx)}} + \frac{4\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{3b^3c^3} \\
&\quad + \frac{4\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{3b^3c^3} \\
&\quad + \frac{12\text{Subst}\left(\int \left(\frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{4\sqrt{x}} - \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{4\sqrt{x}}\right)\,dx, x, a+b\text{arcsinh}(cx)\right)}{b^3c^3}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{2x^2\sqrt{1+c^2x^2}}{3bc(a+b\text{arcsinh}(cx))^{3/2}} - \frac{8x}{3b^2c^2\sqrt{a+b\text{arcsinh}(cx)}} \\
&\quad - \frac{4x^3}{b^2\sqrt{a+b\text{arcsinh}(cx)}} + \frac{8\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{3b^3c^3} \\
&\quad + \frac{8\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{3b^3c^3} \\
&\quad + \frac{3\text{Subst}\left(\int \frac{\cosh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{b^3c^3} \\
&\quad - \frac{3\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{b^3c^3}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2x^2\sqrt{1+c^2x^2}}{3bc(a+b\text{arcsinh}(cx))^{3/2}} - \frac{8x}{3b^2c^2\sqrt{a+b\text{arcsinh}(cx)}} - \frac{4x^3}{b^2\sqrt{a+b\text{arcsinh}(cx)}} \\
&\quad + \frac{4e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c^3} + \frac{4e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c^3} \\
&\quad - \frac{3\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{2b^3c^3} \\
&\quad - \frac{3\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{2b^3c^3} \\
&\quad + \frac{3\text{Subst}\left(\int \frac{e^{-i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{2b^3c^3} \\
&\quad + \frac{3\text{Subst}\left(\int \frac{e^{i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{2b^3c^3}
\end{aligned}
$$

$$= -\frac{2x^2\sqrt{1+c^2x^2}}{3bc(a+b\text{arcsinh}(cx))^{3/2}} - \frac{8x}{3b^2c^2\sqrt{a+b\text{arcsinh}(cx)}} - \frac{4x^3}{b^2\sqrt{a+b\text{arcsinh}(cx)}} + \frac{4e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c^3} + \frac{4e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c^3} + \frac{3\text{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{b^3c^3} - \frac{3\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{b^3c^3} - \frac{3\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{b^3c^3} + \frac{3\text{Subst}\left(\int e^{-\frac{3a}{b}+\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{b^3c^3}$$

$$= -\frac{2x^2\sqrt{1+c^2x^2}}{3bc(a+b\text{arcsinh}(cx))^{3/2}} - \frac{8x}{3b^2c^2\sqrt{a+b\text{arcsinh}(cx)}} - \frac{4x^3}{b^2\sqrt{a+b\text{arcsinh}(cx)}} - \frac{e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{6b^{5/2}c^3} + \frac{e^{\frac{3a}{b}}\sqrt{3\pi}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{2b^{5/2}c^3} - \frac{e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{6b^{5/2}c^3} + \frac{e^{-\frac{3a}{b}}\sqrt{3\pi}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{2b^{5/2}c^3}$$

Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.25

$$\int \frac{x^2}{(a+b\text{arcsinh}(cx))^{5/2}}\,dx = \frac{e^{-3\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}\Bigg(2e^{\frac{4a}{b}+3\text{arcsinh}(cx)}\sqrt{\frac{a}{b}+\text{arcsinh}(cx)}(a+b\text{arcsinh}(cx))\Gamma\left(\frac{1}{2},\frac{a}{b}+\right.}{}$$

```
[In] Integrate[x^2/(a + b*ArcSinh[c*x])^(5/2),x]
```

```
[Out] (2*E^((4*a)/b + 3*ArcSinh[c*x])*Sqrt[a/b + ArcSinh[c*x]]*(a + b*ArcSinh[c*x
])*Gamma[1/2, a/b + ArcSinh[c*x]] - 6*Sqrt[3]*b*E^(3*ArcSinh[c*x])*(-((a +
b*ArcSinh[c*x])/b))^(3/2)*Gamma[1/2, (-3*(a + b*ArcSinh[c*x]))/b] + 2*b*E^(
(2*a)/b + 3*ArcSinh[c*x])*(-((a + b*ArcSinh[c*x])/b))^(3/2)*Gamma[1/2, -((a
 + b*ArcSinh[c*x])/b)] - E^((3*a)/b)*((-1 + E^(2*ArcSinh[c*x]))*(b*(-1 + E^
(4*ArcSinh[c*x])) + a*(6 + 4*E^(2*ArcSinh[c*x]) + 6*E^(4*ArcSinh[c*x])) + 2
*b*(3 + 2*E^(2*ArcSinh[c*x]) + 3*E^(4*ArcSinh[c*x]))*ArcSinh[c*x]) + 6*Sqrt
[3]*E^(3*(a/b + ArcSinh[c*x]))*Sqrt[a/b + ArcSinh[c*x]]*(a + b*ArcSinh[c*x]
)*Gamma[1/2, (3*(a + b*ArcSinh[c*x]))/b]))/(12*b^2*c^3*E^(3*(a/b + ArcSinh[
c*x]))*(a + b*ArcSinh[c*x])^(3/2))
```

Maple [F]

$$\int \frac{x^2}{\left(a+b\ \mathrm{arcsinh}\left(cx\right)\right)^{\frac{5}{2}}}dx$$

```
[In] int(x^2/(a+b*arcsinh(c*x))^(5/2),x)
```

```
[Out] int(x^2/(a+b*arcsinh(c*x))^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{(a+b\mathrm{arcsinh}(cx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2/(a+b*arcsinh(c*x))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^2}{(a+b\mathrm{arcsinh}(cx))^{5/2}}\,dx = \int \frac{x^{2}}{\left(a + b \operatorname{asinh}{\left(c x \right)}\right)^{\frac{5}{2}}}\, dx$$

```
[In] integrate(x**2/(a+b*asinh(c*x))**(5/2),x)
```

```
[Out] Integral(x**2/(a + b*asinh(c*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{x^2}{(a+b\mathrm{arcsinh}(cx))^{5/2}}\,dx = \int \frac{x^{2}}{{\left(b \operatorname{arsinh}\left(c x\right) + a\right)}^{\frac{5}{2}}}\,{d x}$$

```
[In] integrate(x^2/(a+b*arcsinh(c*x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b*arcsinh(c*x) + a)^(5/2), x)
```

Giac [F]

$$\int \frac{x^2}{(a+b\text{arcsinh}(cx))^{5/2}}\,dx=\int \frac{x^2}{(b\operatorname{arsinh}(cx)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^2/(a+b*arcsinh(c*x))^(5/2),x, algorithm="giac")
[Out] integrate(x^2/(b*arcsinh(c*x) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+b\text{arcsinh}(cx))^{5/2}}\,dx=\int \frac{x^2}{(a+b\operatorname{asinh}(c\,x))^{5/2}}\,dx$$

```
[In] int(x^2/(a + b*asinh(c*x))^(5/2),x)
[Out] int(x^2/(a + b*asinh(c*x))^(5/2), x)
```

3.152 $\int \frac{x}{(a+b\text{arcsinh}(cx))^{5/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 183

$$\begin{aligned}\int \frac{x}{(a+b\text{arcsinh}(cx))^{5/2}}\,dx &= -\frac{2x\sqrt{1+c^2x^2}}{3bc(a+b\text{arcsinh}(cx))^{3/2}} \\ &- \frac{4}{3b^2c^2\sqrt{a+b\text{arcsinh}(cx)}} - \frac{8x^2}{3b^2\sqrt{a+b\text{arcsinh}(cx)}} \\ &- \frac{2e^{\frac{2a}{b}}\sqrt{2\pi}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c^2} + \frac{2e^{-\frac{2a}{b}}\sqrt{2\pi}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c^2}\end{aligned}$$

```
[Out] -2/3*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1
/2)/b^(5/2)/c^2+2/3*erfi(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(1/2)*
Pi^(1/2)/b^(5/2)/c^2/exp(2*a/b)-2/3*x*(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(c*
x))^(3/2)-4/3/b^2/c^2/(a+b*arcsinh(c*x))^(1/2)-8/3*x^2/b^2/(a+b*arcsinh(c*x
))^(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {5779, 5818, 5780, 5556, 12, 3389, 2211, 2236, 2235, 5783}

$$\int \frac{x}{(a+b\text{arcsinh}(cx))^{5/2}}\,dx = -\frac{2\sqrt{2\pi}e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c^2} + \frac{2\sqrt{2\pi}e^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c^2} - \frac{4}{3b^2c^2\sqrt{a+b\text{arcsinh}(cx)}} - \frac{8x^2}{3b^2\sqrt{a+b\text{arcsinh}(cx)}} - \frac{2x\sqrt{c^2x^2+1}}{3bc(a+b\text{arcsinh}(cx))^{3/2}}$$

```
[In] Int[x/(a + b*ArcSinh[c*x])^(5/2),x]
```

```
[Out] (-2*x*Sqrt[1 + c^2*x^2])/(3*b*c*(a + b*ArcSinh[c*x])^(3/2)) - 4/(3*b^2*c^2*
Sqrt[a + b*ArcSinh[c*x]]) - (8*x^2)/(3*b^2*Sqrt[a + b*ArcSinh[c*x]]) - (2*E
^((2*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(3*b
^(5/2)*c^2) + (2*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]
])/(3*b^(5/2)*c^2*E^((2*a)/b))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/S
qrt[1 + c^2*x^2]), x], x] - Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Sinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5780

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/(b*c^(m + 1)), Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b], x], x,
a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2x\sqrt{1+c^2x^2}}{3bc(a+b\text{arcsinh}(cx))^{3/2}}+\frac{2\int \frac{1}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^{3/2}}\,dx}{3bc}\\
&\quad+\frac{(4c)\int \frac{x^2}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^{3/2}}\,dx}{3b}\\
&= -\frac{2x\sqrt{1+c^2x^2}}{3bc(a+b\text{arcsinh}(cx))^{3/2}}-\frac{4}{3b^2c^2\sqrt{a+b\text{arcsinh}(cx)}}\\
&\quad-\frac{8x^2}{3b^2\sqrt{a+b\text{arcsinh}(cx)}}+\frac{16\int \frac{x}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx}{3b^2}\\
&= -\frac{2x\sqrt{1+c^2x^2}}{3bc(a+b\text{arcsinh}(cx))^{3/2}}-\frac{4}{3b^2c^2\sqrt{a+b\text{arcsinh}(cx)}}-\frac{8x^2}{3b^2\sqrt{a+b\text{arcsinh}(cx)}}\\
&\quad-\frac{16\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx,x,a+b\text{arcsinh}(cx)\right)}{3b^3c^2}\\
&= -\frac{2x\sqrt{1+c^2x^2}}{3bc(a+b\text{arcsinh}(cx))^{3/2}}-\frac{4}{3b^2c^2\sqrt{a+b\text{arcsinh}(cx)}}\\
&\quad-\frac{8x^2}{3b^2\sqrt{a+b\text{arcsinh}(cx)}}-\frac{16\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{2\sqrt{x}}\,dx,x,a+b\text{arcsinh}(cx)\right)}{3b^3c^2}\\
&= -\frac{2x\sqrt{1+c^2x^2}}{3bc(a+b\text{arcsinh}(cx))^{3/2}}-\frac{4}{3b^2c^2\sqrt{a+b\text{arcsinh}(cx)}}\\
&\quad-\frac{8x^2}{3b^2\sqrt{a+b\text{arcsinh}(cx)}}-\frac{8\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{\sqrt{x}}\,dx,x,a+b\text{arcsinh}(cx)\right)}{3b^3c^2}\\
&= -\frac{2x\sqrt{1+c^2x^2}}{3bc(a+b\text{arcsinh}(cx))^{3/2}}-\frac{4}{3b^2c^2\sqrt{a+b\text{arcsinh}(cx)}}\\
&\quad-\frac{8x^2}{3b^2\sqrt{a+b\text{arcsinh}(cx)}}-\frac{4\text{Subst}\left(\int \frac{e^{-i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx,x,a+b\text{arcsinh}(cx)\right)}{3b^3c^2}\\
&\quad+\frac{4\text{Subst}\left(\int \frac{e^{i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx,x,a+b\text{arcsinh}(cx)\right)}{3b^3c^2}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2x\sqrt{1+c^2x^2}}{3bc(a+b\text{arcsinh}(cx))^{3/2}} - \frac{4}{3b^2c^2\sqrt{a+b\text{arcsinh}(cx)}} \\
&\quad - \frac{8x^2}{3b^2\sqrt{a+b\text{arcsinh}(cx)}} - \frac{8\text{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{3b^3c^2} \\
&\quad + \frac{8\text{Subst}\left(\int e^{-\frac{2a}{b}+\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{3b^3c^2} \\
&= -\frac{2x\sqrt{1+c^2x^2}}{3bc(a+b\text{arcsinh}(cx))^{3/2}} - \frac{4}{3b^2c^2\sqrt{a+b\text{arcsinh}(cx)}} - \frac{8x^2}{3b^2\sqrt{a+b\text{arcsinh}(cx)}} \\
&\quad - \frac{2e^{\frac{2a}{b}}\sqrt{2\pi}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c^2} + \frac{2e^{-\frac{2a}{b}}\sqrt{2\pi}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.09

$$
\int \frac{x}{(a+b\text{arcsinh}(cx))^{5/2}}\,dx = \frac{e^{-2\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}\left(-4\sqrt{2}be^{2\text{arcsinh}(cx)}\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{3/2}\Gamma\left(\frac{1}{2}, -\frac{2(a+b\text{arcsinh}(}{b}\right.\right.}{}
$$

```
[In] Integrate[x/(a + b*ArcSinh[c*x])^(5/2),x]
```

```
[Out] (-4*Sqrt[2]*b*E^(2*ArcSinh[c*x])*(-((a + b*ArcSinh[c*x])/b))^(3/2)*Gamma[1/
2, (-2*(a + b*ArcSinh[c*x]))/b] + E^((2*a)/b)*(-4*a + b - 4*a*E^(4*ArcSinh[
c*x]) - b*E^(4*ArcSinh[c*x]) - 4*b*(1 + E^(4*ArcSinh[c*x]))*ArcSinh[c*x] +
4*Sqrt[2]*E^(2*(a/b + ArcSinh[c*x]))*Sqrt[a/b + ArcSinh[c*x]]*(a + b*ArcSin
h[c*x])*Gamma[1/2, (2*(a + b*ArcSinh[c*x]))/b]))/(6*b^2*c^2*E^(2*(a/b + Arc
Sinh[c*x]))*(a + b*ArcSinh[c*x])^(3/2))
```

Maple [F]

$$
\int \frac{x}{(a+b\ \text{arcsinh}\,(cx))^{\frac{5}{2}}}dx
$$

```
[In] int(x/(a+b*arcsinh(c*x))^(5/2),x)
```

```
[Out] int(x/(a+b*arcsinh(c*x))^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{(a+b\text{arcsinh}(cx))^{5/2}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(x/(a+b*arcsinh(c*x))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x}{(a+b\text{arcsinh}(cx))^{5/2}}\,dx=\int \frac{x}{(a+b\operatorname{asinh}(cx))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x/(a+b*asinh(c*x))**(5/2),x)
```

```
[Out] Integral(x/(a + b*asinh(c*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{x}{(a+b\text{arcsinh}(cx))^{5/2}}\,dx=\int \frac{x}{(b\operatorname{arsinh}(cx)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x/(a+b*arcsinh(c*x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(b*arcsinh(c*x) + a)^(5/2), x)
```

Giac [F]

$$\int \frac{x}{(a+b\text{arcsinh}(cx))^{5/2}}\,dx=\int \frac{x}{(b\operatorname{arsinh}(cx)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x/(a+b*arcsinh(c*x))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x/(b*arcsinh(c*x) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+b\text{arcsinh}(cx))^{5/2}}\,dx=\int \frac{x}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^{5/2}} \,d x$$

```
[In] int(x/(a + b*asinh(c*x))^(5/2),x)
[Out] int(x/(a + b*asinh(c*x))^(5/2), x)
```

3.153 $\int \frac{1}{(a+b\textbf{arcsinh}(cx))^{5/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 143

$$\int \frac{1}{(a+b\operatorname{arcsinh}(cx))^{5/2}}\,dx = -\frac{2\sqrt{1+c^2x^2}}{3bc(a+b\operatorname{arcsinh}(cx))^{3/2}} - \frac{4x}{3b^2\sqrt{a+b\operatorname{arcsinh}(cx)}} + \frac{2e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c} + \frac{2e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c}$$

```
[Out] 2/3*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)/c+2/3*e
rfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(5/2)/c/exp(a/b)-2/3*(c^2*
x^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))^(3/2)-4/3*x/b^2/(a+b*arcsinh(c*x))^(1/2
)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5773, 5818, 5774, 3388, 2211, 2236, 2235}

$$\int \frac{1}{(a+b\operatorname{arcsinh}(cx))^{5/2}}\,dx = \frac{2\sqrt{\pi}e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c} + \frac{2\sqrt{\pi}e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c} - \frac{4x}{3b^2\sqrt{a+b\operatorname{arcsinh}(cx)}} - \frac{2\sqrt{c^2x^2+1}}{3bc(a+b\operatorname{arcsinh}(cx))^{3/2}}$$

```
[In] Int[(a + b*ArcSinh[c*x])^(-5/2),x]
```

```
[Out] (-2*Sqrt[1 + c^2*x^2])/(3*b*c*(a + b*ArcSinh[c*x])^(3/2)) - (4*x)/(3*b^2*Sq
rt[a + b*ArcSinh[c*x]]) + (2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/
Sqrt[b]])/(3*b^(5/2)*c) + (2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]
])/(3*b^(5/2)*c*E^(a/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5774

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[1/(b*c), Su
bst[Int[x^n*Cosh[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b
, c, n}, x]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
```

```
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2\sqrt{1+c^2x^2}}{3bc(a+b\text{arcsinh}(cx))^{3/2}}+\frac{(2c)\int\frac{x}{\sqrt{1+c^2x^2}(a+b\text{arcsinh}(cx))^{3/2}}\,dx}{3b}\\
&= -\frac{2\sqrt{1+c^2x^2}}{3bc(a+b\text{arcsinh}(cx))^{3/2}}-\frac{4x}{3b^2\sqrt{a+b\text{arcsinh}(cx)}}+\frac{4\int\frac{1}{\sqrt{a+b\text{arcsinh}(cx)}}\,dx}{3b^2}\\
&= -\frac{2\sqrt{1+c^2x^2}}{3bc(a+b\text{arcsinh}(cx))^{3/2}}-\frac{4x}{3b^2\sqrt{a+b\text{arcsinh}(cx)}}\\
&\quad+\frac{4\text{Subst}\left(\int\frac{\cosh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx,x,a+b\text{arcsinh}(cx)\right)}{3b^3c}\\
&= -\frac{2\sqrt{1+c^2x^2}}{3bc(a+b\text{arcsinh}(cx))^{3/2}}-\frac{4x}{3b^2\sqrt{a+b\text{arcsinh}(cx)}}\\
&\quad+\frac{2\text{Subst}\left(\int\frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx,x,a+b\text{arcsinh}(cx)\right)}{3b^3c}\\
&\quad+\frac{2\text{Subst}\left(\int\frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx,x,a+b\text{arcsinh}(cx)\right)}{3b^3c}\\
&= -\frac{2\sqrt{1+c^2x^2}}{3bc(a+b\text{arcsinh}(cx))^{3/2}}-\frac{4x}{3b^2\sqrt{a+b\text{arcsinh}(cx)}}\\
&\quad+\frac{4\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx,x,\sqrt{a+b\text{arcsinh}(cx)}\right)}{3b^3c}\\
&\quad+\frac{4\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+b\text{arcsinh}(cx)}\right)}{3b^3c}\\
&= -\frac{2\sqrt{1+c^2x^2}}{3bc(a+b\text{arcsinh}(cx))^{3/2}}-\frac{4x}{3b^2\sqrt{a+b\text{arcsinh}(cx)}}\\
&\quad+\frac{2e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c}+\frac{2e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.27

$$\int \frac{1}{(a+b\operatorname{arcsinh}(cx))^{5/2}}\,dx = \frac{e^{-\frac{a+b\operatorname{arcsinh}(cx)}{b}}\left(-e^{a/b}\left(b+2a\left(-1+e^{2\operatorname{arcsinh}(cx)}\right)-2b\operatorname{arcsinh}(cx)+be^{2\operatorname{arcsinh}(cx}\right.\right.}{}$$

```
[In] Integrate[(a + b*ArcSinh[c*x])^(-5/2),x]
```

```
[Out] (-(E^(a/b)*(b + 2*a*(-1 + E^(2*ArcSinh[c*x])) - 2*b*ArcSinh[c*x] + b*E^(2*A
rcSinh[c*x])*(1 + 2*ArcSinh[c*x]))) - 2*E^((2*a)/b + ArcSinh[c*x])*Sqrt[a/b
 + ArcSinh[c*x]]*(a + b*ArcSinh[c*x])*Gamma[1/2, a/b + ArcSinh[c*x]] - 2*b*
E^ArcSinh[c*x]*(-((a + b*ArcSinh[c*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcSin
h[c*x])/b)])/(3*b^2*c*E^((a + b*ArcSinh[c*x])/b)*(a + b*ArcSinh[c*x])^(3/2)
)
```

Maple [F]

$$\int \frac{1}{\left(a+b\operatorname{arcsinh}\left(cx\right)\right)^{\frac{5}{2}}}dx$$

```
[In] int(1/(a+b*arcsinh(c*x))^(5/2),x)
```

```
[Out] int(1/(a+b*arcsinh(c*x))^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\operatorname{arcsinh}(cx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*arcsinh(c*x))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^{5/2}}\,dx=\int \frac{1}{\left(a+b\operatorname{asinh}{\left(cx\right)}\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a+b*asinh(c*x))**(5/2),x)
[Out] Integral((a + b*asinh(c*x))**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^{5/2}}\,dx=\int \frac{1}{\left(b\operatorname{arsinh}\left(cx\right)+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(c*x))^(5/2),x, algorithm="maxima")
[Out] integrate((b*arcsinh(c*x) + a)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^{5/2}}\,dx=\int \frac{1}{\left(b\operatorname{arsinh}\left(cx\right)+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(c*x))^(5/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^{5/2}}\,dx=\int \frac{1}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^{5/2}}\,dx$$

```
[In] int(1/(a + b*asinh(c*x))^(5/2),x)
[Out] int(1/(a + b*asinh(c*x))^(5/2), x)
```

3.154 $\int \frac{x^2}{(a+b\mathbf{arcsinh}(cx))^{7/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 346

$$\begin{aligned}\int \frac{x^2}{(a+b\operatorname{arcsinh}(cx))^{7/2}}\,dx = & -\frac{2x^2\sqrt{1+c^2x^2}}{5bc(a+b\operatorname{arcsinh}(cx))^{5/2}} \\ & -\frac{8x}{15b^2c^2(a+b\operatorname{arcsinh}(cx))^{3/2}}-\frac{4x^3}{5b^2(a+b\operatorname{arcsinh}(cx))^{3/2}} \\ & -\frac{16\sqrt{1+c^2x^2}}{15b^3c^3\sqrt{a+b\operatorname{arcsinh}(cx)}}-\frac{24x^2\sqrt{1+c^2x^2}}{5b^3c\sqrt{a+b\operatorname{arcsinh}(cx)}} \\ & +\frac{e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{15b^{7/2}c^3}-\frac{3e^{\frac{3a}{b}}\sqrt{3\pi}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{5b^{7/2}c^3} \\ & -\frac{e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{15b^{7/2}c^3}+\frac{3e^{-\frac{3a}{b}}\sqrt{3\pi}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{5b^{7/2}c^3}\end{aligned}$$

```
[Out] -8/15*x/b^2/c^2/(a+b*arcsinh(c*x))^(3/2)-4/5*x^3/b^2/(a+b*arcsinh(c*x))^(3/
2)+1/15*exp(a/b)*erf((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(7/2)/c^3
-1/15*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(7/2)/c^3/exp(a/b)-
3/5*exp(3*a/b)*erf(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/
2)/b^(7/2)/c^3+3/5*erfi(3^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*3^(1/2)*P
i^(1/2)/b^(7/2)/c^3/exp(3*a/b)-2/5*x^2*(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(c
*x))^(5/2)-16/15*(c^2*x^2+1)^(1/2)/b^3/c^3/(a+b*arcsinh(c*x))^(1/2)-24/5*x^
2*(c^2*x^2+1)^(1/2)/b^3/c/(a+b*arcsinh(c*x))^(1/2)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {5779, 5818, 5778, 3389, 2211, 2236, 2235, 5773, 5819}

$$\begin{aligned}\int \frac{x^2}{(a+b\text{arcsinh}(cx))^{7/2}}\,dx = {} & \frac{\sqrt{\pi}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{15b^{7/2}c^3} \\ & -\frac{3\sqrt{3\pi}e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{5b^{7/2}c^3}-\frac{\sqrt{\pi}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{15b^{7/2}c^3} \\ & +\frac{3\sqrt{3\pi}e^{-\frac{3a}{b}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{5b^{7/2}c^3}-\frac{24x^2\sqrt{c^2x^2+1}}{5b^3c\sqrt{a+b\text{arcsinh}(cx)}} \\ & -\frac{16\sqrt{c^2x^2+1}}{15b^3c^3\sqrt{a+b\text{arcsinh}(cx)}}-\frac{8x}{15b^2c^2(a+b\text{arcsinh}(cx))^{3/2}} \\ & -\frac{4x^3}{5b^2(a+b\text{arcsinh}(cx))^{3/2}}-\frac{2x^2\sqrt{c^2x^2+1}}{5bc(a+b\text{arcsinh}(cx))^{5/2}}\end{aligned}$$

```
[In] Int[x^2/(a + b*ArcSinh[c*x])^(7/2),x]
```

```
[Out] (-2*x^2*Sqrt[1 + c^2*x^2])/(5*b*c*(a + b*ArcSinh[c*x])^(5/2)) - (8*x)/(15*b
^2*c^2*(a + b*ArcSinh[c*x])^(3/2)) - (4*x^3)/(5*b^2*(a + b*ArcSinh[c*x])^(3
/2)) - (16*Sqrt[1 + c^2*x^2])/(15*b^3*c^3*Sqrt[a + b*ArcSinh[c*x]]) - (24*x
^2*Sqrt[1 + c^2*x^2])/(5*b^3*c*Sqrt[a + b*ArcSinh[c*x]]) + (E^(a/b)*Sqrt[Pi
]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(15*b^(7/2)*c^3) - (3*E^((3*a)/b)*
Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(5*b^(7/2)*c^3)
 - (Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(15*b^(7/2)*c^3*E^(a/b
)) + (3*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(5*b^(
7/2)*c^3*E^((3*a)/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/S
qrt[1 + c^2*x^2]), x], x] - Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Sinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = -\frac{2x^2\sqrt{1+c^2x^2}}{5bc(a+b\operatorname{arcsinh}(cx))^{5/2}} + \frac{4\int \frac{x}{\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^{5/2}}\,dx}{5bc} + \frac{(6c)\int \frac{x^3}{\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^{5/2}}\,dx}{5b}$$

$$= -\frac{2x^2\sqrt{1+c^2x^2}}{5bc(a+b\operatorname{arcsinh}(cx))^{5/2}} - \frac{8x}{15b^2c^2(a+b\operatorname{arcsinh}(cx))^{3/2}} - \frac{4x^3}{5b^2(a+b\operatorname{arcsinh}(cx))^{3/2}} + \frac{12\int \frac{x^2}{(a+b\operatorname{arcsinh}(cx))^{3/2}}\,dx}{5b^2} + \frac{8\int \frac{1}{(a+b\operatorname{arcsinh}(cx))^{3/2}}\,dx}{15b^2c^2}$$

$$= -\frac{2x^2\sqrt{1+c^2x^2}}{5bc(a+b\operatorname{arcsinh}(cx))^{5/2}} - \frac{8x}{15b^2c^2(a+b\operatorname{arcsinh}(cx))^{3/2}} - \frac{4x^3}{5b^2(a+b\operatorname{arcsinh}(cx))^{3/2}} - \frac{16\sqrt{1+c^2x^2}}{15b^3c^3\sqrt{a+b\operatorname{arcsinh}(cx)}} - \frac{24x^2\sqrt{1+c^2x^2}}{5b^3c\sqrt{a+b\operatorname{arcsinh}(cx)}} + \frac{24\text{Subst}\left(\int \left(-\frac{3\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{4\sqrt{x}} + \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{4\sqrt{x}}\right)dx, x, a+b\operatorname{arcsinh}(cx)\right)}{5b^4c^3} + \frac{16\int \frac{x}{\sqrt{1+c^2x^2}\sqrt{a+b\operatorname{arcsinh}(cx)}}\,dx}{15b^3c}$$

$$= -\frac{2x^2\sqrt{1+c^2x^2}}{5bc(a+b\operatorname{arcsinh}(cx))^{5/2}} - \frac{8x}{15b^2c^2(a+b\operatorname{arcsinh}(cx))^{3/2}} - \frac{4x^3}{5b^2(a+b\operatorname{arcsinh}(cx))^{3/2}} - \frac{16\sqrt{1+c^2x^2}}{15b^3c^3\sqrt{a+b\operatorname{arcsinh}(cx)}} - \frac{24x^2\sqrt{1+c^2x^2}}{5b^3c\sqrt{a+b\operatorname{arcsinh}(cx)}} - \frac{16\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{15b^4c^3} + \frac{6\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{5b^4c^3} - \frac{18\text{Subst}\left(\int \frac{\sinh\left(\frac{3a}{b}-\frac{3x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\operatorname{arcsinh}(cx)\right)}{5b^4c^3}$$

$$
\begin{aligned}
&= -\frac{2x^2\sqrt{1+c^2x^2}}{5bc(a+b\text{arcsinh}(cx))^{5/2}} - \frac{8x}{15b^2c^2(a+b\text{arcsinh}(cx))^{3/2}} \\
&\quad - \frac{4x^3}{5b^2(a+b\text{arcsinh}(cx))^{3/2}} - \frac{16\sqrt{1+c^2x^2}}{15b^3c^3\sqrt{a+b\text{arcsinh}(cx)}} \\
&\quad - \frac{24x^2\sqrt{1+c^2x^2}}{5b^3c\sqrt{a+b\text{arcsinh}(cx)}} - \frac{8\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{15b^4c^3} \\
&\quad + \frac{8\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{15b^4c^3} \\
&\quad + \frac{3\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{5b^4c^3} \\
&\quad - \frac{3\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{5b^4c^3} \\
&\quad - \frac{9\text{Subst}\left(\int \frac{e^{-i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{5b^4c^3} \\
&\quad + \frac{9\text{Subst}\left(\int \frac{e^{i\left(\frac{3ia}{b}-\frac{3ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{5b^4c^3}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2x^2\sqrt{1+c^2x^2}}{5bc(a+b\text{arcsinh}(cx))^{5/2}} - \frac{8x}{15b^2c^2(a+b\text{arcsinh}(cx))^{3/2}} \\
&\quad - \frac{4x^3}{5b^2(a+b\text{arcsinh}(cx))^{3/2}} - \frac{16\sqrt{1+c^2x^2}}{15b^3c^3\sqrt{a+b\text{arcsinh}(cx)}} \\
&\quad - \frac{24x^2\sqrt{1+c^2x^2}}{5b^3c\sqrt{a+b\text{arcsinh}(cx)}} - \frac{16\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{15b^4c^3} \\
&\quad + \frac{16\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{15b^4c^3} \\
&\quad + \frac{6\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{5b^4c^3} \\
&\quad - \frac{6\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{5b^4c^3} \\
&\quad - \frac{18\text{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{5b^4c^3} \\
&\quad + \frac{18\text{Subst}\left(\int e^{-\frac{3a}{b}+\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{5b^4c^3}
\end{aligned}
$$

$$= -\frac{2x^2\sqrt{1+c^2x^2}}{5bc(a+b\text{arcsinh}(cx))^{5/2}} - \frac{8x}{15b^2c^2(a+b\text{arcsinh}(cx))^{3/2}} - \frac{4x^3}{5b^2(a+b\text{arcsinh}(cx))^{3/2}} - \frac{16\sqrt{1+c^2x^2}}{15b^3c^3\sqrt{a+b\text{arcsinh}(cx)}} - \frac{24x^2\sqrt{1+c^2x^2}}{5b^3c\sqrt{a+b\text{arcsinh}(cx)}} + \frac{e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{15b^{7/2}c^3} - \frac{3e^{\frac{3a}{b}}\sqrt{3\pi}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{5b^{7/2}c^3} - \frac{e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{15b^{7/2}c^3} + \frac{3e^{-\frac{3a}{b}}\sqrt{3\pi}\text{erfi}\left(\frac{\sqrt{3}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{5b^{7/2}c^3}$$

Mathematica [A] (verified)

Time = 1.18 (sec) , antiderivative size = 417, normalized size of antiderivative = 1.21

$$\int \frac{x^2}{(a+b\text{arcsinh}(cx))^{7/2}}\,dx = \frac{3b^2e^{\text{arcsinh}(cx)} + e^{-\text{arcsinh}(cx)}\bigl(4a^2-2ab+3b^2+2(4a-b)b\text{arcsinh}(cx)+4b^2\text{arcsi}}{}$$

```
[In] Integrate[x^2/(a + b*ArcSinh[c*x])^(7/2),x]
```

```
[Out] (3*b^2*E^ArcSinh[c*x] + (4*a^2 - 2*a*b + 3*b^2 + 2*(4*a - b)*b*ArcSinh[c*x]
 + 4*b^2*ArcSinh[c*x]^2 - 4*E^(a/b + ArcSinh[c*x])*Sqrt[a/b + ArcSinh[c*x]]
*(a + b*ArcSinh[c*x])^2*Gamma[1/2, a/b + ArcSinh[c*x]])/E^ArcSinh[c*x] - 3*
(b^2*E^(3*ArcSinh[c*x]) + (2*(a + b*ArcSinh[c*x])*(E^(3*(a/b + ArcSinh[c*x]
))*(6*a + b + 6*b*ArcSinh[c*x]) + 6*Sqrt[3]*b*(-((a + b*ArcSinh[c*x])/b))^(
3/2)*Gamma[1/2, (-3*(a + b*ArcSinh[c*x]))/b]))/E^((3*a)/b)) + (2*(a + b*Arc
Sinh[c*x])*(E^(a/b + ArcSinh[c*x])*(2*a + b + 2*b*ArcSinh[c*x]) + 2*b*(-((a
 + b*ArcSinh[c*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcSinh[c*x])/b)]))/E^(a/b
) - (3*(b^2 + 2*(a + b*ArcSinh[c*x])*(6*a - b + 6*b*ArcSinh[c*x] - 6*Sqrt[3
]*E^(3*(a/b + ArcSinh[c*x]))*Sqrt[a/b + ArcSinh[c*x]]*(a + b*ArcSinh[c*x])*
Gamma[1/2, (3*(a + b*ArcSinh[c*x]))/b])))/E^(3*ArcSinh[c*x]))/(60*b^3*c^3*(
a + b*ArcSinh[c*x])^(5/2))
```

Maple [F]

$$\int \frac{x^2}{(a+b\ \text{arcsinh}\,(cx))^{\frac{7}{2}}}dx$$

```
[In] int(x^2/(a+b*arcsinh(c*x))^(7/2),x)
```

```
[Out] int(x^2/(a+b*arcsinh(c*x))^(7/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{(a+b\text{arcsinh}(cx))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2/(a+b*arcsinh(c*x))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^2}{(a+b\text{arcsinh}(cx))^{7/2}}\,dx = \int \frac{x^2}{(a+b\,\text{asinh}\,(cx))^{\frac{7}{2}}}\,dx$$

```
[In] integrate(x**2/(a+b*asinh(c*x))**(7/2),x)
```

```
[Out] Integral(x**2/(a + b*asinh(c*x))**(7/2), x)
```

Maxima [F]

$$\int \frac{x^2}{(a+b\text{arcsinh}(cx))^{7/2}}\,dx = \int \frac{x^2}{(b\,\text{arsinh}\,(cx)+a)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(x^2/(a+b*arcsinh(c*x))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b*arcsinh(c*x) + a)^(7/2), x)
```

Giac [F]

$$\int \frac{x^2}{(a+b\text{arcsinh}(cx))^{7/2}}\,dx = \int \frac{x^2}{(b\,\text{arsinh}\,(cx)+a)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(x^2/(a+b*arcsinh(c*x))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*arcsinh(c*x) + a)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+b\text{arcsinh}(cx))^{7/2}}\,dx=\int \frac{x^2}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^{7/2}} \,d x$$

```
[In] int(x^2/(a + b*asinh(c*x))^(7/2),x)
```

```
[Out] int(x^2/(a + b*asinh(c*x))^(7/2), x)
```

3.155 $\int \frac{x}{(a+b\,\mathbf{arcsinh}(cx))^{7/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 219

$$\int \frac{x}{(a+b\operatorname{arcsinh}(cx))^{7/2}}\,dx=-\frac{2x\sqrt{1+c^2x^2}}{5bc(a+b\operatorname{arcsinh}(cx))^{5/2}}-\frac{4}{15b^2c^2(a+b\operatorname{arcsinh}(cx))^{3/2}}-\frac{8x^2}{15b^2(a+b\operatorname{arcsinh}(cx))^{3/2}}-\frac{32x\sqrt{1+c^2x^2}}{15b^3c\sqrt{a+b\operatorname{arcsinh}(cx)}}+\frac{8e^{\frac{2a}{b}}\sqrt{2\pi}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{15b^{7/2}c^2}+\frac{8e^{-\frac{2a}{b}}\sqrt{2\pi}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{15b^{7/2}c^2}$$

```
[Out] -4/15/b^2/c^2/(a+b*arcsinh(c*x))^(3/2)-8/15*x^2/b^2/(a+b*arcsinh(c*x))^(3/2
)+8/15*exp(2*a/b)*erf(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(1/2)*Pi^
(1/2)/b^(7/2)/c^2+8/15*erfi(2^(1/2)*(a+b*arcsinh(c*x))^(1/2)/b^(1/2))*2^(1/
2)*Pi^(1/2)/b^(7/2)/c^2/exp(2*a/b)-2/5*x*(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh
(c*x))^(5/2)-32/15*x*(c^2*x^2+1)^(1/2)/b^3/c/(a+b*arcsinh(c*x))^(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used

$= \{5779, 5818, 5778, 3388, 2211, 2236, 2235, 5783\}$

$$\int \frac{x}{(a+b\operatorname{arcsinh}(cx))^{7/2}}\,dx = \frac{8\sqrt{2\pi}e^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{15b^{7/2}c^2} + \frac{8\sqrt{2\pi}e^{-\frac{2a}{b}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{15b^{7/2}c^2} - \frac{32x\sqrt{c^2x^2+1}}{15b^3c\sqrt{a+b\operatorname{arcsinh}(cx)}} - \frac{4}{15b^2c^2(a+b\operatorname{arcsinh}(cx))^{3/2}} - \frac{8x^2}{15b^2(a+b\operatorname{arcsinh}(cx))^{3/2}} - \frac{2x\sqrt{c^2x^2+1}}{5bc(a+b\operatorname{arcsinh}(cx))^{5/2}}$$

```
[In] Int[x/(a + b*ArcSinh[c*x])^(7/2),x]
```

```
[Out] (-2*x*Sqrt[1 + c^2*x^2])/(5*b*c*(a + b*ArcSinh[c*x])^(5/2)) - 4/(15*b^2*c^2
*(a + b*ArcSinh[c*x])^(3/2)) - (8*x^2)/(15*b^2*(a + b*ArcSinh[c*x])^(3/2))
- (32*x*Sqrt[1 + c^2*x^2])/(15*b^3*c*Sqrt[a + b*ArcSinh[c*x]]) + (8*E^((2*a
)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(15*b^(7/2
)*c^2) + (8*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcSinh[c*x]])/Sqrt[b]])/(1
5*b^(7/2)*c^2*E^((2*a)/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5778

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Di
st[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Sinh[-a
/b + x/b]^(m - 1)*(m + (m + 1)*Sinh[-a/b + x/b]^2), x], x], x, a + b*ArcSin
h[c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1
]
```

Rule 5779

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
x^m*Sqrt[1 + c^2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] + (-
Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcSinh[c*x])^(n + 1)/S
qrt[1 + c^2*x^2]), x], x] - Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Sinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IG
tQ[m, 0] && LtQ[n, -2]
```

Rule 5783

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_
Symbol] :> Simp[(1/(b*c*(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]]*(
a + b*ArcSinh[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c
^2*d] && NeQ[n, -1]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2x\sqrt{1+c^2x^2}}{5bc(a+b\operatorname{arcsinh}(cx))^{5/2}}+\frac{2\int\frac{1}{\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^{5/2}}\,dx}{5bc} \\
&\quad+\frac{(4c)\int\frac{x^2}{\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^{5/2}}\,dx}{5b} \\
&= -\frac{2x\sqrt{1+c^2x^2}}{5bc(a+b\operatorname{arcsinh}(cx))^{5/2}}-\frac{4}{15b^2c^2(a+b\operatorname{arcsinh}(cx))^{3/2}} \\
&\quad-\frac{8x^2}{15b^2(a+b\operatorname{arcsinh}(cx))^{3/2}}+\frac{16\int\frac{x}{(a+b\operatorname{arcsinh}(cx))^{3/2}}\,dx}{15b^2}
\end{aligned}$$

$$= -\frac{2x\sqrt{1+c^2x^2}}{5bc(a+b\text{arcsinh}(cx))^{5/2}} - \frac{4}{15b^2c^2(a+b\text{arcsinh}(cx))^{3/2}} - \frac{8x^2}{15b^2(a+b\text{arcsinh}(cx))^{3/2}} - \frac{32x\sqrt{1+c^2x^2}}{15b^3c\sqrt{a+b\text{arcsinh}(cx)}} + \frac{32\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}-\frac{2x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{15b^4c^2}$$

$$= -\frac{2x\sqrt{1+c^2x^2}}{5bc(a+b\text{arcsinh}(cx))^{5/2}} - \frac{4}{15b^2c^2(a+b\text{arcsinh}(cx))^{3/2}} - \frac{8x^2}{15b^2(a+b\text{arcsinh}(cx))^{3/2}} - \frac{32x\sqrt{1+c^2x^2}}{15b^3c\sqrt{a+b\text{arcsinh}(cx)}} + \frac{16\text{Subst}\left(\int \frac{e^{-i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{15b^4c^2} + \frac{16\text{Subst}\left(\int \frac{e^{i\left(\frac{2ia}{b}-\frac{2ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\text{arcsinh}(cx)\right)}{15b^4c^2}$$

$$= -\frac{2x\sqrt{1+c^2x^2}}{5bc(a+b\text{arcsinh}(cx))^{5/2}} - \frac{4}{15b^2c^2(a+b\text{arcsinh}(cx))^{3/2}} - \frac{8x^2}{15b^2(a+b\text{arcsinh}(cx))^{3/2}} - \frac{32x\sqrt{1+c^2x^2}}{15b^3c\sqrt{a+b\text{arcsinh}(cx)}} + \frac{32\text{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{15b^4c^2} + \frac{32\text{Subst}\left(\int e^{-\frac{2a}{b}+\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\text{arcsinh}(cx)}\right)}{15b^4c^2}$$

$$= -\frac{2x\sqrt{1+c^2x^2}}{5bc(a+b\text{arcsinh}(cx))^{5/2}} - \frac{4}{15b^2c^2(a+b\text{arcsinh}(cx))^{3/2}} - \frac{8x^2}{15b^2(a+b\text{arcsinh}(cx))^{3/2}} - \frac{32x\sqrt{1+c^2x^2}}{15b^3c\sqrt{a+b\text{arcsinh}(cx)}} + \frac{8e^{\frac{2a}{b}}\sqrt{2\pi}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{15b^{7/2}c^2} + \frac{8e^{-\frac{2a}{b}}\sqrt{2\pi}\,\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{15b^{7/2}c^2}$$

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.95

$$\int \frac{x}{(a+b\text{arcsinh}(cx))^{7/2}}\,dx =$$

$$-\frac{(a+b\text{arcsinh}(cx))\left(e^{-\frac{2a}{b}}\left(2e^{2\left(\frac{a}{b}+\text{arcsinh}(cx)\right)}(4a+b+4b\text{arcsinh}(cx))+8\sqrt{2}b\left(-\frac{a+b\text{arcsinh}(cx)}{b}\right)^{3/2}\Gamma\left(\frac{1}{2},-\right.\right.\right.}{}$$

```
[In] Integrate[x/(a + b*ArcSinh[c*x])^(7/2),x]
```

```
[Out] -1/15*((a + b*ArcSinh[c*x])*((2*E^(2*(a/b + ArcSinh[c*x]))*(4*a + b + 4*b*A
rcSinh[c*x]) + 8*Sqrt[2]*b*(-((a + b*ArcSinh[c*x])/b))^(3/2)*Gamma[1/2, (-2
*(a + b*ArcSinh[c*x]))/b])/E^((2*a)/b) + (-8*a + 2*b - 8*b*ArcSinh[c*x] + 8
*Sqrt[2]*E^(2*(a/b + ArcSinh[c*x]))*Sqrt[a/b + ArcSinh[c*x]]*(a + b*ArcSinh
[c*x])*Gamma[1/2, (2*(a + b*ArcSinh[c*x]))/b])/E^(2*ArcSinh[c*x])) + 3*b^2*
Sinh[2*ArcSinh[c*x]])/(b^3*c^2*(a + b*ArcSinh[c*x])^(5/2))
```

Maple [F]

$$\int \frac{x}{(a+b\ \mathrm{arcsinh}\,(cx))^{\frac{7}{2}}}dx$$

```
[In] int(x/(a+b*arcsinh(c*x))^(7/2),x)
```

```
[Out] int(x/(a+b*arcsinh(c*x))^(7/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{(a+b\text{arcsinh}(cx))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x/(a+b*arcsinh(c*x))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x}{(a+b\text{arcsinh}(cx))^{7/2}}\,dx = \int \frac{x}{(a+b\,\text{asinh}\,(cx))^{\frac{7}{2}}}\,dx$$

```
[In] integrate(x/(a+b*asinh(c*x))**(7/2),x)
[Out] Integral(x/(a + b*asinh(c*x))**(7/2), x)
```

Maxima [F]

$$\int \frac{x}{(a+b\text{arcsinh}(cx))^{7/2}}\,dx = \int \frac{x}{(b\,\text{arsinh}\,(cx)+a)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(x/(a+b*arcsinh(c*x))^(7/2),x, algorithm="maxima")
[Out] integrate(x/(b*arcsinh(c*x) + a)^(7/2), x)
```

Giac [F]

$$\int \frac{x}{(a+b\text{arcsinh}(cx))^{7/2}}\,dx = \int \frac{x}{(b\,\text{arsinh}\,(cx)+a)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(x/(a+b*arcsinh(c*x))^(7/2),x, algorithm="giac")
[Out] integrate(x/(b*arcsinh(c*x) + a)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+b\text{arcsinh}(cx))^{7/2}}\,dx = \int \frac{x}{{\left(a+b\,\mathrm{asinh}\left(c\,x\right)\right)}^{7/2}}\,dx$$

```
[In] int(x/(a + b*asinh(c*x))^(7/2),x)
[Out] int(x/(a + b*asinh(c*x))^(7/2), x)
```

3.156 $\int \frac{1}{(a+b\text{arcsinh}(cx))^{7/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 178

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^{7/2}}\,dx = -\frac{2\sqrt{1+c^2x^2}}{5bc(a+b\text{arcsinh}(cx))^{5/2}} - \frac{4x}{15b^2(a+b\text{arcsinh}(cx))^{3/2}} - \frac{8\sqrt{1+c^2x^2}}{15b^3c\sqrt{a+b\text{arcsinh}(cx)}} - \frac{4e^{a/b}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{15b^{7/2}c} + \frac{4e^{-\frac{a}{b}}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{15b^{7/2}c}$$

```
[Out] -4/15*x/b^2/(a+b*arcsinh(c*x))^(3/2)-4/15*exp(a/b)*erf((a+b*arcsinh(c*x))^(
1/2)/b^(1/2))*Pi^(1/2)/b^(7/2)/c+4/15*erfi((a+b*arcsinh(c*x))^(1/2)/b^(1/2)
)*Pi^(1/2)/b^(7/2)/c/exp(a/b)-2/5*(c^2*x^2+1)^(1/2)/b/c/(a+b*arcsinh(c*x))^
(5/2)-8/15*(c^2*x^2+1)^(1/2)/b^3/c/(a+b*arcsinh(c*x))^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5773, 5818, 5819, 3389, 2211, 2236, 2235}

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^{7/2}}\,dx = -\frac{4\sqrt{\pi}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{15b^{7/2}c} + \frac{4\sqrt{\pi}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\text{arcsinh}(cx)}}{\sqrt{b}}\right)}{15b^{7/2}c} - \frac{8\sqrt{c^2x^2+1}}{15b^3c\sqrt{a+b\text{arcsinh}(cx)}} - \frac{4x}{15b^2(a+b\text{arcsinh}(cx))^{3/2}} - \frac{2\sqrt{c^2x^2+1}}{5bc(a+b\text{arcsinh}(cx))^{5/2}}$$

```
[In] Int[(a + b*ArcSinh[c*x])^(-7/2),x]
```

```
[Out] (-2*Sqrt[1 + c^2*x^2])/(5*b*c*(a + b*ArcSinh[c*x])^(5/2)) - (4*x)/(15*b^2*(
a + b*ArcSinh[c*x])^(3/2)) - (8*Sqrt[1 + c^2*x^2])/(15*b^3*c*Sqrt[a + b*Arc
Sinh[c*x]]) - (4*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(1
5*b^(7/2)*c) + (4*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcSinh[c*x]]/Sqrt[b]])/(15*b^(7
/2)*c*E^(a/b))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5773

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Sqrt[1 + c^
2*x^2]*((a + b*ArcSinh[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1)
), Int[x*((a + b*ArcSinh[c*x])^(n + 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 5818

```
Int[(((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
 + (e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 + c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcSinh[c*x])^(n + 1), x] - Dist[f*(m/(b*c*
(n + 1)))*Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b
*ArcSinh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e,
 c^2*d] && LtQ[n, -1]
```

Rule 5819

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(1/(b*c^(m + 1)))*Simp[(d + e*x^2)^p/(1 + c^2*
x^2)^p], Subst[Int[x^n*Sinh[-a/b + x/b]^m*Cosh[-a/b + x/b]^(2*p + 1), x], x
, a + b*ArcSinh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[e, c^2*d]
&& IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\sqrt{1+c^2x^2}}{5bc(a+b\operatorname{arcsinh}(cx))^{5/2}}+\frac{(2c)\int\frac{x}{\sqrt{1+c^2x^2}(a+b\operatorname{arcsinh}(cx))^{5/2}}\,dx}{5b}\\
&= -\frac{2\sqrt{1+c^2x^2}}{5bc(a+b\operatorname{arcsinh}(cx))^{5/2}}-\frac{4x}{15b^2(a+b\operatorname{arcsinh}(cx))^{3/2}}+\frac{4\int\frac{1}{(a+b\operatorname{arcsinh}(cx))^{3/2}}\,dx}{15b^2}\\
&= -\frac{2\sqrt{1+c^2x^2}}{5bc(a+b\operatorname{arcsinh}(cx))^{5/2}}-\frac{4x}{15b^2(a+b\operatorname{arcsinh}(cx))^{3/2}}\\
&\quad-\frac{8\sqrt{1+c^2x^2}}{15b^3c\sqrt{a+b\operatorname{arcsinh}(cx)}}+\frac{(8c)\int\frac{x}{\sqrt{1+c^2x^2}\sqrt{a+b\operatorname{arcsinh}(cx)}}\,dx}{15b^3}\\
&= -\frac{2\sqrt{1+c^2x^2}}{5bc(a+b\operatorname{arcsinh}(cx))^{5/2}}-\frac{4x}{15b^2(a+b\operatorname{arcsinh}(cx))^{3/2}}\\
&\quad-\frac{8\sqrt{1+c^2x^2}}{15b^3c\sqrt{a+b\operatorname{arcsinh}(cx)}}-\frac{8\operatorname{Subst}\left(\int\frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx,x,a+b\operatorname{arcsinh}(cx)\right)}{15b^4c}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{2\sqrt{1+c^2x^2}}{5bc(a+b\operatorname{arcsinh}(cx))^{5/2}} - \frac{4x}{15b^2(a+b\operatorname{arcsinh}(cx))^{3/2}} \\
&\quad - \frac{8\sqrt{1+c^2x^2}}{15b^3c\sqrt{a+b\operatorname{arcsinh}(cx)}} - \frac{4\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}dx, x, a+b\operatorname{arcsinh}(cx)\right)}{15b^4c} \\
&\quad + \frac{4\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}dx, x, a+b\operatorname{arcsinh}(cx)\right)}{15b^4c} \\
&= -\frac{2\sqrt{1+c^2x^2}}{5bc(a+b\operatorname{arcsinh}(cx))^{5/2}} - \frac{4x}{15b^2(a+b\operatorname{arcsinh}(cx))^{3/2}} \\
&\quad - \frac{8\sqrt{1+c^2x^2}}{15b^3c\sqrt{a+b\operatorname{arcsinh}(cx)}} - \frac{8\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}dx, x, \sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{15b^4c} \\
&\quad + \frac{8\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}dx, x, \sqrt{a+b\operatorname{arcsinh}(cx)}\right)}{15b^4c} \\
&= -\frac{2\sqrt{1+c^2x^2}}{5bc(a+b\operatorname{arcsinh}(cx))^{5/2}} - \frac{4x}{15b^2(a+b\operatorname{arcsinh}(cx))^{3/2}} - \frac{8\sqrt{1+c^2x^2}}{15b^3c\sqrt{a+b\operatorname{arcsinh}(cx)}} \\
&\quad - \frac{4e^{a/b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{15b^{7/2}c} + \frac{4e^{-\frac{a}{b}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{a+b\operatorname{arcsinh}(cx)}}{\sqrt{b}}\right)}{15b^{7/2}c}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.18

$$
\int \frac{1}{(a+b\operatorname{arcsinh}(cx))^{7/2}}\,dx = \frac{-6b^2e^{\operatorname{arcsinh}(cx)} - 2e^{-\operatorname{arcsinh}(cx)}(4a^2+2ab(-1+4\operatorname{arcsinh}(cx))+b^2(3-2\mathrm{arcsin}}{}
$$

```
[In] Integrate[(a + b*ArcSinh[c*x])^(-7/2),x]
```

```
[Out] (-6*b^2*E^ArcSinh[c*x] - (2*(4*a^2 + 2*a*b*(-1 + 4*ArcSinh[c*x]) + b^2*(3 -
 2*ArcSinh[c*x] + 4*ArcSinh[c*x]^2)))/E^ArcSinh[c*x] + 8*E^(a/b)*Sqrt[a/b +
 ArcSinh[c*x]]*(a + b*ArcSinh[c*x])^2*Gamma[1/2, a/b + ArcSinh[c*x]] - (4*(
a + b*ArcSinh[c*x])*(E^(a/b + ArcSinh[c*x])*(2*a + b + 2*b*ArcSinh[c*x]) +
2*b*(-((a + b*ArcSinh[c*x])/b))^(3/2)*Gamma[1/2, -((a + b*ArcSinh[c*x])/b)]
))/E^(a/b))/(30*b^3*c*(a + b*ArcSinh[c*x])^(5/2))
```

Maple [F]

$$\int \frac{1}{(a+b\,\mathrm{arcsinh}\,(cx))^{\frac{7}{2}}}dx$$

```
[In] int(1/(a+b*arcsinh(c*x))^(7/2),x)
```

```
[Out] int(1/(a+b*arcsinh(c*x))^(7/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\mathrm{arcsinh}(cx))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*arcsinh(c*x))^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{(a+b\mathrm{arcsinh}(cx))^{7/2}}\,dx = \int \frac{1}{(a+b\,\mathrm{asinh}\,(cx))^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/(a+b*asinh(c*x))**(7/2),x)
```

```
[Out] Integral((a + b*asinh(c*x))**(-7/2), x)
```

Maxima [F]

$$\int \frac{1}{(a+b\mathrm{arcsinh}(cx))^{7/2}}\,dx = \int \frac{1}{(b\,\mathrm{arsinh}\,(cx)+a)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(c*x))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arcsinh(c*x) + a)^(-7/2), x)
```

Giac [F]

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^{7/2}}\,dx = \int \frac{1}{(b\,\text{arsinh}\,(cx)+a)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/(a+b*arcsinh(c*x))^(7/2),x, algorithm="giac")
[Out] integrate((b*arcsinh(c*x) + a)^(-7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\text{arcsinh}(cx))^{7/2}}\,dx = \int \frac{1}{(a+b\,\text{asinh}\,(c\,x))^{7/2}}\,dx$$

```
[In] int(1/(a + b*asinh(c*x))^(7/2),x)
[Out] int(1/(a + b*asinh(c*x))^(7/2), x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count i
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
```

```
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well")
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_count
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])   #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
```

```
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_result


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```